1)

178(1): Quantized Forces in Relativistic Quantum Mechanics

The relativistic hamiltonian is defined in special relativity as:

$$H = T + U + E_0 \quad -(1)$$

as in eq. (14.115) of Marion and Thornton. Here T is the relativistic kinetic energy:

$$T = (\gamma - 1) mc^2 \quad -(2)$$

Therefore:

$$H = \gamma mc^2 + V$$
$$= (c^2 p^2 + m^2 c^4)^{1/2} + V. \quad -(3)$$

The classical non-relativistic hamiltonian is:

$$H_{nr} = H - mc^2. \quad -(4)$$

In the non-relativistic limit:

$$T \rightarrow \frac{p^2}{2m}. \quad -(5)$$

In the approximation:

$$E = (c^2 p^2 + m^2 c^4)^{1/2} \rightarrow mc^2 \quad -(6)$$

the fermion equation in UFT 174 gives:

$$\hat{H} \phi = E \phi \quad -(7)$$

where

$$\hat{H} = mc^2 + V + \frac{\hat{p}^2}{2m} - \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B}$$
$$- \frac{e^2 \hbar \, \underline{\sigma} \cdot \underline{L}}{16 \pi \epsilon_0 m^2 c^2 r^3} \quad -(8)$$

in the notation of UFT 174.

2) Eq. (7) is a Schroedinger type equation which in the non-relativistic approximation reduces to the Schroedinger Pauli equation:

$$\hat{H}_{SP}\,\phi = E\phi \quad -(9)$$

where
$$\hat{H}_{SP} = \frac{1}{2m}\,\underline{\sigma}\cdot\underline{\hat{p}}\;\underline{\sigma}\cdot\underline{\hat{p}} + V \quad -(10)$$

using eq. (4). In the presence of a magnetic field:

$$\underline{\hat{p}} \rightarrow \underline{\hat{p}} - e\underline{A} \quad -(11)$$

and
$$\hat{H}_{SP} \rightarrow \frac{1}{2m}\,\hat{p}^2 + V - \frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B} \quad -(12)$$

This hamiltonian describes Zeeman splitting and ESR, NMR and MRI. Thus:

$$\hat{H}_{SP}\,\phi = E\phi \quad -(13)$$

where ϕ are wavefunctions of the Schroedinger equation.

The relevant quantum force equation is:

$$(\hat{H}_{SP} - E)\frac{d\phi}{dx} = F\phi \quad -(14)$$

and the quantum force associated with the ESR, NMR and anomalous Zeeman effects is:

3)

$$\boxed{-\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}\,\frac{d\phi}{dx} = F\phi} \quad -(15)$$

i.e.

$$-\frac{e}{m}\,\underline{S}\cdot\underline{B}\,\frac{d\phi}{dx} = F\phi \quad -(16)$$

where

$$\underline{S} = \frac{1}{2}\hbar\,\underline{\sigma} \quad -(17)$$

The force eigenvalues can be worked out for any atomic or molecular wavefunction. In this approximation the Schroedinger wavefunctions can be used, although they do not correctly describe spin as is well known. This is the Schroedinger Pauli approximation.

In spherical polar coordinates eq. (15) is:

$$-\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}\,\underline{\nabla}\phi = \underline{F}\phi \quad -(18)$$

where:

$$\underline{\nabla}\phi = \frac{\partial\phi}{\partial r}\,\underline{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\,\underline{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\phi}\,\underline{e}_\phi \quad -(19)$$

This gives the quantized force on the electron or proton in ESR and NMR.