1) 177(5): Force Eigenvalue for 2p Orbital of H.

The wavefunction in this case has the structure:

$$\phi_{21} = A r \exp\left(-\frac{r}{2a}\right) \quad -(1)$$

with

$$E_{21} = -\frac{\hbar^2}{8ma^2}, \quad -(2)$$

and

$$V = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2mr^2} \quad -(3)$$

with

$$l = 2. \quad -(4)$$

Eq. (1) is the radial part of the complete function for $2p_z$. So:

$$\frac{\partial \phi_{21}}{\partial r} = \frac{\phi_{21}}{r} - \frac{1}{2a}\phi_{21}, \quad -(5)$$

$$\frac{\partial^3 \phi_{21}}{\partial r^3} = \frac{3}{4ra^2}\phi_{21} - \frac{1}{8a^3}\phi_{21} \quad -(6)$$

Therefore:

$$\left(\hat{H} - E\right)\frac{\partial \phi_{21}}{\partial r} = F\phi_{21}$$

$$= -\frac{\hbar^2}{2m}\left(\frac{3}{4ra^2} - \frac{1}{8a^3}\right)\phi_{21} - \left(\frac{e^2}{4\pi\epsilon_0 r} - \frac{3\hbar^2}{2mr^2}\right)\left(\frac{\phi_{21}}{r} - \frac{\phi_{21}}{2a}\right)$$

$$+ \frac{\hbar^2}{8ma^2}\left(\frac{\phi_{21}}{r} - \frac{1}{2a}\phi_{21}\right)$$

$$= \left(-\frac{3\hbar^2}{8mra^2} + \frac{\hbar^2}{16ma^3} - \frac{e^2}{4\pi\epsilon_0 r^2} + \frac{e^2}{8\pi\epsilon_0 ra} + \frac{3\hbar^2}{2mr^3} - \frac{3\hbar^2}{4mar^2} + \frac{\hbar^2}{8mra^2} - \frac{\hbar^2}{16ma^3}\right)\phi_{21} \quad -(7)$$

So:

$$\boxed{F_{21} = -\frac{e^2}{4\pi\epsilon_0 r^2} + \frac{e^2}{8\pi\epsilon_0 r a} - \frac{\hbar^2}{4 m r a^2} + \frac{3\hbar^2}{2 m r^3} - \frac{3\hbar^2}{4 m a r^2}}$$

$$-(8)$$

In this case the classical force:

$$F = -\frac{e^2}{4\pi\epsilon_0 r^2} \quad -(9)$$

is augmented by four quantum corrections.

There will also be quantum corrections due to force from the angle dependent part of ψ_{21}, a spherical harmonic.