1) 177(2): Force Eigenvalue for Zero of the Wavefunction of the Harmonic Oscillator.

Consider the quantum force equation:

$$\boxed{(\hat{H} - E)\frac{d\phi}{dx} = F\phi} \quad -(1)$$

for the harmonic oscillator. The zero'th wavefunction is:

$$\phi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right) \quad -(2)$$

In eq. (1):

$$\hat{H}\frac{\partial\phi_0}{\partial x} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\right)\frac{\partial\phi_0}{\partial x} \quad -(3)$$

where:

$$\frac{\partial\phi_0}{\partial x} = -\alpha x\phi_0 \quad -(4)$$

$$\frac{\partial^2\phi_0}{\partial x^2} = -\alpha\phi_0 + \alpha^2 x^2\phi_0 \quad -(5)$$

$$\frac{\partial^3\phi_0}{\partial x^3} = \alpha^2 x\phi_0\left(3 - \alpha x^2\right) \quad -(6)$$

So:

$$\hat{H}\frac{\partial\phi_0}{\partial x} = \left(-\frac{\hbar^2 m^2\omega^2 x}{2m\hbar^2}\left(3 - \frac{m\omega}{\hbar}x^2\right) - \frac{1}{2}\frac{m^2\omega^3 x^3}{\hbar}\right)\phi_0$$

$$= -\frac{3}{2}m\omega^2 x\phi_0 \quad -(7)$$

Also

$$E\frac{\partial\phi_0}{\partial x} = -\frac{1}{2}m\omega^2 x\phi_0 \quad -(8)$$

2) From eqs. (1), (7) and (8):

$$F_0 \phi_0 = -m\omega^2 x \phi_0 \quad -(9)$$

and

$$\boxed{F_0 = -kx} \quad -(10)$$

where

$$k = m\omega^2. \quad -(11)$$

The restoring force of the wavefunction ϕ_0 is the classical result of Hooke's law, eq. (10).

The energy of the zeroth wavefunction ϕ_0 is:

$$E_0 = \frac{1}{2}\hbar\omega \quad -(12)$$

This result is a remarkable one because it shows that the zero point energy (12) is accompanied by a hitherto unknown restoring force F_0.

Computer algebra can now be used to apply the new fundamental equation (1) of quantum mechanics to an essentially limitless number of problems. For example, the harmonic oscillator wavefunctions are given by

$$\phi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{y^2}{2}\right) \quad -(13)$$

$$\phi_1 = \left(\frac{\alpha}{\pi}\right)^{1/4} \sqrt{2}\, y \exp\left(-\frac{y^2}{2}\right) \quad -(14)$$

3)

$$\psi_2 = \frac{1}{\sqrt{2}}\left(\frac{\alpha}{\pi}\right)^{1/4}\left(2y^2 - 1\right)\exp\left(-\frac{y^2}{2}\right) \quad -(15)$$

$$\psi_3 = \frac{1}{\sqrt{3}}\left(\frac{\alpha}{\pi}\right)^{1/4}\left(2y^3 - 3y\right)\exp\left(-\frac{y^2}{2}\right) \quad -(16)$$

where

$$\alpha = \frac{m\omega}{\hbar}, \quad y = \alpha^{1/2}x \quad -(17)$$

The radial H wavefunctions are:

n	l	$R_{nl}(r)$
1	0 (1s)	$2\left(\frac{1}{a}\right)^{3/2}\exp\left(-\frac{\rho}{2}\right)$
2	0 (2s)	$\frac{1}{2\sqrt{2}}\left(\frac{1}{a}\right)^{3/2}(2-\rho)\exp\left(-\frac{\rho}{2}\right)$
2	1 (2p)	$\frac{1}{2\sqrt{6}}\left(\frac{1}{a}\right)^{3/2}\rho\exp\left(-\frac{\rho}{2}\right)$

where a is the Bohr radius and

$$\rho = \frac{2r}{na} \quad -(18)$$

The Bohr radius is:

$$a = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 5.29177 \times 10^{-11}\,m \quad -(19)$$

So the fake eigenvalues F_n can be worked out for the harmonic oscillator and radial wavefunctions of the H atom.