1) 175(5): Evaluating the Counter Example to the Heisenberg Uncertainty Principle

In note 175(4) it was shown that

$$[\hat{x}^2, \hat{p}^2]\phi = 2\hbar^2\phi + 4i\hbar\hat{x}\hat{p}\phi \quad -(1)$$

The Heisenberg uncertainty principle can be evaluated as follows, (P. W. Atkins, "Molecular Quantum Mechanics", Oxford University Press, 2nd. ed., 1983, pp. 93 ff.).

If:

$$[\hat{A}, \hat{B}]\phi = i\hat{C}\phi \quad -(2)$$

then:

$$\delta\hat{A}\,\delta\hat{B} \geq \frac{1}{2}|\langle\hat{C}\rangle| \quad -(3)$$

where

$$\delta\hat{A} = \left(\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2\right)^{1/2} \quad -(4)$$

$$\delta\hat{B} = \left(\langle\hat{B}^2\rangle - \langle\hat{B}\rangle^2\right)^{1/2}, \quad -(5)$$

and $\langle\hat{C}\rangle$ is the expectation value of $\hat{C}$.

The usual Copenhagen interpretation is that if $\langle\hat{C}\rangle$ is zero, $\hat{A}$ and $\hat{B}$ are knowable to arbitrary precision. If $\langle\hat{C}\rangle$ is non-zero, then $\hat{A}$ and $\hat{B}$ are not simultaneously knowable, a contradiction of Bacon's ideas of the causal philosophy, the very basis of natural philosophy. In the causal interpretation

) of ECE theory, there is nothing in natural philosophy
... is unknowable. Commutator relations such as eq. (1)
... mathematical outcomes of:

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}, \quad -(6)$$

... the x direction.
... the Schroedinger operator is

In eq. (1):

$\hat{A} = \hat{x}^2, \quad \hat{B} = \hat{p}^2, \quad \hat{C}$ is chosen to be ... in agreement with the definition (2), in which i is there for convenience (see Atkins). So:

$$\hat{C} = 2\hbar^2 + 4\hbar \hat{x} \hat{p} i - (7)$$

and

$$\langle \hat{C} \rangle = \int \psi^* \hat{C} \psi \, d\tau \Big/ \int \psi^* \psi \, d\tau \quad -(8)$$

In the previous note it was shown that $\langle \hat{C} \rangle$ can be zero or non-zero, depending on the exact solution chosen of the Schroedinger equation. This is a reductio ad absurdum refutation of the Copenhagen interpretation, because it shows that $\hat{A}$ and $\hat{B}$ can be simultaneously knowable or not for the same equation, the Schroedinger equation. In the ECE interpretation this result is a consequence of eq. (6), no more or no less than that.

3) In the previous note, only the zero'th wavefunction of the harmonic oscillator was used. The first four normalized wave functions are:

$$\phi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-y^2/2\right) \quad -(9)$$

$$\phi_1 = \left(\frac{\alpha}{\pi}\right)^{1/4} \sqrt{2}\, y \exp\left(-y^2/2\right) \quad -(10)$$

$$\phi_2 = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{1}{\sqrt{2}} \left(2y^2 - 1\right) \exp\left(-y^2/2\right) \quad -(11)$$

$$\phi_3 = \left(\frac{\alpha}{4}\right)^{1/4} \frac{1}{\sqrt{3}} \left(2y^3 - 3y\right) \exp\left(-y^2/2\right) \quad -(12)$$

where

$$\alpha = \frac{m\omega}{\hbar}, \quad y = \alpha^{1/2} x \quad -(13)$$

It will be interesting to work out these wave functions when used to evaluate $\langle \hat{C} \rangle$ from eqs. (7) and (8).

From eq. (9), it was shown in the previous note that

$$\langle \hat{x}\hat{p} \rangle = \left(\frac{\alpha}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-\alpha x^2/2}\, x \hat{p}\, e^{-\alpha x^2/2}\, dx \quad -(14)$$

$$= i\hbar/2$$

so from eq. (7) and (8):

$$\langle \hat{C} \rangle = 0 \quad -(15)$$

for this wave function, ϕ_0.

4) Computer Algebra Problem

1) Check the result (14) by computer using:

$$\langle \hat{x}\hat{p} \rangle = -i\hbar\left(\frac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{\infty} e^{-\alpha x^2/2}\, x \frac{\partial}{\partial x}\left(e^{-\alpha x^2/2}\right) dx \quad \text{— (16)}$$

2) Evaluate $\langle \hat{C} \rangle$ using the wavefunctions (10) to (12).

If $\langle \hat{C} \rangle$ is different for different wavefunctions of the harmonic oscillator, the Copenhagen interpretation is refuted. Such a result would mean that $\hat{x}^2$ and $\hat{p}^2$ can be specified simultaneously sometimes, but not at other times, for the same exact solution of the Schroedinger equation. This is the harmonic oscillator, used throughout physics.