1) 175(3): Simple Counter-Example to Indeterminacy Using the Schroedinger Equation.

Consider the Schroedinger equation in the x dimension:

$$\hat{H}\phi = E\phi \quad -(1)$$

where:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}. \quad -(2)$$

Then

$$\phi(x) = A\exp(ikx) + B\exp(-ikx) \quad -(3)$$

where

$$E = \frac{p^2}{2m} = \frac{\hbar^2 \kappa^2}{2m}. \quad -(4)$$

$$p = \hbar\kappa \quad -(5)$$

Therefore.

if

$$B = 0, \quad -(6)$$

and

$$\phi = Ae^{i\kappa x}, \phi^* = Ae^{-i\kappa x}. \quad -(7)$$

Now calculate:

$$\delta x = \left(\langle\hat{x}^2\rangle - \langle\hat{x}\rangle^2\right)^{1/2} \quad -(8)$$

$$\delta p = \left(\langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2\right)^{1/2}. \quad -(9)$$

According to Bohr and Heisenberg:

$$\delta x\,\delta p \geq \hbar/2. \quad -(10)$$

The symbol $\langle\ \rangle$ denotes "expectation value"

2) defined by:

$$\langle \hat{\alpha} \rangle = \int_{-\infty}^{\infty} \phi^* \hat{\alpha} \phi \, dx \Big/ \int_{-\infty}^{\infty} \phi^* \phi \, dx \quad -(11)$$

Using eq. (7):

$$\int_{-\infty}^{\infty} \phi^* \phi \, dx = \int_{-\infty}^{\infty} A^2 \, dx \rightarrow \infty \quad -(12)$$

The subject of quantum mechanics avoids this problem by assuming

$$\int_{-L/2}^{L/2} A^2 \, dx = 1, \quad -(13)$$

so

$$A = \frac{1}{L^{1/2}} \quad -(14)$$

This is a rather dubious procedure right at the outset, but accepting it, we proceed to evaluate the relevant expectation values. Firstly:

$$\langle \hat{x} \rangle = \int_{-L/2}^{L/2} \frac{x}{L} \, dx = \frac{1}{L} \left. \frac{x^2}{2} \right|_{-L/2}^{L/2} \quad -(15)$$

$$= 0$$

Secondly: $$\langle \hat{x}^2 \rangle = \frac{1}{L} \int_{-L/2}^{L/2} x^2 \, dx = \frac{1}{L} \left. \frac{x^3}{3} \right|_{-L/2}^{L/2}$$

$$= \frac{L^2}{12} \quad -(16)$$

3) So:

$$\delta x = \left( \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2 \right)^{1/2} = \frac{L}{\sqrt{12}} \quad -(17)$$

Thirdly:

$$\langle \hat{p} \rangle = -i\frac{\hbar}{L} \int_{-L/2}^{L/2} \phi^* \frac{\partial \phi}{\partial x} \, dx \quad -(18)$$

where

$$\frac{\partial \phi}{\partial x} = i\kappa \phi \quad -(19)$$

So:

$$\langle \hat{p} \rangle = \frac{\hbar \kappa}{L} \Big|_{-L/2}^{L/2} = \hbar \kappa \quad -(20)$$

i.e

$$p = \hbar \kappa \quad -(21)$$

Finally:

$$\langle \hat{p}^2 \rangle = -\frac{\hbar^2}{L} \int_{-L/2}^{L/2} \phi^* \frac{\partial^2 \phi}{\partial x^2} \, dx \quad -(21)$$

where

$$\frac{\partial^2 \phi}{\partial x^2} = -\kappa^2 \phi \quad -(22)$$

So

$$\langle \hat{p}^2 \rangle = \frac{\hbar^2 \kappa^2}{L} \Big|_{-L/2}^{L/2} = \hbar^2 \kappa^2 \quad -(23)$$

So:

$$\delta p = \left( \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2 \right)^{1/2} = 0 \quad -(24)$$

and

$$\boxed{\delta x \, \delta p = 0} \quad -(25)$$

<u>which violates the claim (10)</u>, <u>QED</u>

The standard protagonists would argue that:

$$L \to \infty. \quad -(26)$$

but this is entirely arbitrary. According to eqn. (12), if $L \to \infty$, then:

$$\langle \hat{x}^2 \rangle \to \frac{\infty}{\infty} \quad -(27)$$

and is indeterminate. So:

$$\delta x \, \delta p = \text{indeterminate}. \quad -(28)$$

Note that:

$$[\hat{p}, \hat{x}]\phi = -i\hbar \left[\frac{\partial}{\partial x}, x\right]\phi.$$

$$= -i\hbar \left(\frac{\partial}{\partial x}(x\phi) - x\frac{\partial \phi}{\partial x}\right)$$

$$= -i\hbar \left(x\frac{\partial \phi}{\partial x} + \phi - x\frac{\partial \phi}{\partial x}\right)$$

$$= -i\hbar\phi$$

so

$$[\hat{x}, \hat{p}]\phi = i\hbar\phi \quad -(29)$$

However, this does not mean that:

$$\delta\hat{x}\,\delta\hat{p} \geq \hbar/2. \quad -(30)$$