1) 174(5): Simple Example of the Method of 174(1)

Consider the fermion equation in wave format:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad — (1)$$

where

$$\phi = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix}. \quad — (2)$$

Multiply both sides of eq. (1) by ϕ^{-1}:

$$(\Box \phi)\phi^{-1} + \left(\frac{mc}{\hbar} \right)^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \quad — (3)$$

The inverse matrix is:

$$\phi^{-1} = \frac{\text{adjoint } \phi}{\text{determinant } \phi} \quad — (4)$$

where:

$$\text{adjoint } \phi = \begin{bmatrix} \phi_2^L & -\phi_2^R \\ -\phi_1^L & \phi_1^R \end{bmatrix} \quad — (5)$$

and

$$\text{determinant } \phi = \phi_1^R \phi_2^L - \phi_2^R \phi_1^L. \quad — (6)$$

If it is assumed that:

$$\phi = \phi^{(0)} e^{i(\omega t - kZ)} \quad — (7)$$

then

$$\phi^{-1} = \left(\phi^{(0)} \right)^{-1} e^{-i(\omega t - kZ)} \quad — (8)$$

so

$$\phi \phi^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad — (9)$$

In this case:

2)

$$\Box \phi = \Box\left(\phi^{(0)} e^{i(\omega t - kz)}\right)$$
$$= \left(k^2 - \frac{\omega^2}{c^2}\right)\phi \quad -(10)$$

Therefore eq. (3) is:

$$\left(\left(k^2 - \frac{\omega^2}{c^2}\right) + \left(\frac{mc}{\hbar}\right)^2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \quad -(11)$$

i.e.

$$\frac{\omega^2}{c^2} = k^2 + \left(\frac{mc}{\hbar}\right)^2 \quad -(12)$$

Finally use:

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{k} \quad -(13)$$

and eq. (12) becomes the Einstein energy equation:

$$E^2 = p^2c^2 + m^2c^4. \quad -(14)$$

Therefore a solution of eq. (1) is:

$$\phi = \phi^{(0)} \exp\left(i(\omega t - kz)\right) \quad -(15)$$

in which ω and k are related by eq. (12). If, in eq. (14):

$$E = i\hbar\frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar\underline{\nabla} \quad -(16)$$

then eq. (1) is obtained again.