1) 172(6): Derivation of the g Factor of the Electron from the ECE Fermion Equation

The ECE fermion equation is equivalent to the chiral representation of the Dirac equation. In the presence of the scalar potential ϕ and vector potential $\underline{A}$ the Dirac equation in chiral rep is:

$$\gamma^0 (E - e\phi)\psi - c\underline{\gamma}\cdot(\underline{p} - e\underline{A})\psi = mc^2\psi \quad -(1)$$

where $\psi = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix}$, $\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\gamma^i = \begin{bmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{bmatrix}$ $-(2)$

so:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}(E - e\phi)\begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} - c\begin{bmatrix} 0 & -\underline{\sigma} \\ \underline{\sigma} & 0 \end{bmatrix}\cdot(\underline{p} - e\underline{A})\begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix}$$
$$= mc^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \quad -(3)$$

The basic structure is:

$$mc^2\phi^R = (E + c\underline{\sigma}\cdot\underline{p})\phi^L \quad -(4)$$
$$mc^2\phi^L = (E - c\underline{\sigma}\cdot\underline{p})\phi^R \quad -(5)$$

Note carefully that the eigenvalue mc^2 is positive. In the presence of ϕ and $\underline{A}$, eqs. (4) and (5) are:

$$((E - e\phi) + c\underline{\sigma}\cdot(\underline{p} - e\underline{A}))\phi^L = mc^2\phi^R \quad -(6)$$
$$((E - e\phi) - c\underline{\sigma}\cdot(\underline{p} - e\underline{A}))\phi^R = mc^2\phi^L \quad -(7)$$

so

$$((E - e\phi)^2 - c^2(\underline{\sigma}\cdot\underset{\sim}{\pi})(\underline{\sigma}\cdot\underset{\sim}{\pi}))\phi^R = (mc^2)^2\phi^R \quad -(8)$$

where

$$\underset{\sim}{\pi} = \underline{p} - e\underline{A} \quad -(9)$$

2) Therefore:

$$\left((E-e\phi)^2 - m^2c^4\right)\phi^R = c^2(\underline{\sigma}\cdot\underline{\Pi})(\underline{\sigma}\cdot\underline{\Pi})\phi^R \quad -(10)$$

Now use:

$$(\underline{\sigma}\cdot\underline{\Pi})(\underline{\sigma}\cdot\underline{\Pi})\phi^R = \left((\underline{p}-e\underline{A})^2 \mp ie\underline{\sigma}\cdot\left((\underline{p}\times\underline{A})+(\underline{A}\times\underline{p})\right)\right)\phi^R \quad -(11)$$

The operator equation is:

$$\underline{p} = -i\hbar\underline{\nabla} \quad -(12)$$

So

$$-i^2e\hbar\underline{\sigma}\cdot\left(\underline{\nabla}\times(\underline{A}\phi^R) + \underline{A}\times\underline{\nabla}\phi^R\right)$$
$$= e\hbar\underline{\sigma}\cdot\left((\underline{\nabla}\times\underline{A})\phi^R + (\underline{\nabla}\phi^R)\times\underline{A} + \underline{A}\times(\underline{\nabla}\phi^R)\right) \quad -(13)$$
$$= \hbar(e\underline{\sigma}\cdot\underline{\nabla}\times\underline{A})\phi^R \quad -(14)$$
$$= (e\hbar\underline{\sigma}\cdot\underline{B})\phi^R$$

So

$$\left((E-e\phi)^2 - m^2c^4\right)\phi^R = \left((\underline{p}-e\underline{A})^2 \mp e\hbar\underline{\sigma}\cdot\underline{B}\right)\phi^R$$

i.e.

$$\boxed{\begin{gathered}\frac{1}{2m}\left((E-e\phi)^2 - m^2c^4\right)\phi^R = \hat{H}\phi^R \\ \text{where} \quad \hat{H} = \frac{1}{2m}\left((\underline{p}-e\underline{A})^2 \mp e\hbar\underline{\sigma}\cdot\underline{B}\right)\end{gathered}} \quad -(15)$$

For simplicity and illustration consider the case:

$$\phi = 0 \quad -(16)$$

3) where there are no electric fields present. Then:

$$\frac{p^2}{2m}\phi^R = \hat{H}\phi^R \quad -(17)$$

where

$$p^2 = E^2 - m^2c^4 \quad -(18)$$

$$= \gamma^2 m^2 v^2$$

and

$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(19)$$

In the limit:

$$v \ll c \quad -(20)$$

$$\frac{p^2}{2m} = \frac{1}{2}mv^2 = T \quad -(21)$$

so

$$\boxed{\hat{H}\phi^R = T\phi^R} \quad -(22)$$

The interaction of the fermion with the magnetic field $\underline{B}$ is given by:

$$\boxed{\hat{H}_{int} = -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}} \quad -(23)$$

This is the result verified experimentally in ESR and NMR, the famous spin 1/2 and − 1/2.

Conclusion

The ECE fermion equation produces the g factor 2 of the electron without the use of negative energy.

4) Note that eqs. (4) and (5) can be written as:

$$\left((E + c\underline{\sigma}\cdot\underline{p})(E - c\underline{\sigma}\cdot\underline{p})\right)\phi^R = m^2c^4\phi^R \quad -(24)$$

$$\left((E - c\underline{\sigma}\cdot\underline{p})(E + c\underline{\sigma}\cdot\underline{p})\right)\phi^L = m^2c^4\phi^L \quad -(25)$$

and this is a <u>factorization</u> of the Einstein energy equation in the basis set defined by the Pauli matrices.

In the <u>standard</u> representation used originally by Dirac:

$$\gamma^0_S = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \underline{\gamma}_S = \begin{bmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{bmatrix} \quad -(26)$$

and this gives from eq. (1):

$$(E - e\phi)\phi^R - c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi^L = mc^2\phi^R \quad -(27)$$

$$\boxed{-(E - e\phi)\phi^L} + c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi^R = mc^2\phi^L \quad -(28)$$

"Negative energy" ?

<u>This is an incorrect choice of γ matrices by Dirac</u>. For some reason Ryder switches to the standard rep. on page 53 of the second edition of "Quantum Field Theory" having prior to that used the chiral rep. In the standard rep the spinor is made up of combinations of the chiral rep spinors as follows:

$$\phi_S = S\,\phi_C \quad -(29)$$

where

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad -(30)$$

so

$$\phi_S = \frac{1}{\sqrt{2}}\begin{bmatrix} \phi^R + \phi^L \\ \phi^R - \phi^L \end{bmatrix}. \quad -(31)$$

Also

$$\gamma_{0S} = S\,\gamma_{0C}\,S^{-1} \quad -(32)$$

where

$$S = S^{-1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad -(33)$$

It becomes clear that the chiral representation is the correct one. The ECE fermion equation is equivalent to the chiral rep but simpler and more powerful. The derivation of the g factor 2 of the electron given in this note is original and simpler than the derivation from the standard rep. Negative energy eigenvalues do not occur in the chiral ~~standard~~ rep.
