172(3): Definition of the Tetrad Used in the ECE / Dirac Equations.

The general definition of the tetrad in a space of any dimension is:

$$V^a = q^a_\mu V^\mu \quad -(1)$$

In Cartan's original definition, a denotes a different space from μ. In the ECE theory a and μ can also denote different representations of the same space, e.g. complex circular and Cartesian. The key point is that in ECE theory covariant derivatives of V^a and V^μ are in general different from four derivatives. In other words there exist connections denoted:

$$D_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda} V^\lambda \quad -(2)$$

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(3)$$

so from the beginning, ECE is a theory of physics developed in non-Minkowski spacetimes. Eqns (1) to (3) imply the tetrad postulate:

$$\partial_\mu q^a_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \quad -(4)$$

where:

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda, \quad \omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu. \quad -(5)$$

Eq. (4) can be rewritten as:

$$(\square + R)\, q^a_\mu = 0 \quad -(6)$$

$$R = q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) \quad -(7)$$

The space appropriate to the ECE fermion equation and Dirac equation is two dimensional. The tetrad

2) is defined by:

$$\begin{bmatrix} \psi^R \\ \psi^L \end{bmatrix} = \begin{bmatrix} \psi^R_1 & \psi^R_2 \\ \psi^L_1 & \psi^L_2 \end{bmatrix} \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix} \quad -(8)$$

so:

$$V^a = \begin{bmatrix} \psi^R \\ \psi^L \end{bmatrix}, \quad V_\mu = \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix} \quad -(9)$$

In direct analogy to eq. (8) consider the unit vectors of the complex circular basis, and restrict attention to the transverse components. Then:

$$\underline{e}^{R(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) \quad -(10)$$

$$\underline{e}^{L(2)} = \frac{1}{\sqrt{2}} (\underline{i} + i\underline{j}) \quad -(11)$$

Therefore

$$\begin{bmatrix} \underline{e}^{R(1)} \\ \underline{e}^{L(2)} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(12)$$

so

$$q^a_\mu = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \quad -(13)$$

If the tetrad propagates then:

$$q^a_\mu = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} e^{i\phi} \quad -(14)$$

where ϕ is the phase. Eq. (12) is part of a four dimensional space (the spacetime) and eq. (8) is a two dimensional space of spinors. The unit vectors $\underline{e}^{R(1)}$ and $\underline{e}^{L(2)}$ may be made to propagate as follows:

$$\underline{e}^{R(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(15)$$

$$\underline{e}^{L(2)} = \frac{1}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\phi} \quad -(16)$$

so the frame is propagating and a connection exists.

3) The unit vectors $\underline{e}^R$ and $\underline{e}^L$ are right and left circularly polarized.

Note carefully that $\underline{e}^R$ and $\underline{e}^L$ are moving, i.e translating in Z and spinning. So the frame of reference is moving and a connection exists. So the labels in eq. (1) are:

$$\left.\begin{array}{l} a = R, L, \\ \mu = 1, 2. \end{array}\right\} \quad -(17)$$

In exact analogy the tetrad in eq. (8) may be made to propagate:

$$q^a_\mu = \begin{bmatrix} q^R_1(0) & q^R_2(0) \\ q^L_1(0) & q^L_2(0) \end{bmatrix} e^{i\phi} \quad -(18)$$

i.e

$$q^R_1 = q^R_1(0)\, e^{i\phi} \quad -(19)$$

and so on. In eq. (12), we may write:

$$\underline{e}^R = \underline{e}^R(0)\, e^{i\phi} \quad -(20)$$

$$\underline{e}^L = \underline{e}^L(0)\, e^{i\phi} \quad -(21)$$

so by analogy:

$$q^R = q^R(0)\, e^{i\phi} \quad -(22)$$

$$q^L = q^L(0)\, e^{i\phi} \quad -(23)$$

and a connection exists for the representation labelled by a and for the representation labelled by μ. Thus:

$$(\Box + R) \begin{bmatrix} q^R_1 & q^R_2 \\ q^L_1 & q^L_2 \end{bmatrix} = 0 \quad -(24)$$

4) and this is automatically written in the general 2-D space with torsion and curvature, respectively:

$$T^a_{\mu\nu} = \left(d \wedge q^a + \omega^a{}_b \wedge q^b \right)_{\mu\nu} \quad -(25)$$

for all μ, ν, and:

$$R^a{}_{b\mu\nu} = \left(d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \right)_{\mu\nu} \quad -(26)$$

for all μ, ν.

The Dirac equation is obtained by writing eq. (24) as:

$$(\square + R) \begin{bmatrix} \phi^R_1 \\ \phi^R_2 \\ \phi^L_1 \\ \phi^L_2 \end{bmatrix} = 0 \quad -(27)$$

and taking the limit:

$$R \to \left(\frac{mc}{\hbar} \right)^2 \quad -(28)$$

to obtain:

$$\left(\square + (mc/\hbar)^2 \right) \phi = 0 \quad -(29)$$

which factorizes to:

$$\left(i \gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) \phi = 0 \quad -(30)$$

The momentum representation of eq. (30) is:

$$\left(\gamma^\mu p_\mu - mc \right) \phi = 0 \quad -(31)$$

The effect of gravitation on eq. (24) is to change R and the eigenfunction. The effect of electromagnetism is similar. For example the

5) minimal prescription method for electromagnetism is:

$$p_\mu \rightarrow p_\mu + eA_\mu \quad -(32)$$

so

$$(\gamma^\mu(p_\mu + eA_\mu) - mc)\psi = 0 \quad -(33)$$

It is seen that ψ of eq. (33) is different from ψ of eq. (31). In general, eq. (31) is:

$$(\gamma^\mu p_\mu - R^{1/2})\psi = 0 \quad -(34)$$

so electromagnetism has the effect of changing eq. (34) to:

$$(\gamma^\mu p_\mu - R_1^{1/2})\psi = 0 \quad -(35)$$

where

$$R_1^{1/2} = R^{1/2} - e\gamma^\mu A_\mu \quad -(36)$$

From comparison of eqns. (7) and (36) it is seen that e/m has changed the connections. Similarly gravitation or the strong or weak field will change the connections of the fermion field.

Any permutation or combination of fields can be considered in this way, so ECE is a generally covariant unified field theory.