172(11): Darwin Term from Fermion Equation

The Darwin term emerges from the spin orbit coupling term as follows:

$$(\underline{\sigma} \cdot \underline{p}\, \phi\, \underline{\sigma} \cdot \underline{p})\, \phi^R = -\hbar^2\, \underline{\sigma} \cdot \underline{\nabla} \phi\, \underline{\sigma} \cdot \underline{\nabla} \phi^R \quad -(1)$$

and is given as:

$$H_{Darwin} = -\frac{\hbar^2}{4m^2c^2}\, \underline{\nabla} \phi\, \underline{\nabla} \phi^R \quad -(2)$$

so it emerges together with the spin orbit term from the ECE fermion equation. It is observed in the fine structure of spectra.

In general all the known features of atomic and molecular spectra are described by the fermion equation. The description is in general:

$$\boxed{\sigma^0 (E - e\phi)\psi + c\sigma^3 \left((\underline{p} - e\underline{A})\psi \cdot \underline{\sigma} \right) = mc^2 \sigma^1 \psi} \quad -(3)$$

where

$$\psi = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \quad -(4)$$

$$\left.\begin{aligned} \sigma^0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \\ \sigma^2 &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{aligned}\right\} \quad -(5)$$