172(8): The Wave Format of the ECE Fermion Equation

The ECE fermion equation is:

$$\boxed{\sigma^0 \hat{E}\psi - c\sigma^3\left(\hat{p}_x \psi \sigma^1 - \hat{p}_y \psi \sigma^2 + \hat{p}_Z \psi \sigma^3\right) = mc^2 \sigma^1 \psi} \quad -(1)$$

Here:

$$\psi = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix}, \quad \sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad -(2)$$

$$\sigma^2 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Written out in full, eq. (1) is:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \hat{E} \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} - c \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \left(\hat{p}_x \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \hat{p}_y \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \right.$$

$$\left. + \hat{p}_Z \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right) = mc^2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \quad -(3)$$

i.e.

$$\hat{E} \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} - c\hat{p}_x \begin{bmatrix} \psi_2^R & \psi_1^R \\ -\psi_2^L & -\psi_1^L \end{bmatrix} + c\hat{p}_y \begin{bmatrix} -i\psi_2^R & i\psi_1^R \\ i\psi_2^L & -i\psi_1^L \end{bmatrix} \quad -(4)$$

$$- c\hat{p}_Z \begin{bmatrix} \psi_1^R & -\psi_2^R \\ -\psi_1^L & \psi_2^L \end{bmatrix} = mc^2 \begin{bmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{bmatrix}$$

Eq. (4) is equivalent to:

$$\left.\begin{aligned}
\hat{E}\psi_1^R - c\left(\hat{p}_Z \psi_1^R + (\hat{p}_x + i\hat{p}_y)\psi_2^R\right) &= mc^2 \psi_1^L \\
\hat{E}\psi_2^R - c\left((\hat{p}_x - i\hat{p}_y)\psi_1^R - \hat{p}_Z \psi_2^R\right) &= mc^2 \psi_2^L \\
\hat{E}\psi_1^L + c\left(\hat{p}_Z \psi_1^L + (\hat{p}_x + i\hat{p}_y)\psi_2^L\right) &= mc^2 \psi_1^R \\
\hat{E}\psi_2^L + c\left((\hat{p}_x - i\hat{p}_y)\psi_1^L - \hat{p}_Z \psi_2^L\right) &= mc^2 \psi_2^R
\end{aligned}\right\} -(5)$$

2) Eq. (5) is equivalent to the two equations:

$$\left(\hat{E}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - c\begin{bmatrix}\hat{p}_Z & \hat{p}_X + i\hat{p}_Y\\ \hat{p}_X - i\hat{p}_Y & -\hat{p}_Z\end{bmatrix}\right)\begin{bmatrix}\psi_1^R\\ \psi_2^R\end{bmatrix} = mc^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\psi_1^L\\ \psi_2^L\end{bmatrix} \quad -(6)$$

and

$$\left(\hat{E}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + c\begin{bmatrix}\hat{p}_Z & \hat{p}_X + i\hat{p}_Y\\ \hat{p}_X - i\hat{p}_Y & -\hat{p}_Z\end{bmatrix}\right)\begin{bmatrix}\psi_1^L\\ \psi_2^L\end{bmatrix} = mc^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\psi_1^R\\ \psi_2^R\end{bmatrix} \quad -(7)$$

which may be written in condensed format as:

$$(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})\,\phi^R = mc^2\,\phi^L \quad -(8)$$

$$(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})\,\phi^L = mc^2\,\phi^R \quad -(9)$$

From eqs. (8) and (9):

$$(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})\,\phi^L = m^2c^4\,\phi^L \quad -(10)$$

$$(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})\,\phi^R = m^2c^4\,\phi^R \quad -(11)$$

Now use:

$$\hat{E} = i\hbar\frac{\partial}{\partial t}, \quad \hat{p} = -i\hbar\underline{\nabla} \quad -(12)$$

$$\Box = \partial^\mu\partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(13)$$

to find that:

$$\left.\begin{aligned}(\Box + (mc/\hbar)^2)\,\psi_1^R &= 0\\ (\Box + (mc/\hbar)^2)\,\psi_2^R &= 0\\ (\Box + (mc/\hbar)^2)\,\psi_1^L &= 0\\ (\Box + (mc/\hbar)^2)\,\psi_2^L &= 0\end{aligned}\right] \quad -(14)$$

5) i.e.

$$\boxed{\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix} = 0} \quad -(15)$$

which is the wave format of the ECE fermion equation. Eq. (1) is a factorization of eq. (15) using the same eigenfunction. Eq (15) is a limit of the ECE wave equation:

$$(\Box + R)\, q_\mu^a = 0 \quad -(16)$$

with

$$q_\mu^a = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix}, \quad R = \left(\frac{mc}{\hbar}\right)^2. \quad -(17)$$

Eq (16) is equivalent to the tetrad postulate of Cartan:

$$D_\mu q_\nu^a = 0. \quad -(18)$$

The above formalism is based on Cartan geometry and is valid in the general spacetime in su(2) representation space.

It is simpler and more powerful than the chiral representation of the Dirac equation. This is:

$$(\gamma^\mu \hat{p}_\mu - mc)\, \psi_D = 0 \quad -(19)$$

here:

4) $\gamma^\mu \hat{p}_\mu = \gamma^0 \hat{p}_0 + \gamma^i \hat{p}_i$ — (20)

Here:

$$\gamma^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \gamma^3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad -(21)$$

In condensed notation, eq. (21) is:

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \gamma^i = \begin{bmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{bmatrix} \quad -(22)$$

So eq. (19) is:

$$\hat{E}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix} - c\hat{p}_x \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix} - c\hat{p}_y \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix}$$

$$- c\hat{p}_z \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix} = mc^2 \begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix} \quad -(23)$$

where the Dirac spinor is:

$$\psi_D = \begin{bmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{bmatrix} \quad -(24)$$

The wave format of eq. (19) is:

$$\left(\Box + (mc/\hbar)^2\right)\psi_D = 0 \quad -(25)$$

and is equivalent to eq. (15).

5)

The ECE Spinor is:

$$\phi = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix} \quad -(26)$$

and is a Cartan tetrad in SU(2) rep space. Being a tetrad, it can be written in the general spacetime with torsion and curvature. On the other hand the Dirac spinor is not a tetrad and cannot be written in the general spacetime. The ECE spinor is part of a generally covariant unified field theory, but the Dirac spinor is not. The way that the Dirac equation (19) is usually written in the general spacetime is to attempt to generalise $\gamma^\mu p_\mu$, leaving ϕ_D and mc unchanged.

Eq. (1) can be immediately generalized to spacetime with torsion and curvature by writing it:

$$\boxed{\hat{E}\phi - c\sigma^3\left(\hat{p}_x\phi\sigma^1 - \hat{p}_y\phi\sigma^2 + \hat{p}_z\phi\sigma^3\right) = \hbar R^{1/2}\sigma^1\phi}$$

$$-(27)$$