UFT 170(1): Renormalization and the Green Function

The idea of renormalization in QED can be illustrated with the Klein Gordon equation:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad -(1)$$

In QED this is known as a free field equation. If the source of the field is included in eq. (1) then:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi^0 = \rho \quad -(2)$$

where ϕ^0 is known as the bare field. The bare field is evaluated using a series expansion of Green functions:

$$\phi^0 = \phi' + \int G(x-y\,;\,m_0)\,\rho(y)\,dy \quad -(3)$$

where the Green function is defined by:

$$\left(\Box_x + (m_0 c / \hbar)^2 \right) G(x-y) = \delta(x-y) \quad -(4)$$

For a fermion:

$$\left(-i\gamma^\mu \partial_\mu + m_0 c / \hbar \right) G(x-y) = \delta(x-y) \quad -(5)$$

The bare field is expressed in terms of creation and annihilation operators similar to the free field, but the norm is no longer unity. So the field, mass and energy have to be renormalized with a loose parameter Z, $0 < Z < 1$:

$$\phi^R = \frac{\phi^0}{Z}, \quad m_R = \frac{m_0}{Z}, \quad E_n^R = \frac{E_n^0}{Z} \quad -(6)$$

The physical mass is considered to be m_R, but the bare mass m_0 is an unspecified parameter which

2) together with the parameter Z are varied by fitting to produce an agreement with experiment. For a point source $1/Z$ is infinite, so m_0 has to be very close to zero to produce a finite m_R.

So QED is not a fundamental theory, or more accurately, ceases to be one as soon as renormalization is introduced. Its claims to accuracy are based on fitting to data.

The basic equation of QED is simply the modified Dirac equation:

$$\left(i\gamma^\mu \partial_\mu - \frac{e}{\hbar}\gamma_\mu\left(A^\mu + B^\mu\right) - \frac{mc}{\hbar}\right)\phi = 0 \quad -(7)$$

where the potential A^μ is generated by the electron itself and where B^μ is generated by the source. Therefore QED is:

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right)\phi = \frac{e}{\hbar}\gamma_\mu\left(A^\mu + B^\mu\right)\phi \quad -(8)$$

The Lamb shift is well known to be a definite indication of the fact that the fluctuating electric and magnetic fields of the vacuum perturb the Coulomb potential in an atom or molecule. Therefore there is electric power available from this vacuum. It is claimed that QED can describe this effect with accuracy, but this is "hocus pocus" in the words of Feynman, one of the developers of QED. What actually happens is that the parameters

3)

Z, ϕ°, m_0 and E_n are adjusted to produce agreement with the Lamb shift.

For the purposes of engineering it is much more helpful to adopt a far simpler approach which nevertheless has the necessary accuracy and physics built into it.

The change of potential due to the vacuum fluctuations is:

$$\Delta V = V(\underset{\sim}{r} + \delta \underset{\sim}{r}) - V(\underset{\sim}{r}) \quad - (9)$$

This is expanded as:

$$\Delta V = \delta \underset{\sim}{r} \cdot \underline{\nabla} V + \frac{1}{2} \left(\delta \underset{\sim}{r} \cdot \underline{\nabla} \right)^2 V(\underline{r}) + \cdots$$

$$- (10)$$

It is now assumed that the fluctuations are isotropic.

$$\langle \delta \underset{\sim}{r} \rangle_{vac} = \underline{0}, \quad - (11)$$

$$\left\langle \left(\delta \underset{\sim}{r} \cdot \underline{\nabla} \right)^2 \right\rangle_{vac} = \frac{1}{3} \left\langle \left(\delta \underset{\sim}{r} \right)^2 \right\rangle_{vac} \nabla^2 - (12)$$

so:

$$\boxed{\langle \Delta V \rangle = \frac{1}{6} \left\langle \left(\delta \underset{\sim}{r} \right)^2 \right\rangle_{vac} \left\langle \nabla^2 \left(- \frac{e^2}{4\pi \epsilon_0 r} \right) \right\rangle_{at}}$$

$$- (13)$$

This is a very useful result for engineering, because [it] shows exactly how vacuum energy is transferred [t]o an atom. The process is split into a product of [va]cuum fluctuations and atomic property.

Using methods to be developed in the next [no]te it can be shown that:

4)

$$\left\langle \left(\delta \underset{\sim}{r}\right)^2 \right\rangle_{vac} = \frac{1}{2\epsilon_0 \pi^2}\left(\frac{e^2}{\hbar c}\right)\left(\frac{\hbar}{mc}\right)^2 \log_e\left(\frac{4\epsilon_0 \hbar c}{e^2}\right) \quad -(14)$$

which is based on the Newton equation:

$$m\frac{d^2}{dt^2}\left(\delta \underset{\sim}{r}\right) = -e\underline{E} \quad -(15)$$

with:

$$\delta r(t) = \delta r(0)\left(\exp(-i\omega t) + \exp(i\omega t)\right) \quad -(16)$$

Inside the atom or molecule:

$$\left\langle \nabla^2\left(-\frac{e^2}{4\pi\epsilon_0 r}\right)\right\rangle_{at} = -\frac{e^2}{4\pi\epsilon_0}\int \phi^*(\underset{\sim}{r})\, \nabla^2\left(\frac{1}{r}\right)\phi(\underset{\sim}{r})\, d\underline{r} \quad -(17)$$

where

$$\nabla^2 \frac{1}{r} = -4\pi\,\delta(\underset{\sim}{r}) \quad -(18)$$

so

$$\left\langle \nabla^2\left(-\frac{e^2}{4\pi\epsilon_0 r}\right)\right\rangle_{at} = \frac{e^2}{\epsilon_0}\left|\phi(0)\right|^2 \quad -(19)$$

For the 2s orbital of the H atom:

$$\left\langle \nabla^2\left(-\frac{e^2}{4\pi\epsilon_0 r}\right)\right\rangle_{at} = \frac{e^2}{\epsilon_0}\left|\phi_{2s}(0)\right|^2 \quad -(20)$$

$$= \frac{e^2}{8\pi\epsilon_0 a_0^3}$$

where the Bohr radius is:

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \quad -(21)$$

so

$$\langle \Delta V\rangle = \frac{4}{3}\frac{e^2}{4\pi\epsilon_0}\frac{e^2}{4\pi\epsilon_0 \hbar c}\left(\frac{\hbar}{mc}\right)^2 \frac{1}{8\pi a_0^3}\log_e\frac{4\epsilon_0 \hbar c}{e^2} \quad -(22)$$

5)

This result is already of sufficient accuracy to design a circuit which is able to trap the vacuum $\underline{E}$ field of eq. (15). The vacuum $\underline{E}$ field comes from the vacuum potential of ECE theory.

In order to design a circuit, the isotropic averaging of the Coulomb law in eq. (19) within an atom is replaced by a mechanism based on an electric circuit. The potential within the circuit is then increased by:

$$\langle \Delta V \rangle_{circuit} = \frac{1}{6} \langle (\delta \underline{r})^2 \rangle_{vac} \left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle_{circuit} \quad -(23)$$

Spin connection resonance can be introduced into the Coulomb law of eq. (23), so $\langle \Delta V \rangle_{circuit}$ is amplified. If it is assumed that there is no need for isotropic averaging then:

$$\langle \Delta V \rangle_{circuit} = \frac{1}{6} \langle (\delta \underline{r})^2 \rangle_{vac} \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right), \quad -(24)$$

an equation which can be generalized for various types of circuit potential A_μ^a, not just a scalar potential.
