170(2). Poynting Theorem for Vacuum Electric Field Strength.

This is:

$$\underline{E}_{vac} \cdot \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) = \underline{J} \cdot \underline{E}_{vac} \quad -(1)$$

where

$$\underline{\nabla} \times \underline{E}_{vac} + \frac{\partial \underline{B}_{vac}}{\partial t} = \underline{0} \quad -(2)$$

The vacuum electric field strength $\underline{E}_{vac}$ is defined by the vacuum fluctuations $\delta \underline{r}$:

$$m \frac{d^2}{dt^2} (\delta \underline{r}) = -e \underline{E}_{vac} \quad -(3)$$

Eq (3) is a combination of the Newton law and Lorentz force law, and describes the effect of $\underline{E}_{vac}$ on an electron of mass m and charge $-e$. Eq. (3) is observed in the Lamb shift.

Using:

$$\underline{E}_{vac} \cdot \underline{\nabla} \times \underline{H} = -\underline{\nabla} \cdot (\underline{E}_{vac} \times \underline{H}) - \underline{H} \cdot \frac{\partial \underline{B}_{vac}}{\partial t} \quad -(4)$$

then:

$$-\int \underline{J} \cdot \underline{E}_{vac} \, d^3x = \frac{\partial U}{\partial t} + \underline{\nabla} \cdot \underline{S} \quad -(5)$$

where:

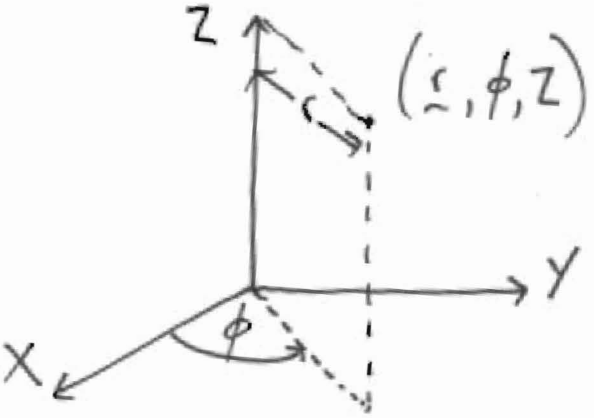
$$U = \frac{1}{2} \left(\underline{E}_{vac} \cdot \underline{D} + \underline{B}_{vac} \cdot \underline{H} \right) \quad -(6)$$

$$\underline{S} = \underline{E}_{vac} \times \underline{H} \quad -(7)$$

The total work done by the vacuum $\underline{E}_{vac}$ within the volume V is $\int_V \underline{J} \cdot \underline{E}_{vac}\, d^3x$ in watts or joules per second. This work is always done on a circuit, because $\underline{E}_{vac}$ is ubiquitous.

For example consider a straight wire of radius r_0 along the Z axis carrying a current I in amps or coulombs per second. Determine the power in watts entering the wire due to $\underline{E}_{vac}$. ("Electromagnetics" (REA, New York, 1993), problem 12.2.

Use the cylindrical polar coordinates:



$$x = r\cos\phi$$
$$y = r\sin\phi$$
$$Z = Z$$
$$\underline{e}_r = \underline{i}\cos\phi + \underline{j}\sin\phi$$
$$\underline{e}_\phi = -\underline{i}\sin\phi + \underline{j}\cos\phi$$
$$\underline{e}_Z = \underline{k}$$

The magnetic field strength at the surface of the wire is

$$\underline{H} = \frac{I}{2\pi r_0}\,\underline{e}_\phi \quad -(8)$$

The intrinsic electric field strength in the Z direction is:

$$\underline{E} = \frac{1}{\sigma}\,\underline{J} = \frac{1}{\sigma}\,J\,\underline{k} \quad -(9)$$

here $\underline{J}$ is the electric current density in amps per square metre or $Cs^{-1}m^{-2}$, and σ is the conductivity.

∴ The intrinsic Poynting vector at the surface of

3) the wire is:

$$\underline{S}(\text{intrinsic}) = \underline{E} \times \underline{H} \quad -(8)$$

$$= \frac{I^2}{2\pi^2 r_0^3 \sigma} \underline{k} \times \underline{e}_\phi = -\frac{I^2}{2\pi^2 r_0^3 \sigma} \underline{e}_r$$

The power in watts entering a unit length of wire is:

$$P = \oint \underline{S} \cdot d\underline{A} = \oint \underline{S} \cdot \underline{e}_r \, dA \quad -(9)$$

$$= \int \underline{\nabla} \cdot \underline{S} \, d^3x$$

Therefore: $$P(\text{intrinsic}) = \frac{2\pi r_0 I^2}{2\pi^2 r_0^3 \sigma} = \frac{I^2}{\pi r_0^2 \sigma} = I^2 R \quad -(10)$$

This means that the intrinsic electric field supplies the power that is lost as heat ($I^2 R$ watts).

Now repeat the calculation with the vacuum electric field $\underline{E}$ field. Without loss of generality consider:

$$\underline{E}_{vac} = \frac{I_{vac}}{\pi r_0^2} \underline{e}_z \quad -(11)$$

so: $$\underline{S}(\text{vacuum}) = \underline{E}_{vac} \times \underline{H} = -\frac{I_{vac}^2}{2\pi^2 r_0^3 \sigma} \underline{e}_r \quad -(12)$$ (Cs^{-1}),

where I_{vac} is the vacuum current in amps, and where:

$$J_{vac} = \frac{I_{vac}}{\pi r_0^2} \quad -(13)$$

:)

Therefore the vacuum power [in watts] entering a unit length of wire is:

$$\boxed{\begin{aligned} \underline{P}\,(\text{vacuum}) &= \oint \underline{S}_{vac} \cdot d\underline{A} \\ &= \underline{I}^2_{vac}\, R \end{aligned}} \quad -(14)$$

This is always present in the wire. The reason is that the vacuum current I_{vac} in amps or $J s^{-1}$ is always present.

Conclusion

The heat generated in a wire of radius r_0 ~~generated~~ aligned along Z should be slightly greater than the intrinsic heat $\underline{I}^2 R$ where I the intrinsic current in amps and R the resistance in ohms. The engineering problem consists of amplifying the effect of $\underline{E}_{vac}$ by circuit design.