69(2): Poynting Theorem and Deflection of Light by Gravitation.

The firm experimental evidence that gravitation and em may be mutually influential is the deflection of light by mass. In this note we make a new approach to this problem by constructing a new type of Poynting theorem from the Newton and Coulomb laws:

$$\nabla \cdot \mathbf{d} = \rho_m \quad -(1)$$

$$\nabla \cdot \mathbf{D} = \rho. \quad -(2)$$

First multiply eq. (2) by $\mathbf{E}$ in volt per metre or $JC^{-1}m^{-1}$:

$$\mathbf{E}(\nabla \cdot \mathbf{D}) = \rho \mathbf{E}. \quad -(3)$$

Secondly multiply by c:

$$\boxed{c\mathbf{E}(\nabla \cdot \mathbf{D}) = c\rho\mathbf{E}} \quad -(4)$$

Similarly:

$$\boxed{c\mathbf{g}(\nabla \cdot \mathbf{d}) = c\rho_m \mathbf{g}} \quad -(5)$$

Both sides of eqs. (4) and (5) have units of watts per cubic metre.

Units Check

$$c\rho\mathbf{E} = ms^{-1}Cm^{-3}JC^{-1}m^{-1} = Js^{-1}m^{-3}$$

$$\begin{aligned} c\rho_m \mathbf{g} &= ms^{-1}kgm^{-3}ms^{-2} = kgm^{-1}s^{-3} \\ &= kg\,m^2 s^{-2} s^{-1} m^{-3} \\ &= J s^{-1} m^{-3} = \text{watt}\, m^{-3} \end{aligned}$$

In the process of deflection of light by gravitation, the latter can be thought of as transferring a power per unit volume to the power per unit volume of the e/m field of the light beam. This process can be represented by:

$$c\underline{E}\,(\underline{\nabla}\cdot\underline{D}) = c\rho_m\,\underline{g} \quad -(6)$$

so

$$\boxed{\rho\,\underline{E} = \rho_m\,\underline{g}} \quad -(7)$$

In terms of field magnitudes:

$$\rho E = \rho_m g \quad -(8)$$

So the process of deflection of light by gravitation is described by:

$$\underline{\nabla}\cdot\underline{D} = \left(\frac{g}{E}\right)\rho_m \quad -(9)$$

and

$$\underline{\nabla}\cdot\underline{d} = \rho_m \quad -(10)$$

In eq. (9), assume:

$$\underline{D} = \epsilon_0\,\underline{E} \quad -(11)$$

then:

$$\boxed{\underline{\nabla}\cdot\underline{E} = \frac{\rho_m}{\epsilon_0}\,\frac{g}{E}} \quad -(12)$$

The electric field strength $\underline{E}$ of the light beam is deflected by the mass density ρ_m, as observed experimentally.

If it is assumed for simplicity that:

$$\underline{E} = E_z\,\underline{k} \quad -(13)$$

then:

$$\frac{\partial E_Z}{\partial Z} = \frac{\rho_m}{\epsilon_0}\frac{g}{E_Z} \quad -(14)$$

If the unit vector $\underline{k}$ is thought of as developing a Z dependence, i.e. if the spacetime is curved by ρ_m, then:

$$\boxed{\underline{\nabla} \cdot \underline{k}(Z) = \frac{\rho_m g}{\epsilon_0 E^2} = \frac{\rho_m g}{U}} \quad -(15)$$

here

$$U = \epsilon_0 E^2 \quad -(16)$$

, the energy per unit volume of the electromagnetic field. Its power density, I, in watts per square metre, is

$$I = cU \quad -(17)$$

If it is assumed as in UFT 167 that:

$$D_Z = \epsilon_0 g^{00} g_{ZZ} E_Z \quad -(18)$$

then in eq. (9):

$$\frac{\partial}{\partial Z}\left(\epsilon_0 g^{00} g_{ZZ} E_Z\right) = \left(\frac{g}{E_Z}\right)\rho_m \quad -(19)$$

Assume for simplicity that

$$g^{00} = 1 \quad -(20)$$

then:

$$\boxed{\frac{\partial g_{ZZ}}{\partial Z} = \frac{g\rho_m}{U}} \quad -(21)$$

or

$$\boxed{\frac{\partial \epsilon_Z}{\partial Z} = \frac{\epsilon_0 g\rho_m}{U}} \quad -(22)$$

4) where ϵ_Z is the permittivity defined by:

$$\epsilon_Z = \epsilon_0 \, g_{ZZ}. \quad -(23)$$

The final result is therefore:

$$\boxed{\frac{\partial \epsilon_Z}{\partial Z} = \frac{g \rho_m}{E_Z^2}} \quad -(24)$$

These equations are formally correct, but there is a need to relate them to the notical method of developing the interaction of gravitation and electromagnetism. In particular there is a need to introduce a formalism based on direct interaction of the $\underline{E}$ of an electromagnetic field with the mass density:

$$\underline{E} \, (\underline{\nabla} \cdot \underline{d}) = \underline{E} \rho_m \quad -(25)$$