59(1): Application of the Poynting Theorem

Application of the electromagnetic and gravitational Poynting theorems:

Consider the electromagnetic and gravitational Poynting theorems:

$$\underline{E} \cdot \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) = \underline{J} \cdot \underline{E} \quad -(1)$$

$$\underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} \right) = \underline{J}_m \cdot \underline{g} \quad -(2)$$

Here:

$\underline{J} \cdot \underline{E}$ = total work done by the e/m field on a source within a given volume, in watts per cubic metre.

$\underline{J}_m \cdot \underline{g}$ = total work done by the gravitational field on a source within a given volume, in watts per cubic metre.

If both the gravitational and e/m fields are present at the same time then:

$$\underline{J} \cdot \underline{E} + \underline{J}_m \cdot \underline{g} = \underline{E} \cdot \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) + \underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} \right) \quad (3)$$

where the LHS is the total work done by both fields. The total work done is a conversion of energy. If the total work done is converted completely into gravitational energy (per unit time), then

$$\underline{J} \cdot \underline{E} + \underline{J}_m \cdot \underline{g} = \underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} \right) \quad -(4)$$

If the total electromagnetic work done, $\underline{J} \cdot \underline{E}$, is converted completely into gravitational energy per unit time then:

$$\underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} \right) = \underline{J} \cdot \underline{E} \quad -(5)$$

In order to find the effect of $\underline{J} \cdot \underline{E}$ on $\underline{g}$, eq. (5) is more useful than the usual form of the Poynting

2) theorem:

$$\frac{\partial U_{grav}}{\partial t} + \underline{\nabla} \cdot \underline{S}_{grav} = - \underline{J} \cdot \underline{E} \quad -(6)$$

where

$$U_{grav} = \frac{1}{2}\left(\underline{g} \cdot \underline{d} + \underline{b} \cdot \underline{h}\right) \quad -(7)$$

$$\underline{S}_{grav} = \underline{g} \times \underline{h}. \quad -(8)$$

Eq. (6) means that the time rate of change of gravitational energy within a given volume ($\partial U_{grav} / \partial t$) plus the energy flowing out through the boundary surface of the volume per unit time ($\underline{\nabla} \cdot \underline{S}_{grav}$) is equal to the ~~work done~~ negative of the work done by the electromagnetic field on the sources within the volume. Therefore the work done per unit time per unit volume by the e/m field $\underline{J} \cdot \underline{E}$ is converted into gravitational form (LHS of eq. (6)). Then eq. (6) is a statement of conservation of energy of e/m plus gravitational fields. <u>The total energy is conserved</u>.

The conservation of total energy is made clear by denoting:

$$\int_V \underline{J} \cdot \underline{E}\, d^3x = \frac{dE_{em}}{dt} \quad -(9)$$

$$\int_V \frac{\partial U_{grav}}{\partial t} d^3x = \frac{dE_{grav}}{dt} \quad -(10)$$

so

$$\frac{d}{dt}\left(E_{grav} + E_{em}\right) = - \int_V \underline{\nabla} \cdot \underline{S}_{grav} \quad -(11)$$

∴ Eq. (11) may be re-expressed as:

$$\frac{d}{dt}\left(E_{grav} + E_{em}\right) = -\oint_A \underline{n} \cdot \underline{S}_{grav} \, dA \quad -(12)$$

using the Stokes theorem.

Using eq. (8):

$$\frac{d}{dt}\left(E_{grav} + E_{em}\right) = -\oint \underline{n} \cdot \underline{g} \times \underline{h} \, dA \quad -(13)$$

If the gravitomagnetic field strength $\underline{h}$ is zero, or if:

$$\underline{g} \times \underline{h} = \underline{0} \quad -(14)$$

then

$$\frac{d}{dt}\left(E_{grav} + E_{em}\right) = 0 \quad -(15)$$

and the total energy $E_{grav} + E_{em}$ does not change with time.

In UFT 168 the result (15) was expressed as:

$$\boxed{\underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t}\right) = \underline{E} \cdot \underline{J}} \quad -(16)$$

If:

$$\underline{\nabla} \times \underline{h} = \underline{0} \quad -(17)$$

and

$$\underline{d} = \frac{1}{c^2 k} \underline{g} \quad -(18)$$

then

$$\boxed{\underline{g} \cdot \frac{\partial \underline{g}}{\partial t} = -c^2 k \, \underline{E} \cdot \underline{J}} \quad -(19)$$

4) Eq. (16) means that the electromagnetic power per cubic metre, $\underline{E} \cdot \underline{J}$, in $J s^{-1} m^{-3}$ or watts m^{-3}, has been transformed into the gravitational power per cubic metre:

$$\boxed{\frac{P}{V}(\text{grav}) = \underline{g} \cdot \left(\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} \right)} \quad -(20)$$

If eq. (17) is assumed for simplicity, then:

$$\boxed{\begin{aligned} \frac{P}{V}(\text{grav}) &= -\frac{1}{c^2 k} \underline{g} \cdot \frac{\partial \underline{g}}{\partial t} \\ &= -8\pi G \, \underline{g} \cdot \frac{\partial \underline{g}}{\partial t} \end{aligned}} \quad -(21)$$

Thus the process is summarized by:

$$\frac{P}{V}(\text{em}) \rightarrow \frac{P}{V}(\text{grav.}) \quad -(22)$$

This is a completely general result of thermodynamics which leads to eq. (40) of ~~note~~ UFT 168:

$$\boxed{\frac{\partial g_Z}{\partial t} = -\left(\frac{c^2 k}{g_Z} \right) E_Z J_Z} \quad -(23)$$

which means that the time rate of change of the acceleration due to gravity can be changed by the electromagnetic power per unit volume $E_Z J_Z$.

Units check

$\underline{E} \cdot \underline{J} = \underline{g} \cdot \underline{J}_m =$ ~~$kg\, m^{-1} s^{-3}$~~ $= J s^{-1} m^{-3} =$ watts m^{-3}

$\underline{J}_m = kg\, s^{-1} m^{-2}$, $\underline{g} = m s^{-2}$, $\underline{h} = kg\, m^{-1} s^{-1}$,

$\underline{J} = C s^{-1} m^{-2}$,

$\underline{d} = kg\, m\, m^{-3}$, $k = m\, kg\, m^{-1}$