1) 169(3): Interaction of Free Space Electromagnetism with Gravitation.

The free space electromagnetic field is:

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{0} \quad -(1)$$

where:

$$\underline{H} = \frac{1}{\mu_0} \underline{B}, \quad \underline{D} = \epsilon_0 \underline{E}. \quad -(2)$$

Therefore the free space Poynting theorem is:

$$\underline{E} \cdot \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) = \underline{0} \quad -(3)$$

i.e.

$$\frac{\partial U}{\partial t} + \underline{\nabla} \cdot \underline{S} = 0 \quad -(4)$$

where

$$U = \frac{1}{2} \left(\underline{E} \cdot \underline{D} + \underline{B} \cdot \underline{H} \right) \quad -(5)$$

$$\underline{S} = \underline{E} \times \underline{H} \quad -(6)$$

Eq. (4) is

$$\boxed{\partial_\mu p^\mu = 0} \quad -(7)$$

where

$$p^\mu = \left(\frac{En}{c}, \underline{p} \right) \quad -(8)$$

and

$$En = \int U \, d^3x, \quad \underline{p} = \frac{1}{c^2} \int \underline{S} \, d^3x \quad -(9)$$

Here p^μ is the energy-momentum four vector of the electromagnetic field.

When the e/m field interacts with gravitation

2) eq. (3) is changed to:

$$\underline{E} \cdot \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) = \underline{g} \cdot \underline{J}_m \quad -(10)$$

i.e.

$$\frac{\partial u}{\partial t} + \underline{\nabla} \cdot \underline{S} = - \underline{g} \cdot \underline{J}_m \quad -(11)$$

or

$$\boxed{\partial_\mu p^\mu = - \frac{1}{c^2} \int \underline{g} \cdot \underline{J}_m \, d^3x} \quad -(12)$$

where the gravitational current density $\underline{J}_m$ is defined by:

$$\underline{\nabla} \times \underline{h} - \frac{\partial \underline{d}}{\partial t} = \underline{J}_m \quad -(13)$$

Introducing the Planck Einstein hypothesis:

$$p^\mu = \hbar \kappa^\mu \quad -(14)$$

then

$$\partial_\mu \kappa^\mu = - \frac{1}{\hbar c^2} \int \underline{g} \cdot \underline{J}_m \, d^3x \quad -(15)$$

where

$$\kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa} \right) \quad -(16)$$

Eq. (15) shows that the free space relation:

$$\partial_\mu \kappa^\mu = 0 \quad -(17)$$

is changed by gravitation.

In situation where

$$\underline{\nabla} \times \underline{h} = \underline{0} \quad -(18)$$

3) then from eq. (13):

$$\underline{J}_m = -\frac{\partial d}{\partial t} = -\frac{1}{c^2 k}\frac{\partial g}{\partial t} \quad -(19)$$

So:

$$\boxed{\partial_\mu \kappa^\mu = \frac{1}{c^4 \hbar k}\int \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}\, d^3x} \quad -(20)$$

Units check

RHS of eq. (20) is:

$$(m s^{-2} m s^{-3} m^3 kgm) / (m^4 s^{-4} J s m)$$
$$= kgm / (J s^2) = kgm / (kgm\, m^2 s^{-2} s^2) = m^{-2}$$

If the quantum hypothesis is not used then:

$$\partial_\mu p^\mu = \frac{1}{c^4 k}\int \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}\, d^3x \quad -(21)$$

where

$$k = 1.86595 \times 10^{-26}\ m\, kg^{-1} \quad -(22)$$
$$c = 2.997925 \times 10^{8}\ m s^{-1} \quad -(23)$$

So

$$\boxed{\partial_\mu p^\mu = 6.63463 \times 10^{-9}\int \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}\, d^3x} \quad -(24)$$

In vector format, eq. (24) is:

$$\frac{1}{c}\frac{\partial E_n}{\partial t} + \underline{\nabla} \cdot \underline{p} = 6.63463 \times 10^{-9} A$$

where

$$A = \int \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}\, d^3x \quad -(25)$$

4) where E_n = electromagnetic energy, $\underline{p}$ = e/m momentum. In terms of energy and momentum densities:

$$\boxed{\frac{1}{c}\frac{\partial U}{\partial t} + \underline{\nabla}\cdot\underline{\Pi} = 6.63463\times10^{-9}\,\underline{g}\cdot\frac{\partial \underline{g}}{\partial t}} \quad -(26)$$

where

$$E_n = \int u\, d^3x, \quad \underline{p} = \int \underline{\Pi}\, d^3x \quad -(27)$$

It is seen from eq. (27) that the momentum density $\underset{\sim}{\Pi}$ develops a dependence on distance.

From the metric theory of UFT 150 and 155 and similar:

$$p = m\frac{dr}{dt} = mcb\left(\left(1-\frac{r_0}{r}\right)\left(\frac{1}{b^2}-\left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)\right)^{1/2} \quad -(28)$$

so

$$\boxed{\frac{dp}{dr} \neq 0} \quad -(29)$$

Finally if it is assumed that in eq. (25):

$$\partial E_n / \partial t = 0 \quad -(30)$$

then, in the radial direction:

$$\boxed{\begin{aligned}\frac{\partial p}{\partial r} &= \frac{1}{c^4 k}\int \underline{g}\cdot\frac{\partial \underline{g}}{\partial t}\, d^3x \\ &= mcb\left(\left(1-\frac{r_0}{r}\right)\left(\frac{1}{b^2}-\left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)\right)^{1/2}\end{aligned}} \quad -(31)$$

5) so the rate of change $\partial \underline{g} / \partial t$ can be found, together with the gravitational current $\underline{J_m}$.

Technological Applications

This argument may be reversed for tech. applications, where we wish to find the effect of the e/m field on the gravitational field $\underline{g}$. The answer is given in principle by eq. (26). In situations where eq. (30) applies, then:

$$\boxed{\underline{\nabla} \cdot \underline{\Pi} = \frac{1}{c^4 k} \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}} \quad -(32)$$

and the Poynting vector associated with $\underset{\sim}{\Pi}$ is

$$\underset{\sim}{\pi} = \frac{1}{c^2} \underline{E} \times \underline{H} \quad -(33)$$

so

$$\boxed{\underline{\nabla} \cdot \underline{E} \times \underline{H} = \frac{1}{c^2 k} \underline{g} \cdot \frac{\partial \underline{g}}{\partial t}} \quad -(34)$$

Units Check

$$LHS = J\, m^{-3} s^{-1} = kgm\, m^{-1} s^{-3}$$

$$RHS = kgm\, m^{2} s^{-5} m^{-3} s^{2} = kgm\, m^{-1} s^{-3} \checkmark$$