168(1): Definition of Units

The basic geometrical structures of ECE 167 are:

$$\partial_\mu T^{a\mu\nu} = j_I^{a\nu} = R^{a\ \mu\nu}_{\mu} - \omega^a_{\mu b} T^{b\mu\nu} \quad -(1)$$

and

$$\partial_\mu \tilde{T}^{a\mu\nu} = j_H^{a\nu} = \tilde{R}^{a\ \mu\nu}_{\mu} - \omega^a_{\mu b} \tilde{T}^{b\mu\nu} \quad -(2)$$

Define:

$$G^{a\mu\nu} = \epsilon_0 A^{(0)} g^{\mu\rho} g^{\nu\sigma} T^a_{\rho\sigma} \quad -(3)$$
$$= \epsilon_0 g^{\mu\rho} g^{\nu\sigma} F^a_{\rho\sigma}$$

Then:

$$\boxed{\partial_\mu G^{a\mu\nu} = J^{a\nu}} \quad -(4)$$

where:

$$J^{a\nu} = \epsilon_0 c A^{(0)} j^{a\nu} \quad -(5)$$

Units

$$\epsilon_0 = J^{-1} C^2 m^{-1}, \quad A^{(0)} = J s C^{-1} m^{-1}$$
$$j^{a\nu} = m^{-2}$$

so

$$J^{a\nu} = J^{-1} C^2 m^{-1} m s^{-1} J s C^{-1} m^{-1} m^{-2}$$
$$= C m^{-3}$$

These are the units of charge density $\rho$. So:

$$\boxed{J^{a\nu} = \left(\rho^a, \frac{\underline{J}^a}{c}\right)} \quad -(6)$$

For each $a$:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \underline{D} &= \rho \\ \underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} &= \underline{J} \end{aligned}} \quad -(7)$$

In eq. (7):

2) $\underline{D} = C m^{-2}$, $\underline{H} = A m^{-1} = C s^{-1} m^{-1}$,

$\underline{\nabla} \times \underline{H} = C s^{-1} m^{-2}$; $\partial \underline{D} / \partial t = C s^{-1} m^{-2}$; $\underline{J} = C s^{-1} m^{-2}$

Therefore:

$$\boxed{J^{a\nu} = \epsilon_0 c A^{(0)} \left( R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \right)} \quad -(8)$$

and

$$G^{\mu\nu} = \begin{bmatrix} 0 & -D_x & -D_y & -D_z \\ D_x & 0 & -H_z/c & H_y/c \\ D_y & H_z/c & 0 & -H_x/c \\ D_z & -H_y/c & H_x/c & 0 \end{bmatrix} \quad -(9)$$

with metric:

$$g^{\mu\nu} = \begin{bmatrix} g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & -g^{22} & 0 \\ 0 & 0 & 0 & -g^{33} \end{bmatrix} \quad -(10)$$

The homogeneous field equation is:

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0 \quad -(11)$$

with metric

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(12)$$

made up of plane wave tetrads. So, for each $a$:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(13)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0}. \quad -(14)$$

So the metrical structures of ~~the~~ field equations

4) and (11) are different.