1) 167(5): New Resonance Structures.

Start with consideration of the torsion tensor:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(1)$$

and antisymmetry law:

$$0 = \partial_\mu q^a_\nu + \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu + \omega^a_{\nu b} q^b_\mu . \quad -(2)$$

Adding (1) and (2):

$$T^a_{\mu\nu} = 2\left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu\right) \quad -(3).$$

In vector notation:

$$\underline{E}^a = -c\underline{\nabla} A^a_0 - \frac{\partial \underline{A}^a}{\partial t} - c\,\omega^a_{0b}\underline{A}^b + c A^b_0 \underline{\omega}^a_b \quad -(4)$$

$$= -2\left(c\underline{\nabla} A^a_0 + c\,\omega^a_{0b}\underline{A}^b\right)$$

In general:

$$-c\underline{\nabla} A^a_0 - c\,\omega^a_{0b}\underline{A}^b = -\frac{\partial \underline{A}^a}{\partial t} + c A^b_0 \underline{\omega}^a_b \quad -(5)$$

If there are no vector potentials present:

$$-c\underline{\nabla} A^a_0 = c A^b_0 \underline{\omega}^a_b \quad -(6)$$

and

$$\underline{E}^a = -c\underline{\nabla} A^a_0 + c A^b_0 \underline{\omega}^a_b \quad -(7)$$

Denote for each a:

$$\underline{E} = -\underline{\nabla}\phi + \phi^b \underline{\omega}_b \quad -(8)$$

where

$$\phi := c A_0 . \quad -(9)$$

Then:

So:

$$\underline{E}^a = -\underline{\nabla}\phi^a + \phi^b \underline{\omega}^a{}_b \quad -(10)$$

i.e. $\underline{E}^a = -\underline{\nabla}\phi^a + \phi^0\underline{\omega}^a{}_0 + \phi^1\underline{\omega}^a{}_1 + \phi^2\underline{\omega}^a{}_2 + \phi^3\underline{\omega}^a{}_3$

$$-(11)$$

The polarization indices a are spacelike. Label them 1, 2, 3. Then for example:

$$\underline{E}^1 = -\underline{\nabla}\phi^1 + \phi^0\underline{\omega}^1{}_0 + \phi^1\underline{\omega}^1{}_1 + \phi^2\underline{\omega}^1{}_2 + \phi^3\underline{\omega}^1{}_3$$

$$-(12)$$

In this equation, $\underline{E}^1$ can be associated only with ϕ^1,

so:

$$\underline{E}^1 = -\underline{\nabla}\phi^1 + \underline{\omega}^1{}_1\,\phi^1 \quad -(13)$$

Similarly:

$$\underline{E}^2 = -\underline{\nabla}\phi^2 + \underline{\omega}^2{}_2\,\phi^2 \quad -(14)$$

$$\underline{E}^3 = -\underline{\nabla}\phi^3 + \underline{\omega}^3{}_3\,\phi^3 \quad -(15)$$

By antisymmetry:

$$-\underline{\nabla}\phi^1 = \underline{\omega}^1{}_1\,\phi^1 \quad -(16)$$

$$-\underline{\nabla}\phi^2 = \underline{\omega}^2{}_2\,\phi^2 \quad -(17)$$

$$-\underline{\nabla}\phi^3 = \underline{\omega}^3{}_3\,\phi^3 \quad -(18)$$

Therefore if ϕ^1, ϕ^2 and ϕ^3 are positive-valued, $\underline{\omega}^1{}_1$, $\underline{\omega}^2{}_2$ and $\underline{\omega}^3{}_3$ are negative valued.

The displacements are defined by:

$$D^1 = \epsilon_0\, g^{00} g^{11} E^1 \quad -(19)$$

$$D^2 = \epsilon_0\, g^{00} g^{22} E^2 \quad -(20)$$

$$D^3 = \epsilon_0\, g^{00} g^{33} E^3 \quad -(21)$$

3) and

$$\partial_1 D^1 + \partial_2 D^2 + \partial_3 D^3 = \rho \quad -(22)$$

Any coordinate system can be introduced at this point. If attention is restricted to D^3 for simplicity:

$$\partial_3 D^3 = \rho \quad -(23)$$

Denote:

$$\underline{\omega}^3{}_3 = -\omega \underline{e}^3 \quad -(24)$$

If the Cartesian coordinate system is used:

$$\underline{\omega}^3{}_3 = -\omega \underline{k} \quad -(25)$$

and

$$\frac{\partial D_Z}{\partial Z} = \rho \quad -(26)$$

where

$$D_Z = \epsilon_0 g^{00} g_{ZZ} E_Z \quad -(27)$$

$$E_Z = -\frac{\partial \phi_Z}{\partial Z} - \omega \phi_Z . \quad -(28)$$

Therefore

$$\boxed{\frac{\partial^2}{\partial Z^2}\left(g^{00} g_{ZZ} \phi_Z\right) + \frac{\partial}{\partial Z}\left(g^{00} g_{ZZ} \phi_Z \omega\right) = -\frac{\rho}{\epsilon_0}}$$

$$-(29)$$

This is a resonance structure:

$$\frac{\partial^2}{\partial Z^2}\left(g^{00} g_{ZZ} \phi_Z\right) + \frac{\partial}{\partial Z}\left(g^{00} g_{ZZ} \omega\right) \phi_Z + g^{00} g_{ZZ} \omega \frac{\partial \phi_Z}{\partial Z} = -\rho / \epsilon_0 \quad -(30)$$

4) If g^{00} and g_{ZZ} are assumed to be independent of Z, then:

$$\boxed{\frac{\partial^2 \phi_Z}{\partial Z^2} + \left(\frac{\partial \omega}{\partial Z}\right)\phi_Z + \omega \frac{\partial \phi_Z}{\partial Z} = \frac{-\rho}{\epsilon_0 g^{00} g_{ZZ}} \qquad -(31)}$$

This is a damped Euler-Bernoulli resonance equation. The metric elements are incorporated into the driving term's denominator.

As $\omega \rightarrow 0$ $\quad -(32)$

the Poisson equation is recovered in the format:

$$\frac{\partial^2 \phi_Z}{\partial Z^2} = \frac{-\rho}{\epsilon_0 g^{00} g_{ZZ}} \qquad -(33)$$

In general:

$$D^3 = -\epsilon_0 g^{00} g^{33} (\partial_3 + \omega) \phi^3 \qquad -(34)$$

$$\partial_3 D^3 = \rho \qquad -(35)$$

These equations also apply to gravitational theory and modify the Newton law.