167(1): The Geometrical Origin of $\underline{H}$, $\underline{m}$, $\underline{D}$ and $\underline{P}$.

Consider the fundamental definition of torsion and curvature:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\kappa\mu\nu} V^\kappa - T^\kappa{}_{\mu\nu} D_\kappa V^\rho \quad -(1)$$

in previously defined notation. In eq. (1) the torsion tensor is:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} \quad -(2)$$

and its Hodge dual is:

$$\tilde{T}^\kappa{}_{\mu\nu} = \tilde{\Gamma}^\kappa{}_{\mu\nu} - \tilde{\Gamma}^\kappa{}_{\nu\mu} \quad -(3)$$

$$= \frac{1}{2} \| g \|^{1/2} \epsilon_{\mu\nu}{}^{\alpha\beta} T_{\alpha\beta}{}^{\kappa}$$

where

$$\epsilon_{\mu\nu}{}^{\alpha\beta} = g^{\alpha\rho} g^{\beta\sigma} \epsilon_{\mu\nu\rho\sigma} \quad -(4)$$

Denote:

$$\Lambda^\kappa{}_{\mu\nu} = \tilde{\Gamma}^\kappa{}_{\mu\nu}. \quad -(5)$$

The covariant derivative defined by $\Lambda^\kappa{}_{\mu\nu}$ is:

$$D_\mu V^\rho = \partial_\mu V^\rho + \Lambda^\rho{}_{\mu\lambda} V^\lambda. \quad -(6)$$

The spin connection is defined by:

$$D_\mu V^a = \partial_\mu V^a + \omega^a{}_{\mu b} V^b. \quad -(7)$$

Eqs. (6) and (7) imply the tetrad postulate:

$$\partial_\mu q^a{}_\nu = \Lambda^a{}_{\mu\nu} - \omega^a{}_{\mu\nu}. \quad -(8)$$

Using eq. (6) in the commutator $[D_\mu, D_\nu]$ acting on V^ρ produces:

$$[D_\mu, D_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_{\kappa\mu\nu} V^\kappa - \tilde{T}^\kappa{}_{\mu\nu} D_\kappa V^\rho \quad -(9)$$

2) Eq. (3) implies:

$$\partial_\mu \alpha^a_\nu + \omega^a_{\mu b}\alpha^b_\nu = \frac{1}{2}\|g\|^{1/2} g^{\alpha\rho} g^{\beta\sigma} \epsilon_{\mu\nu\rho\sigma}\left(\partial_\alpha A^a_\beta + \omega^a_{\alpha b} A^b_\beta\right) \quad -(10)$$

for example:

$$\partial_0 \alpha^a_1 + \omega^a_{0b}\alpha^b_1 = \|g\|^{1/2} g^{22} g^{33}\left(\partial_2 A^a_3 + \omega^a_{2b} A^b_3\right) \quad -(11)$$

Therefore:

$$\tilde{F}^a_{\mu\nu} = \partial_\mu \alpha^a_\nu - \partial_\nu \alpha^a_\mu + \omega^a_{\mu b}\alpha^b_\nu - \omega^a_{\nu b}\alpha^b_\mu \quad -(12)$$

and

$$\left(D \wedge \tilde{F}^a\right) = \alpha^{0}\left(\tilde{R} \wedge q\right) \quad -(13)$$

where

$$\alpha^a_\mu = \alpha^{(0)} q^a_\mu . \quad -(14)$$

In vector notation:

$$\underline{E}^a = \frac{1}{\epsilon_0}\left(\underline{D}^a - \underline{P}^a\right) = -c\underline{\nabla}\alpha^a_0 - \frac{\partial \underline{\alpha}^a}{\partial t} - c\,\omega^a_{0b}\underline{\alpha}^b + c\,\alpha^b_0 \underline{\omega}^a_b \quad -(15)$$

$$\underline{B}^a = \mu_0\left(\underline{H}^a + \underline{M}^a\right) = \underline{\nabla} \times \underline{\alpha}^a - \underline{\omega}^a{}_b \times \underline{\alpha}^b \quad -(16)$$

with

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(17)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(18)$$