1) 167(6): Standard Definitions of Field Tensors

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & B_3 & -B_2 \\ E_2/c & -B_3 & 0 & B_1 \\ E_3/c & B_2 & -B_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^1/c & -E^2/c & -E^3/c \\ E^1/c & 0 & -B^3 & B^2 \\ E^2/c & B^3 & 0 & -B^1 \\ E^3/c & -B^2 & B^1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & -B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -B^1 & -B^2 & -B^3 \\ B^1 & 0 & E^3/c & -E^2/c \\ B^2 & -E^3/c & 0 & E^1/c \\ B^3 & E^2/c & -E^1/c & 0 \end{bmatrix} = \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{bmatrix}$$

$$\tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3/c & E_2/c \\ B_2 & E_3/c & 0 & -E_1/c \\ B_3 & -E_2/c & E_1/c & 0 \end{bmatrix} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z/c & -E_y/c \\ -B_y & -E_z/c & 0 & E_x/c \\ -B_z & E_y/c & -E_x/c & 0 \end{bmatrix}$$

Here

$$F^{\mu\nu} = g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \quad -(1)$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \quad -(2)$$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \mathrm{diag}(1,-1,-1,-1) \quad -(3)$$

$$\left.\begin{aligned} \epsilon^{0123} &= -\epsilon^{1230} = \epsilon^{2301} = -\epsilon^{3012} = 1 \\ \epsilon^{1023} &= -\epsilon^{2130} = \epsilon^{3201} = -\epsilon^{0312} = -1 \\ \epsilon^{1032} &= -\epsilon^{2103} = \epsilon^{3210} = -\epsilon^{0321} = 1 \\ \epsilon^{1302} &= -\epsilon^{2013} = \epsilon^{3120} = -\epsilon^{0231} = -1 \end{aligned}\right\} -(4)$$

2) Define:

$$G^{\mu\nu} = \epsilon_0 g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \quad -(5)$$

then:

$$G^{\mu\nu} = \begin{bmatrix} 0 & -D^1 & -D^2 & -D^3 \\ D^1 & 0 & -H^3/c & H^2/c \\ D^2 & H^3/c & 0 & -H^1/c \\ D^3 & -H^2/c & H^1/c & 0 \end{bmatrix} = \begin{bmatrix} 0 & -D_x & -D_y & -D_x \\ D_x & 0 & -H_z/c & H_y/c \\ D_y & H_z/c & 0 & -H_x/c \\ D_z & -H_y/c & H_x/c & 0 \end{bmatrix} \quad -(6)$$

and

$$\left.\begin{aligned} D_x &= \epsilon_0 g^{00} g^{11} E_x \\ D_y &= \epsilon_0 g^{00} g^{22} E_y \\ D_z &= \epsilon_0 g^{00} g^{33} E_z \\ H_x &= \frac{1}{\mu_0} g^{22} g^{33} B_x \\ H_y &= \frac{1}{\mu_0} g^{11} g^{33} B_y \\ H_z &= \frac{1}{\mu_0} g^{22} g^{11} B_z \end{aligned}\right] \quad -(7)$$

Metric:

$$g^{\mu\nu} = \begin{bmatrix} g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & -g^{22} & 0 \\ 0 & 0 & 0 & -g^{33} \end{bmatrix} \quad -(8)$$

Field Equations

$$\left.\begin{aligned} \underline{\nabla} \cdot \underline{D} &= \rho \\ \underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} &= \underline{J} \end{aligned}\right] \quad -(9)$$