163(8) : Development of R for Elastic Scattering

The covariant mass ratio is defined as:

$$m = \gamma m_0 = \hbar\omega / c^2 \quad -(1)$$

So

$$R = \left(\frac{mc}{\hbar}\right)^2 = \left(\frac{\omega}{c}\right)^2 = \gamma^2\left(\frac{m_0 c}{\hbar}\right)^2 \quad -(2)$$

where

$$(\Box + R) q^a_\mu = 0 \quad -(3)$$

and

$$R = q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right). \quad -(4)$$

Therefore, if we consider only $q^{(3)}_3$ for simplicity:

$$q^{(3)}_3 = \exp(\pm i\omega t) \quad -(5)$$

$$\partial^\mu\left(\omega^{(3)}_{\mu 3} - \Gamma^{(3)}_{\mu 3}\right) = \frac{\omega^2}{c^2} \exp(\mp i\omega t) \quad -(6)$$

The particle of mass $m$ is governed by:

$$\omega^2 = \kappa^2 c^2 + \frac{m_0^2 c^4}{\hbar^2} \quad -(7)$$

$$= \kappa^2 c^2 + \frac{\omega^2}{\gamma^2}. \quad -(8)$$

So

$$\omega^2\left(1 - \frac{1}{\gamma^2}\right) = \kappa^2 c^2 \quad -(9)$$

$$= \frac{\omega^2 v^2}{c^2}$$

So

$$\kappa = \frac{\omega v}{c^2}, \quad \text{Q.E.D.} \quad -(10)$$

2) Therefore if the covariant mass ratio is:

$$\frac{m}{m_0} = \gamma. \quad -(11)$$

the correct relation is obtained between $\kappa$, $\omega$ and $v$, eq. (10). This gives the tetrad component (5) and an equation for the spin connection minus the gamma connection, eq. (6).

The scattering process is described by eqns. (1) to (3) of note 163(7), in which:

$$\gamma'' = 1. \quad -(12)$$

This is because $m_2$ does not move in the elastic scattering. Therefore the covariant mass ratio for $m_2$ is always one.