163(1): General Compton Type Theory.

In a Compton type theory the photon is massless and there is exchange of energy and momentum between a photon and electron.

In general:

$$\hbar(\omega-\omega') = E_2 - E_1 \quad -(1)$$
$$\underbrace{\hbar(\underline{\kappa}-\underline{\kappa}')}_{\text{photon}} = \underbrace{\underline{p}_2 - \underline{p}_1}_{\text{electron}} \quad -(2)$$

The initial energy and momentum of the electron are E_1 and $\underline{p}_1$ respectively. The final energy and momentum of the electron are E_2 and $\underline{p}_2$ respectively. The initial angular frequency and wave vector of the photon are ω_1 and $\underline{\kappa}_1$ respectively, the final angular frequency and wavenumber of the photon are ω_2 and $\underline{\kappa}_2$ respectively.

UFT 158 to 162 revealed severe self inconsistencies in Compton theory when the photon was treated as massive. UFT 162 showed that these inconsistencies occurred in the theory of absorption and Raman scattering even with a massless photon. Therefore it is important to determine when scattering theory starts to become inconsistent. For this purpose the general theory of Compton scattering is developed in UFT 163. Denote:

$$\underline{\Pi} = \underline{p}_2 - \underline{p}_1 . \quad -(3)$$

The de Broglie momentum postulate (wave particle duality) means that:

$$\underline{p}_1 = \hbar \underline{\kappa}_1 \quad - (4)$$
$$\underline{p}_2 = \hbar \underline{\kappa}_2 \quad - (5)$$

so:
$$\pi^2 = \hbar^2 \left(\kappa_1^2 + \kappa_2^2 - 2\kappa_1\kappa_2 \cos\theta' \right) \quad - (6)$$
where θ' is the angle between $\underline{p}_1$ and $\underline{p}_2$ of the electron.

If the photon is assumed massless, then:
$$\pi^2 = \hbar^2 \left(\kappa^2 + \kappa'^2 - 2\kappa\kappa' \cos\theta \right) \quad - (7)$$
where θ is the angle between $\underline{\kappa}$ and $\underline{\kappa}'$ of the photon. Eq. (7) is:
$$\pi^2 = \left(\frac{\hbar}{c}\right)^2 \left(\omega^2 + \omega'^2 - 2\omega\omega' \cos\theta \right) \quad - (8)$$
because for a massless photon:
$$\omega = \kappa c, \quad \omega' = \kappa' c. \quad - (9)$$

Denote
$$E = E_2 - E_1 \quad - (10)$$
then
$$E^2 = \hbar^2 \left(\omega^2 + \omega'^2 - 2\omega\omega' \right) \quad - (11)$$
so
$$\boxed{\underbrace{c^2\pi^2 - E^2}_{\text{electron}} = \underbrace{2\hbar^2\omega\omega'(1 - \cos\theta)}_{\text{photon}}} \quad - (12)$$

<u>Eq. (12)</u> <u>can be used</u> as <u>an experimental</u> <u>test</u>.

') The de Broglie postulate for the electron mean:

$$E_1 = \hbar\omega_1 \quad , \quad E_2 = \hbar\omega_2 \quad - (13)$$

$$\underline{p}_1 = \hbar\underline{\kappa}_1 \quad , \quad \underline{p}_2 = \hbar\underline{\kappa}_2 . \quad - (14)$$

In fact the postulates (13) and (14) are made for any particle of mass m. The energies and momenta are related by the Einstein energy equations:

$$E_1^2 = c^2 p_1^2 + m^2 c^4 \quad - (15)$$

$$E_2^2 = c^2 p_2^2 + m^2 c^4 \quad - (16)$$

with

$$\pi^2 = p_1^2 + p_2^2 - 2 p_1 p_2 \cos\theta' \quad - (17)$$

Define

$$E_0 = mc^2 , \quad - (18)$$

then:

$$p_1^2 + p_2^2 = \frac{1}{c^2}\left(E_1^2 + E_2^2 - 2E_0^2\right) . \quad - (19)$$

lerefore

$$c^2\pi^2 - E^2 = 2\left(E_1 E_2 - E_0^2 - c^2 p_1 p_2 \cos\theta'\right) \quad - (20)$$

d:

$$\boxed{\underbrace{E_1 E_2 - E_0^2 - c^2 p_1 p_2 \cos\theta'}_{\text{electron}} = \underbrace{\hbar^2\omega\omega'\left(1 - \cos\theta\right)}_{\text{photon}}} \quad - (21)$$

ie:

$$p_1 = \frac{1}{c}\left(E_1^2 - E_0^2\right)^{1/2} , \quad - (22)$$

$$p_2 = \frac{1}{c}\left(E_2^2 - E_0^2\right)^{1/2} . \quad - (23)$$

4) If the electron or any particle of mass m is initially stationary, then:

$$\hbar\omega_1 = E_1 = mc^2, \qquad -(24)$$

$$\hbar\omega_2 = E_2 = \hbar(\omega - \omega') + mc^2 \qquad -(25)$$

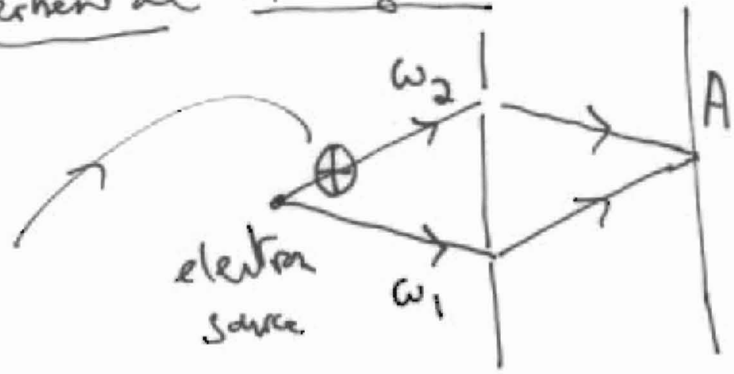
and eq. (21) reduces to the standard Compton formula:

$$\omega - \omega' = \frac{\hbar}{mc^2}\,\omega\omega'(1-\cos\theta) \qquad -(26)$$

Q.E.D.

Experimental Arrangement

γ Rays (beam of angular frequency ω



Interferogram

Fig (1)

The γ Ray beam collides with the electron beam in one arm of a Young interferometer, changing the electron frequency from ω_1 to ω_2. This sets up an interferogram at point A. The frequency ω_2 can be found from the interferogram given the initial frequency ω_1 of the electron beam. The scattering angle θ is known from a Compton experiment. The data can be tested against eqns (21) and (25).