163(3): Theory of Diffraction and Electron Microscopy.

The usual theory of electron microscopy is based on a simplistic equation, which is the non-relativistic limit is:

$$E = eU = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad -(1)$$

where U is the electric potential. The momentum is defined by simplistic use of the de Broglie postulate:

$$p = \hbar \kappa \quad -(2)$$

so

$$\kappa = \frac{1}{\hbar}(2mpU)^{1/2} \quad -(3)$$

and

$$eU = \frac{\hbar^2 \kappa^2}{2m} \quad -(4)$$

The electron wavelength is said incorrectly to be found by this method. What is actually found is κ, and then by incorrect use of the kinetic energy.

The relativistic kinetic energy is then used incorrectly to adjust the result (4). The relativistic kinetic energy is

$$T = (\gamma - 1)mc^2 \quad -(5)$$

$$= E - E_0$$

where E is the total energy and E_0 the rest energy.

Therefore:

$$T = (p^2c^2 + m^2c^4)^{1/2} - mc^2 \quad -(6)$$

Using eq. (2)

$$T = (\hbar^2\kappa^2c^2 + m^2c^4)^{1/2} - mc^2$$

$$= eU \quad -(7)$$

so:

2)

$$\kappa = \frac{1}{\hbar c}\left(eU\left(eU + 2mc^2\right)\right)^{1/2} \quad -(8)$$

This is incorrect because the de Broglie postulate is

$$E = \hbar\omega = \gamma mc^2 \quad -(9)$$

So

$$eU = \left(p^2c^2 + m^2c^4\right)^{1/2} \quad -(10)$$

$$= \left(\hbar^2\kappa^2c^2 + m^2c^4\right)^{1/2}$$

So

$$\hbar^2\kappa^2c^2 = (eU)^2 - m^2c^4$$

$$\kappa = \frac{1}{\hbar c}\left(e^2U^2 - m^2c^4\right)^{1/2} \quad -(11)$$

$$\boxed{\kappa = \frac{1}{\hbar c}\left(eU - E_0\right)^{1/2}\left(eU + E_0\right)^{1/2}} \quad -(12)$$

The relativistic momentum is:

$$\underline{p} = \gamma m\underline{v} \quad -(13)$$

and this momentum is always associated with any moving electron by definition. We have:

$$p^2 = c^2\gamma^2m^2\frac{v^2}{c^2} = c^2\gamma^2m^2\left(1 - \frac{1}{\gamma^2}\right) \quad -(14)$$

$$= \gamma^2c^2m^2 - c^2m^2$$

So

$$E^2 = c^2p^2 + E_0^2 \quad -(15)$$

where

$$E = \gamma mc^2, \quad E_0 = mc^2 \quad -(16)$$

The complete or total energy associated with the relativistic momentum is always E.

Define the four momentum by:

$$p^\mu p_\mu = m^2 c^2, \quad -(17)$$

i.e.

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \quad p_\mu = \left(\frac{E}{c}, -\underline{p}\right) \quad -(18)$$

then eqns. (15) and (17) are the same. The four momentum is defined by the total energy. The minimal prescription is:

$$p^\mu \rightarrow p^\mu + eA^\mu \quad -(19)$$

where

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right) \quad -(20)$$

so

$$E \rightarrow E + e\phi \quad -(21)$$

$$\underline{p} \rightarrow \underline{p} + e\underline{A} \quad -(22)$$

The potential U used in the theory of microscopy is

$$U = \phi \quad -(23)$$

so eU must be associated with the total energy, not with the kinetic energy T. The latter is defined by

$$T = \int \underline{F} \cdot \underline{v} \, dt = \int \frac{d}{dt}(\gamma m \underline{v}) \cdot \underline{v} \, dt \quad -(24)$$

$$\boxed{E = \int \underline{F} \cdot \underline{v} \, dt + mc^2 = \hbar\omega} \quad -(25)$$

4) The usual classical kinetic energy T is the limit

$$v \ll c \qquad -(26)$$

in

$$T = (\gamma - 1) mc^2$$

$$= \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) mc^2$$

$$\sim \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) mc^2$$

$$= \frac{1}{2} mv^2. \qquad -(27)$$

The simplistic association of $e\phi$ (or $e\Pi$) with T in eq. (1) is incorrect.

Therefore the theory of electron microscopy is incorrect. Any theory based on eq. (1) is incorrect. Some unreliable wikipedia articles use eq. (1).