

2(6): Evaluation of the Angle of Scatter in Note 162(5)

The starting equation is:

$$\hbar^2 k^2 = p''^2 + p'^2 - 2p'p'' \cos \theta \quad (1)$$

Eliminate p' and p'' in favour of the energies of the electron in orbitals 1 and 2, E_1 and E_2 :

$$E_1^2 = c^2 p'^2 + m^2 c^4 \quad (2)$$

$$E_2^2 = c^2 p''^2 + m^2 c^4 \quad (3)$$

so eqn (1) becomes

$$\hbar^2 \omega^2 = E_1^2 + E_2^2 - 2E_0^2 - 2(E_1^2 - E_0^2)^{1/2}(E_2^2 - E_0^2)^{1/2} \cos \theta \quad (4)$$

where

$$E_0 = mc^2 \quad (5)$$

is the mass of the electron.

The energy levels E_1 and E_2 are found from the Dirac equation of an atom, and are known and measurable atomic spectra. If it is assumed that the electron mass is the constant mass of the standard laboratory, it is so known experimentally. So the scattering angle can be found:

$$\cos \theta = \frac{E_1^2 + E_2^2 - 2E_0^2 - \hbar^2 \omega^2}{2(E_1^2 - E_0^2)^{1/2}(E_2^2 - E_0^2)^{1/2}} \quad (6)$$

In eq. (6):

$$E_1 + \hbar \omega = E_2 \quad (7)$$

from energy conservation.

2) Therefore:

$$\cos \theta = \frac{E_1 E_2 - E_0^2}{(E_1^2 - E_0^2)^{1/2} (E_2^2 - E_0^2)^{1/2}} \quad - (8)$$

This is a new fundamental characteristic of absorption process of any kind assuming the theory is correct.

Now solve eq. (4) for E_0 as follows:

$$2E_0^2 + 2(E_1^2 - E_0^2)^{1/2} (E_2^2 - E_0^2)^{1/2} \cos \theta = E_1^2 + E_2^2 - \hbar^2 \omega^2 \\ = 2E_1 E_2 \quad - (9)$$

$$\text{i.e. } (E_1^2 - E_0^2)(E_2^2 - E_0^2) \cos^2 \theta = (E_1 E_2 - E_0^2)^2 \quad - (10)$$

which is eq. (8), so:

$$(E_1^2 E_2^2 - E_0^2 (E_1^2 + E_2^2) + E_0^4) \cos^2 \theta = E_1^2 E_2^2 - 2E_0^2 E_1 E_2 + E_0^4 \quad - (11)$$

$$E_0^4 (1 - \cos^2 \theta) + E_0^2 ((E_1^2 + E_2^2) \cos^2 \theta - 2E_1 E_2) + E_1^2 E_2^2 (1 - \cos^2 \theta) = 0 \quad - (12)$$

$$E_0^2 = \frac{1}{2a} \left(-b \pm (b^2 - 4ac')^{1/2} \right) \quad - (13)$$

$$a = 1 - \cos^2 \theta \quad - (14)$$

$$b = (E_1^2 + E_2^2) \cos^2 \theta - 2E_1 E_2 \quad - (15)$$

$$c' = E_1^2 E_2^2 (1 - \cos^2 \theta) \quad - (16)$$

3) Eqs. (8) and (13) - (16) give a rigorous test of standard absorption theory. The two energy levels E_1 and E_2 can be measured experimentally. The two variables are $\cos \theta$ and E_0 . Usually, E_0 is of course a constant, but in the light of recent drastic failures this can no longer be assumed.

The test of the standard theory is simple, first evaluate $\cos \theta$ from eq. (8) for a given E_1 and E_2 , assuming that the mass m is that of the standard laboratories. Then

$$E_0 = mc^2 \quad (17)$$

Having found $\cos \theta$ in this way, use it in eqs. (13) to (16) to find E_0 . Obviously, E_0 should be eq. (17) again. If not the standard theory of absorption fails.
