Equal Mass Compton Scattering at any Angle

In this case the momentum equation (4) of note 159(15) should be used. This equation leads to:

$$\omega''^2 v''^2 = \omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' v v' \cos\theta. \quad -(1)$$

Define:

$$x = mc^2/\hbar \quad -(2)$$

then eq. (1) is:

$$\omega''^2 - x^2 = \omega^2 - x^2 + \omega'^2 - x^2 - 2\left((\omega^2 - x^2)(\omega'^2 - x^2)\right)^{1/2}\cos\theta \quad -(3)$$

i.e.

$$\omega''^2 = \omega^2 + \omega'^2 - x^2 - 2\left((\omega^2 - x^2)(\omega'^2 - x^2)\right)^{1/2}\cos\theta. \quad -(4)$$

From energy conservation:

$$\omega'' = x + \omega - \omega' \quad -(5)$$

Therefore:

$$(x + \omega - \omega')^2 = \omega^2 + \omega'^2 - x^2 - 2\left((\omega^2 - x^2)(\omega'^2 - x^2)\right)^{1/2}\cos\theta$$
$$= x^2 + 2x(\omega - \omega') + \omega^2 - 2\omega\omega' + \omega'^2 \quad -(6).$$

This equation is:

$$x^2 + x(\omega - \omega') - \omega\omega' + \left((\omega^2 - x^2)(\omega'^2 - x^2)\right)^{1/2}\cos\theta = 0 \quad -(7)$$

i.e.

$$(x - \omega')(x + \omega) + \left((\omega^2 - x^2)(\omega'^2 - x^2)\right)^{1/2}\cos\theta = 0 \quad -(8)$$

$$(x - \omega')^2 (x + \omega)^2 = (x^2 - \omega^2)(x^2 - \omega'^2)\cos^2\theta \quad -(9)$$
$$= (x - \omega)(x + \omega)(x - \omega')(x + \omega')\cos^2\theta$$

i.e.

$$(x - \omega)(x + \omega')\cos^2\theta = (x - \omega')(x + \omega). \quad -(10)$$

checked by computer

This means:

so:

$$x^2(1-\cos^2\theta) + x(1+\cos^2\theta)(\omega'-\omega) - \omega\omega'(1-\cos^2\theta) = 0 \quad -(12)$$

The solution is:

$$x = \frac{1}{2(1-\cos^2\theta)}\left[(\omega-\omega')(1+\cos^2\theta) \pm \left((\omega'-\omega)^2(1+\cos^2\theta)^2 + 4\omega\omega'(1-\cos^2\theta)\right)^{1/2}\right] \quad -(13)$$

<u>Scattering at 90°</u>

In eq. (10):

$$(x-\omega)(x+\omega') = 0 \quad -(14)$$

so

$$x = \omega' \quad -(15)$$

This means:

$$M = \frac{\hbar\omega'}{c^2} \quad -(16)$$

which is an <u>absurd result</u> because <u>M is proportional to ω'</u>.

<u>Scattering at 180°</u> — (17)

In eq. (10)

$$(x-\omega)(x+\omega') = (x-\omega')(x+\omega)$$

$$x^2 - \omega x + \omega' x - \omega\omega' = x^2 - \omega' x + \omega x - \omega\omega'$$

i.e.

$$\boxed{\omega = \omega' \text{ for all } M} \quad -(18)$$

3)

It is known from experimental data that eqn. (18) is not true. The data are as follows, from:

1) P. L. Jolivette and N. Rouze, Am. J. Phys., 62, 266 (1994).

At $\theta = 180°$:

$\omega / 10^{21}$ (rad s^{-1})	$\omega' / 10^{21}$ (rad s^{-1})
1.7824	0.3186
2.0244	0.327
1.0053	0.2806
0.7763	0.2613
1.9363	0.3244
0.5408	0.2264

It is obvious that experimentally:

$$\omega \neq \omega'. \quad — (19)$$