1) 160(4): General Compton Scattering at Any Angle

In this case, in the notation of note 160(3):

$$\omega''^2 v''^2 = \omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' v v' \cos\theta \quad -(1)$$

where:

$$v''^2 = 1 - \left(\frac{x_2}{\omega''}\right)^2, \quad v'^2 = 1 - \left(\frac{x_1}{\omega'}\right)^2, \quad v^2 = 1 - \left(\frac{x_1}{\omega}\right)^2 \quad -(2)$$

so:

$$\omega^2 + \omega'^2 - \omega''^2 = 2x_1^2 - x_2^2 + 2\left(\omega^2 - x_1^2\right)^{1/2}\left(\omega'^2 - x_1^2\right)^{1/2}\cos\theta \quad -(3)$$

From energy conservation:

$$\omega'' + \omega' - \omega = x_2. \quad -(4)$$

Now eliminate ω'' between eqs. (3) and (4):

$$x_1^2 + (\omega - \omega')x_2 - \omega\omega' + \left(\omega^2 - x_1^2\right)^{1/2}\left(\omega'^2 - x_1^2\right)^{1/2}\cos\theta = 0 \quad -(5)$$

So:

$$\boxed{x_2 = \frac{\omega\omega'}{\omega - \omega'} - \left(\frac{x_1^2 + \left(\omega^2 - x_1^2\right)^{1/2}\left(\omega'^2 - x_1^2\right)^{1/2}\cos\theta}{\omega - \omega'}\right) \quad -(6)}$$

Here

$$x_2 = \frac{m_2 c^2}{\hbar}, \quad x_1 = \frac{m_1 c^2}{\hbar} \quad -(7)$$

Eq. (6) is a further test of the de Broglie / Einstein theory. If $\cos\theta = 0$, eq. (6) reduces to the result of 160(3)