159(8): Electron - Electron Compton Scattering at 90°

In this case use equations (1) to (5) of note 159(7) with:

$$m = M. \quad -(1)$$

It is shown as follows that this gives a simple quadratic in M, the electron mass. The latter is not constant. The starting equation is eq. (2) of note 159(7):

$$v^2 = \frac{B}{\omega^2} \quad -(2)$$

where:

$$B = \frac{1}{2}\left(A - c^2\left(1 - \left(\frac{\omega}{\omega'}\right)^2\right)\omega'^2\right), \quad -(3)$$

$$A = \Omega^2 \omega^2 c^2 \left(1 + \frac{2Mc^2}{\hbar\omega\Omega}\right) \quad -(4)$$

$$\Omega = 1 - \frac{\omega'}{\omega}. \quad -(5)$$

The mass of the electron is:

$$M = \frac{\hbar\omega}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{1/2}. \quad -(6)$$

Here ω' is the scattered frequency of the electron wave at 90°. The frequency of the incoming electron wave is ω.

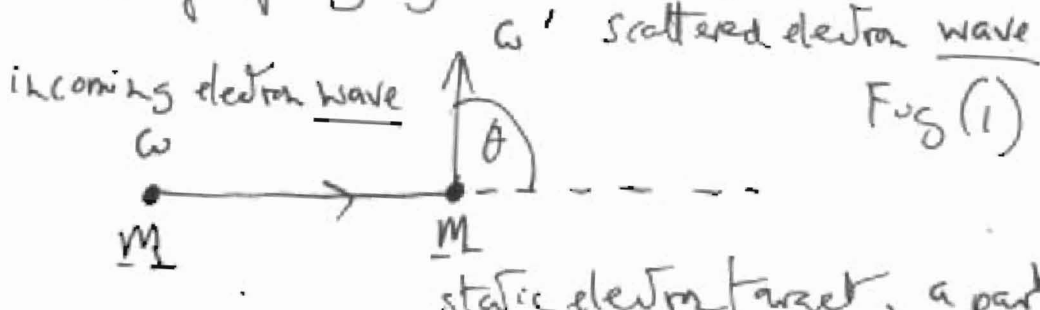

Fig (1)

Therefore the problem reduces to working out the algebra of eqns. (1) - (6) to give M.

2) We have:

$$v^2 = \frac{c^2}{2}\left[\Omega^2\left(1+\frac{2Mc^2}{\Omega\hbar\omega}\right)-\left(\left(\frac{\omega'}{\omega}\right)^2-1\right)\right] \quad -(7)$$

From eq. (7) in eq. (6):

$$\left(\frac{Mc^2}{\hbar\omega}\right)^2 = 1 - \frac{\Omega^2}{2}\left(1+\frac{2Mc^2}{\Omega\hbar\omega}\right)-\frac{1}{2}\left(1-\left(\frac{\omega'}{\omega}\right)^2\right)$$

$$-(8)$$

Define

$$x = \frac{Mc^2}{\hbar\omega} \quad -(9)$$

Eq (8) is:

$$x^2+\Omega x+y = 0, \quad -(10)$$

where

$$y = \frac{1}{2}\left(1+\left(\frac{\omega'}{\omega}\right)^2-\Omega^2\right) \quad -(11)$$

$$= \frac{1}{2}\left(1+\left(\frac{\omega'}{\omega}\right)^2-\left(1-\frac{\omega'}{\omega}\right)^2\right) \quad -(12)$$

$$y = \frac{\omega'}{\omega} \quad -(13)$$

So

$$x^2+\left(1-\frac{\omega'}{\omega}\right)x+\frac{\omega'}{\omega} = 0 \quad -(14)$$

and

$$x = \frac{1}{2}\left[\frac{\omega'}{\omega}-1 \pm \left(\left(1-\frac{\omega'}{\omega}\right)^2-4\frac{\omega'}{\omega}\right)^{1/2}\right]$$

i.e.

$$M = \frac{\hbar\omega}{2c^2}\left[\frac{\omega'}{\omega}-1\pm\left(1+\left(\frac{\omega'}{\omega}\right)^2-6\frac{\omega'}{\omega}\right)^{1/2}\right]$$

$$-(15)$$

3)

i.e.

$$\boxed{\underline{M} = \frac{\hbar}{2c^2}\left[\omega' - \omega \pm \left(\omega^2 + \omega'^2 - 6\omega\omega'\right)^{1/2}\right]} \quad -(16)$$

This equation can be checked by computer algebra, and shows that $\underline{M}$ is not constant.

<u>Conclusion</u>

There is no such thing as a constant $\underline{M}$ in the relativistic equations governing electron - electron scattering. As for photon mass, <u>$\underline{M}$ must be interpreted as a scalar curvature.</u> In order for $\underline{M}$ to be positive and real the following three conditions are needed.

1) Take the + sign in eq. (16).
2) $\omega^2 \geq \omega'(\omega' + 6\omega)$.
3) $(\omega^2 + \omega'^2 - 6\omega\omega')^{1/2} - \omega \geq \omega'$.