2) 159(12): Data on Compton Edge

Data on Compton edge scattering can be found from googling "laboratory Compton scattering", last site on first page, P.L. Jolivette and N. Rouze, Am. J. Phys., 62, 266 (1994). Some of these data are given in the following table.

Compton Scattering at $\theta = 180^\circ$ (Compton Edge)

E (keV)	E − E′ (keV)	ω (rad s^{-1}) / 10^{21}	ω′ (rad s^{-1}) / 10^{21}
1173.2	963.5	1.7824	0.3186
1332.5	1117.0	2.0244	0.327
661.7	477.0	1.0053	0.2806
511.0	339.0	0.7763	0.2613
1274.5	1061.0	1.9363	0.3244
356.0	207.0	0.5408	0.2264

$1\,eV = 1.51924 \times 10^{15}$ rad s^{-1}

As can be seen from Table 1 of the attached, there are many more data available. The above table gives the six first entries.

Compton scattering, the electron mass, and relativity: A laboratory experiment

P. L. Jolivette and N. Rouze
Hope College, Holland, Michigan 49423

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Compton scattering in a semiconductor detector is used to "discover" the relativistic relation between energy and momentum and to demonstrate the dependence of p, E and γ on β. The motivation is to measure the (rest) mass of the electron, and this can be done to within 1 keV with a commonly available set of gamma ray sources. To determine precisely where the Compton edge occurs in a spectrum, a Monte Carlo calculation of detector response is described which also helps the student to understand the physics of the detection process.

I. INTRODUCTION

In modern physics courses Compton scattering is introduced to emphasize the particle nature of photons and the recoiling electron is largely ignored. However, with commonly available gamma ray sources the kinetic energy of the recoiling electron is often on the order of 1 MeV and the electrons are indeed relativistic. Compton scattering is a convenient source of relativistic particles in the undergraduate laboratory, and when it occurs in a detector the kinetic energy of the recoiling electron is directly measured. In this report we describe how such measurements can be used to determine the relativistic relations between energy, momentum, and mass for the recoiling electron simply and precisely. The experiment is initially presented to the students as a measurement of the electron mass to complement an e/m experiment and the Millikan oil drop determination of the charge of the electron. The student's analysis is forced onto an instructive detour due to the obvious inadequacy of the nonrelativistic equations. The process leads to the discovery, based on the data, of the energy–momentum relation of special relativity.

Experiments to measure the mass of the electron[1] and the relativistic relations for the electron[2] using the Compton edge have been published previously, but these reports did not emphasize the discovery approach or the precision possible. Measurements of the Compton angular distribution with the intent of discovering the form of the relativistic relations have also been described,[3] but this is a more complicated experiment and requires a high intensity source of gamma rays.

II. THE ANALYSIS

The experiment consists of measuring the energy of the Compton edge from ordinary gamma ray spectra as a function of the energy E_γ of the incident gamma rays. The Compton edge corresponds to the kinetic energy T of the recoiling electron when the incident gamma ray is scattered through 180°. To achieve the best results, the Compton edge energy must be measured with high precision, and we recommend the use of a germanium detector. Further, to avoid systematic errors it is important to understand the shape of the spectrum, and models to describe the shape are discussed in Sec. III.

The data used in this report are listed in Table I and Fig. 1 shows a plot of T vs E_γ. The best that can be said about these variables is that they are indeed the measured quantities. The relation between them, whether derived classically or relativistically, provides little insight to the student. The fact that the dependent and independent parameters are not simply related is the greatest barrier to using this experiment with unsophisticated students.

There are only four ideas needed to analyze the data; the realization that the gamma ray is a photon, conservation of energy and momentum in the collision, and the classical relation $E_\gamma = p_\gamma c$ between the energy and momentum of an electromagnetic wave or photon. For the Compton edge, the energy of the electron corresponds to an incident photon scattered through 180° and conservation of momentum gives

$$p_\gamma = p - p'_\gamma, \tag{1}$$

where p_γ, p'_γ, and p are the momenta of the incident gamma ray, the scattered gamma ray, and the recoiling electron, respectively. Conservation of energy gives

$$p_\gamma c = p'_\gamma c + T. \tag{2}$$

These relations may be combined to give the electron momentum in terms of the experimental variables,

$$pc = 2E_\gamma - T. \tag{3}$$

The above does not rely in any way on special relativity.

As a first attempt to determine the electron mass, we can assume the nonrelativistic relation, $T = p^2/2m_{nr}$, so that the (rest) energy of the electron is given in terms of the experimental quantities by

$$m_{nr}c^2 = \frac{p^2c^2}{2T} = \frac{(2E_\gamma - T)^2}{2T}. \tag{4}$$

When the calculated values of $m_{nr}c^2$ are plotted as a function of T, Fig. 2, it is immediately obvious that $m_{nr}c^2$ is not constant. But the surprising result is that, to within experimental uncertainties, the dependence on T is linear with a slope of 1/2. Furthermore, the intercept is equal to the value of m_0c^2 which the student should expect from the low energy e/m and Millikan experiments, or from the "book value." Replacing the left-hand side of Eq. (4) with this experimental result, $m_0c^2 + T/2$, gives

$$p^2c^2 = 2Tm_0c^2 + T^2. \tag{5}$$

The addition of $(m_0c^2)^2$ to complete the square gives

$$p^2c^2 + (m_0c^2)^2 = (T + m_0c^2)^2 = E^2, \tag{6}$$

where E is the total energy of the electron. This is the central energy-momentum relationship of special relativity.

Table I. Measured Compton edge energies.

Source	E_γ (keV)	$E_{\rm Compton}$ (keV)
^{60}Co	1173.2	963.5 ± 1.0
^{60}Co	1332.5	1117.0 ± 1.0
^{137}Cs	661.7	477.0 ± 1.0
^{22}Na	511.0	339.0 ± 1.0
^{22}Na	1274.5	1061.0 ± 1.0
^{133}Ba	356.0	207.0 ± 1.0
^{207}Bi	569.7	393.0 ± 1.0
^{207}Bi	1063.7	858.0 ± 1.0
^{207}Bi	1770.2	1547.0 ± 1.0
^{152}Eu	344.3	196.0 ± 1.0
^{152}Eu	1408.0	1190.0 ± 1.0
^{208}Tl	2614.5	2382.0 ± 2.0
^{116}In	416.9	263.0 ± 2.0
^{116}In	1097.3	890.0 ± 2.0
^{116}In	1293.0	1079.0 ± 2.0
^{116}In	2112.1	1883.0 ± 2.0
^{57}Co	122.1	39.0 ± 1.0
^{241}Am[a]	59.5	11.3 ± 0.5
^{57}Co[a]	122.1	39.3 ± 0.5

[a]Measured with a Si(Li) detector.

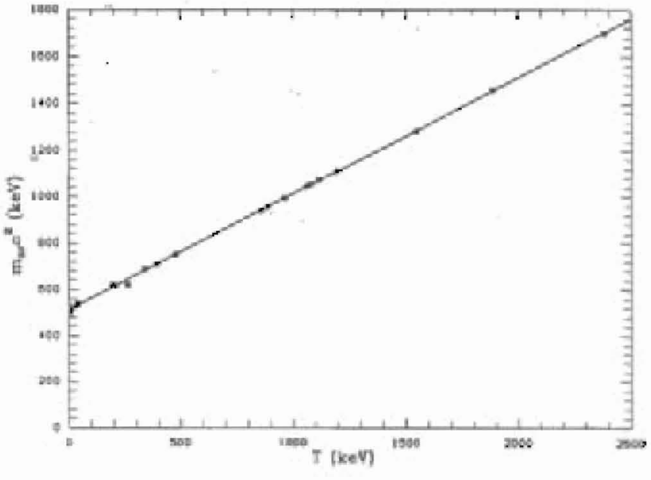

Fig. 2. The electron rest energy calculated nonrelativistically and then fitted with a linear relationship. There are uncertainties in both directions which are usually smaller than the points. The intercept is 513.6 ± 1.9 keV and the slope is 0.4991 ± 0.0016.

Once Eq. (6) has been "discovered" it can be solved to calculate directly the rest energy as originally asked of the student. This procedure differs from using the intercept of Fig. 2 since relativity is now assumed to be the correct theory. In terms of the measured quantities,

$$m_0c^2=\frac{p^2c^2-T^2}{2T}=\frac{2E_\gamma(E_\gamma-T)}{T}. \tag{7}$$

The calculated values of m_0c^2 are plotted as a function of T in Fig. 3, and the mass is clearly seen to be independent of the kinetic energy. Statistical errors of about ±1 keV in m_0c^2 are possible, but systematic errors of about the same magnitude are also present largely due to residual errors in actually locating the Compton edge, see Sec. III below.

The student has now reached the stated goal of measuring the (rest) mass of the electron. In addition the data can be manipulated to verify other common relativistic relations. To determine the electron velocity, or equivalently $\beta=v/c$, we use the relativistic mass m and the relations $p=mv$ and $E=mc^2$ to give

$$\beta=\frac{v}{c}=\frac{mvc}{mc^2}=\frac{pc}{E}=\frac{2E_\gamma-T}{T+m_0c^2}=\frac{T(2E_\gamma-T)}{T^2-2E_\gamma T+2E_\gamma^2}. \tag{8}$$

Figure 4 shows the plot of pc vs β. For the 2.615 MeV gamma ray from ^{208}Tl, $\beta=0.984$, and the plot clearly shows the expected asymptotic behavior as β approaches unity.

A plot of E vs β also shows the expected asymptotic behavior. The total energy E is expressed in terms of the experimental measurements as

$$E=T+m_0c^2=\frac{T^2-2TE_\gamma+2E_\gamma^2}{T}. \tag{9}$$

The relativistic factor γ can be calculated from the relation $E=\gamma m_0c^2$ so that

$$\gamma=\frac{E}{m_0c^2}=\frac{m_0c^2+T}{m_0c^2}=1+\frac{T^2}{2E_\gamma(E_\gamma-T)}. \tag{10}$$

Figure 5 plots γ vs β. For reasons discussed below γ is calculated from the relation $\gamma=E/m_0c^2$ using the value of m_0c^2 determined in Fig. 3 rather than the last form of Eq.

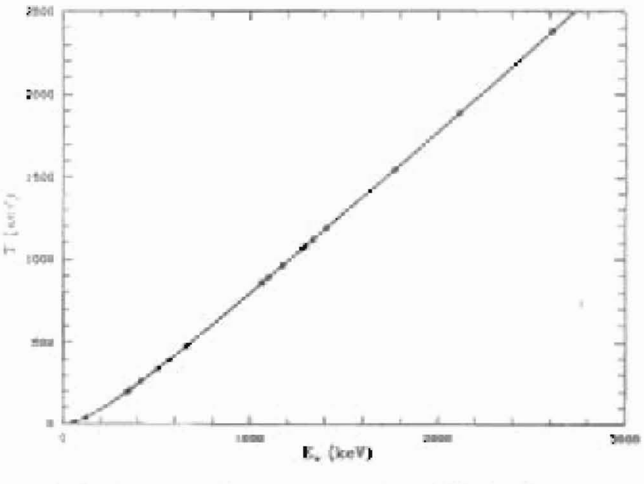

Fig. 1. The electron kinetic energy as a function of E_γ. The line is a fit of $T=2E_\gamma^2/(2E_\gamma+m_0c^2)$ [from Eq. (7)] which yields $m_0c^2=512.7\pm0.9$ keV. The uncertainties are smaller than the data points.

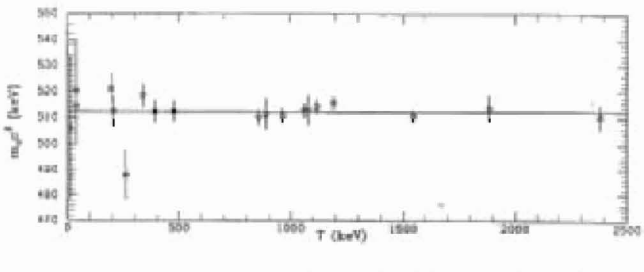

Fig. 3. The electron rest energy vs electron kinetic energy when calculated relativistically. There are uncertainties in both directions. The uncertainties in m_0c^2 are large at low energies since $d(m_0c^2)/dT$ is large there. For this data set $m_0c^2=512.7\pm0.9$ keV identical to that in Fig. 1.

Compton edge

From Wikipedia, the free encyclopedia

In spectrophotometry, the **Compton edge** is a feature of the spectrograph that results from the Compton scattering in the scintillator or detector. When a gamma-ray scatters off the scintillator but escapes, only a fraction of its energy is registered by the detector. This leads to a spectrum of gamma-rays in the data that is not really there. The highest energy that occurs from this process is the Compton edge.

Background

In a Compton scattering process, an incident photon collides with an electron in the scintillator. The amount of energy exchanged varies with angle, and is given by the formula:

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$$

or

$$E' = \frac{E}{1 + \frac{(1-\cos\theta)E}{m_e c^2}}$$

- E is the energy of the incident photon.
- E' is the energy of the outgoing photon, which escapes the detector.
- m_e is the mass of the electron.
- c is the speed of light.
- Θ is the angle of deflection for the photon.

The amount of energy transferred to the scintillator varies with the angle of deflection. As Θ approaches zero, none of the energy is transferred. The maximum amount of energy is transferred when Θ approaches 180 degrees.

$$E_T = E - E'$$

$$E_{\text{Compton}} = E_T(\max) = \frac{2E^2}{m_e c^2 + 2E}$$

It is impossible for the photon to transfer any more energy via this process, hence there is a sharp cutoff at this energy giving rise to the name *Compton edge*.

Spectrograph Feature

See also