159(13): Scattered Frequency at 90°, Electron - Electron Scattering

At 90° angle of scatter the equations are:

$$\omega^2 v^2 + \omega'^2 v'^2 = A \qquad -(1)$$

$$\frac{v'^2}{c^2} = 1 - \left(\frac{\omega}{\omega'}\right)^2 \left(1 - \frac{v^2}{c^2}\right) \qquad -(2)$$

So

$$2\omega^2 v^2 + \omega'^2 c^2 \left(1 - \left(\frac{\omega}{\omega'}\right)^2\right) = A \qquad -(3)$$

$$= \Omega^2 \omega^2 c^2 \left(1 + \frac{2mc^2}{\hbar \omega \Omega}\right)$$

i.e.

$$\frac{2\omega^2 v^2}{c^2} = \omega^2 - \omega'^2 + \Omega^2 \omega^2 + \frac{2mc^2}{\hbar} \Omega \omega \qquad -(4)$$

where

$$\Omega = 1 - \frac{\omega'}{\omega} \qquad -(5)$$

i.e.

$$\frac{v^2}{c^2} = \frac{1}{2}\left(1 - \frac{\omega'^2}{\omega^2} + \left(1 - \frac{\omega'}{\omega}\right)^2 + \frac{2mc^2 \Omega}{\hbar\omega}\right)$$

$$\boxed{\frac{v^2}{c^2} = \left(1 - \frac{\omega'}{\omega}\right)\left(1 + \frac{mc^2}{\hbar \omega}\right)} \qquad -(6)$$

Therefore

$$\frac{\omega'}{\omega} = 1 + \frac{v^2}{c^2}\left(1 + \frac{mc^2}{\hbar\omega}\right)^{-1/2}, \qquad -(7)$$

here

$$\hbar \omega = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} mc^2 \qquad -(8)$$

Therefore:

2)

$$\omega' = \omega\left[1 + \frac{v^2}{c^2\left(1+\left(1-\frac{v^2}{c^2}\right)^{1/2}\right)^{1/2}}\right] \quad -(9)$$

In this case $\boxed{\omega' \geq \omega}$ $-(10)$

Using eq. (6) in eq. (2):

$$\frac{v^{2\prime}}{c^2} = 1 - \left(\frac{\omega}{\omega'}\right)^2\left(1-\left(1-\frac{\omega'}{\omega}\right)\left(1+\frac{mc^2}{\hbar\omega}\right)\right)$$

$$= 1 - \left(\frac{\omega}{\omega'}\right)^2 + \left(\frac{\omega}{\omega'}\right)^2\left(1-\frac{\omega'}{\omega}+\frac{mc^2}{\hbar\omega}-\frac{\omega'}{\omega}\frac{mc^2}{\hbar\omega}\right)$$

$$= 1 - \frac{\omega}{\omega'} + \left(\frac{\omega}{\omega'}\right)^2\frac{mc^2}{\hbar\omega}\left(1-\frac{\omega'}{\omega}\right)$$

$$= 1 - \frac{\omega'}{\omega} + \frac{mc^2}{\hbar\omega}\left(\left(\frac{\omega}{\omega'}\right)^2 - \frac{\omega}{\omega'}\right)$$

$$\boxed{\frac{v'^2}{c^2} = \left(1-\frac{\omega}{\omega'}\right)\left(1+\frac{\omega}{\omega'}\frac{mc^2}{\hbar\omega}\right)} \quad -(10)$$

However:

$$\hbar\omega' = \left(1-\frac{v'^2}{c^2}\right)^{-1/2} mc^2$$

$$1-\frac{v'^2}{c^2} = \left(\frac{mc^2}{\hbar\omega'}\right)^2 \quad -(11)$$

3)

Therefore:

$$\underline{1} - x^2 = 1 - \frac{\omega}{\omega'} + \frac{\omega}{\omega'}\left(1 - \frac{\omega}{\omega'}\right)x \quad -(12)$$

where

$$x = \frac{mc^2}{\hbar\omega'} \quad -(13)$$

$$x^2 + \frac{\omega}{\omega'}\left(1 - \frac{\omega}{\omega'}\right)x - \frac{\omega}{\omega'} = 0 \quad -(14)$$

$$x = \frac{1}{2}\left[\frac{\omega}{\omega'}\left(\frac{\omega}{\omega'} - 1\right) \pm \left(\left(\frac{\omega}{\omega'}\right)^2\left(1 - \frac{\omega}{\omega'}\right)^2 + 4\frac{\omega}{\omega'}\right)^{1/2}\right]$$

$$= \frac{1}{2}\frac{\omega}{\omega'}\left[\frac{\omega}{\omega'} - 1 \pm \left(\left(1 - \frac{\omega}{\omega'}\right)^2 + 4\frac{\omega'}{\omega}\right)^{1/2}\right]$$

So:

$$\boxed{\underline{m} = \frac{\hbar\omega}{2c^2}\left[\frac{\omega}{\omega'} - \underline{1} \pm \left(\left(1 - \frac{\omega}{\omega'}\right)^2 + 4\frac{\omega'}{\omega}\right)^{1/2}\right] \quad -(15)}$$

This is an absurd result because $\underline{m}$ is the electron mass, a constant. So the de Broglie / Einstein theory fails.