159(14): Energy Conservation Relations in Electron - Electron Interaction, a Direct Test of the de Broglie Hypothesis

The energy conservation equation of relevance is:

$$\gamma \underline{m} c^2 + \underline{m} c^2 = \gamma' \underline{m} c^2 + \gamma'' \underline{m} c^2 \quad - (1)$$

An incoming electron of energy $\gamma \underline{m} c^2$ collides with a stationary electron of energy $\underline{m} c^2$ to produce two electrons of energy $\gamma' \underline{m} c^2$ and $\gamma'' \underline{m} c^2$. Therefore:

$$\gamma + 1 = \gamma' + \gamma''. \quad - (2)$$

The de Broglie hypothesis gives:

$$\frac{\gamma'}{\gamma} = \frac{\omega'}{\omega} \quad - (3)$$

$$\frac{\gamma''}{\gamma} = \frac{\omega''}{\omega} \quad - (4)$$

$$\frac{\gamma''}{\gamma'} = \frac{\omega''}{\omega'} \quad - (5)$$

The electron is a wave as well as a particle, so the frequencies ω, ω' and ω'' are experimentally observable. From eqs. (2) to (5), expressions for γ, γ' and γ'' can be obtained as follows.

Firstly:

$$\frac{\omega}{\omega'} \gamma' + 1 = \gamma' + \frac{\omega''}{\omega'} \gamma' \quad - (6)$$

$$\gamma' \left(\frac{\omega - \omega''}{\omega'} \right) + 1 = \gamma' \quad - (7)$$

So:

2)

$$\gamma'\left(1+\frac{\omega''-\omega}{\omega'}\right)=1 \quad -(8)$$

$$\boxed{\gamma'=\left[1+\frac{\omega''-\omega}{\omega'}\right]^{-1}} \quad -(9)$$

Similarly,

$$\frac{\omega}{\omega''}\gamma''+1=\frac{\omega'}{\omega''}\gamma''+\gamma'' \quad -(10)$$

$$\gamma''\left(1+\frac{\omega'-\omega}{\omega''}\right)=1 \quad -(11)$$

$$\boxed{\gamma''=\left[1+\frac{\omega'-\omega}{\omega''}\right]^{-1}} \quad -(12)$$

Thirdly:

$$\gamma+1=\frac{\omega'}{\omega}\gamma+\frac{\omega''}{\omega}\gamma \quad -(13)$$

$$\boxed{\gamma=\left[\frac{\omega'+\omega''}{\omega}-1\right]^{-1}} \quad -(14)$$

By definition:

$$\gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2} \quad -(15)$$

$$\gamma'=\left(1-\frac{v'^2}{c^2}\right)^{-1/2} \quad -(16)$$

$$\gamma''=\left(1-\frac{v''^2}{c^2}\right)^{-1/2} \quad -(17)$$

Therefore:

3) $$1 - \frac{v'^2}{c^2} = \left(1 + \frac{\omega'' - \omega}{\omega'}\right)^2 \quad -(18)$$

$$1 - \frac{v''^2}{c^2} = \left(1 + \frac{\omega' - \omega}{\omega''}\right)^2 \quad -(19)$$

$$1 - \frac{v^2}{c^2} = \left(\frac{\omega' + \omega''}{\omega} - 1\right)^2 \quad -(20)$$

The three electron velocities are therefore:

$$v^2 = c^2\left[1 - \left(\frac{\omega' + \omega''}{\omega} - 1\right)^2\right] \quad -(21)$$

$$v'^2 = c^2\left[1 - \left(\frac{\omega'' - \omega}{\omega'} + 1\right)^2\right] \quad -(22).$$

$$v''^2 = c^2\left[1 - \left(\frac{\omega' - \omega}{\omega''} + 1\right)^2\right] \quad -(23)$$

If the theory is correct the electron mass $\underline{m}$ from eqs. (21) to (23) should be the same:

$$\underline{m} = \frac{\hbar\omega}{\gamma c^2} = \frac{\hbar\omega_1}{\gamma'_1 c^2} = \frac{\hbar\omega_2}{\gamma''_2 c^2} \quad -(24)$$

The three expressions for electron mass are:

1) $$\underline{m} = \frac{\hbar}{c^2}\,\omega\left(\frac{\omega' + \omega''}{\omega} - 1\right)$$

$$\boxed{\underline{m} = \frac{\hbar}{c^2}\left(\omega' + \omega'' - \omega\right)} \quad -(25)$$

4)

2) $$\underline{m} = \frac{\hbar}{c^2}\omega'\left(1 + \frac{\omega'' - \omega}{\omega'}\right)$$

$$\boxed{\underline{m} = \frac{\hbar}{c^2}\left(\omega' + \omega'' - \omega\right)} \quad -(26)$$

3) $$\underline{m} = \frac{\hbar}{c^2}\omega''\left(1 + \frac{\omega' - \omega}{\omega''}\right)$$

$$\boxed{\underline{m} = \frac{\hbar}{c^2}\left(\omega'' + \omega' - \omega\right)} \quad -(27)$$

Therefore the electron mass must be:

$$\boxed{\underline{m} = \frac{\hbar}{c^2}\left(\omega'' + \omega' - \omega\right)} \quad -(28).$$

if the de Broglie / Einstein theory is correct.

The result (28) can be checked with the momentum equation as in the reset note. Using the momentum equation, one of the frequencies (e.g. ω'') can be eliminated.