159(1): Second Expression for Photon Mass from Compton Effect.

From conservation of total energy:

$$\gamma m c^2 + M c^2 = \gamma' m c^2 + \left(M^2 c^4 + c^2 p'^2\right)^{1/2} \quad -(1)$$

From conservation of total momentum:

$$\underline{p}' = \hbar\left(\underline{\kappa} - \underline{\kappa}'\right) \quad -(2)$$

In these equations, the de Broglie Einstein equation give:

$$\frac{\omega}{\omega'} = \frac{\gamma}{\gamma'} \quad -(3)$$

Now eliminate γ:

$$\gamma = \frac{\omega}{\omega'}\gamma' \quad -(4)$$

Eqs. (1) to (4) give:

$$\omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' v v' \cos\theta = A' \quad -(5)$$

where

$$A' = c^2 \omega'^2 \Omega'^2 \left(1 + \frac{2Mc^2}{\hbar\omega'\Omega'}\right) \quad -(6)$$

with:

$$\Omega' = \frac{\omega}{\omega'} - 1. \quad -(7)$$

We have used:

$$\kappa = \frac{\omega v}{c^2}, \quad \kappa' = \frac{\omega' v'}{c^2}. \quad -(8)$$

Now use:

$$1 - \frac{v'^2}{c^2} = \left(\frac{\omega}{\omega'}\right)^2\left(1 - \frac{v^2}{c^2}\right) \quad -(9)$$

2)
$$\frac{v'^2}{c^2} = 1 - \left(\frac{\omega}{\omega'}\right)^2\left(1-\frac{v^2}{c^2}\right) \quad -(10)$$

$$1-\frac{v^2}{c^2} = \left(\frac{\omega'}{\omega}\right)^2\left(1-\frac{v'^2}{c^2}\right) \quad -(11)$$

$$\frac{v^2}{c^2} = 1-\left(\frac{\omega'}{\omega}\right)^2\left(1-\frac{v'^2}{c^2}\right) \quad -(12)$$

From eq. (12) in eq. (5):

$$\omega'^2 v'^2 - \omega\omega' v v' \cos\theta = B' \quad -(13)$$

where
$$B' = \frac{1}{2}\left(A' - \omega^2 c^2\left(1-\left(\frac{\omega'}{\omega}\right)^2\right)\right) \quad -(14)$$

so
$$v = \frac{\omega'^2 v'^2 - B'}{\omega\omega' v' \cos\theta} \quad -(15)$$

Use eq. (15) in eq. (10) to give:

$$\frac{v'^4}{c^2}\left(1-\cos^2\theta\right) + v'^2\left[\left(1-\left(\frac{\omega}{\omega'}\right)^2\right)\cos^2\theta - \frac{2B'}{c^2\omega'^2}\right] + \left(\frac{B'}{c\omega'^2}\right)^2 = 0 \quad -(16)$$

The solution of eq. (16) is:

$$v'^2 = \frac{1}{2a}\left(-b \pm \left(b^2 - 4ac'\right)^{1/2}\right) \quad -(17)$$

where
$$a = \frac{1}{c^2}\left(1-\cos^2\theta\right) \quad -(18)$$

$$b = \left(1 - \left(\frac{\omega}{\omega'}\right)^2 \cos^2\theta - \frac{2B'}{c^2\omega'^2}\right) \quad -(19)$$

$$c' = \left(\frac{B'}{c\omega'^2}\right)^2 \quad -(20)$$

$$B' = \frac{1}{2}\left(A' - \omega^2 c^2\left(1 - \left(\frac{\omega'}{\omega}\right)^2\right)\right) \quad -(21)$$

$$A' = c^2\omega'^2\Omega'^2\left(1 + \frac{2mc^2}{\hbar\omega'\Omega'}\right) \quad -(22)$$

$$\Omega' = \frac{\omega}{\omega'} - 1 \quad -(23)$$

The mass of the photon is:

$$m = \frac{\hbar\omega'}{c^2}\left(1 - \frac{v'^2}{c^2}\right)^{1/2} \quad -(24)$$

The mass m should be the same as that in UFT 158 if de Broglie is correct.
