1)

159(3) : Theory of the Photo-Electric Effect

Consider a photon of mass m colliding with an electron of mass M bound to a material with initial binding energy V_1. The electron is initially at rest with rest energy Mc^2.

Conservation of Energy

$$\gamma mc^2 + Mc^2 + V_1 = \gamma' mc^2 + \left(M^2c^4 + c^2p'^2\right)^{1/2} + V_2 \quad -(1)$$

Conservation of Momentum

$$\underline{p}' = \hbar\left(\underline{\kappa} - \underline{\kappa}'\right) \quad -(2)$$

The relativistic kinetic energy of the electron after collision is:

$$T = \left(M^2c^4 + c^2p'^2\right)^{1/2} - Mc^2 \quad -(3)$$

$$= \left(\gamma_e - 1\right)Mc^2 \quad -(4)$$

where

$$\gamma_e = \left(1 - \frac{v_e^2}{c^2}\right)^{-1/2}, \quad -(5)$$

and where v_e is the velocity of the electron after collision. In the non-relativistic limit:

$$v_e \ll c \quad -(6)$$

and

$$\gamma_e \sim 1 + \frac{1}{2}\frac{v_e^2}{c^2} + \ldots \quad (7)$$

2)

So

$$T \sim \left(1 + \frac{1}{2} \frac{v_e^2}{c^2} - 1 \right) M c^2 \quad -(8)$$

$$= \frac{1}{2} M v_e^2,$$

which is the non-relativistic kinetic energy.

Therefore:

$$\boxed{T = (\gamma_e - 1) M c^2 = (\gamma - \gamma') m c^2 + V_1 - V_2 \quad -(9)}$$

<u>This</u> is <u>the correct</u> <u>theory</u> of <u>the photoelectric effect.</u>

The usual theory is almost always written as:

$$T = \frac{1}{2} M v_e^2 = \hbar \omega - \Phi \quad -(10)$$

where Φ is the binding energy or work function.

From comparison of eq. (9) and (10):

$$\Phi = V_2 - V_1 \quad -(11)$$

and the usual theory uses the non-relativistic limit for the kinetic energy T. If:

$$V_1 = V_2 \quad -(12)$$

<u>then</u> <u>the Compton effect</u> is <u>regained</u>:

$$T = (\gamma_e - 1) M c^2 = (\gamma - \gamma') m c^2 \quad -(13)$$

Eq. (13) shows that the kinetic energy gained

3) by the electron is the kinetic energy lost by the photon. Denote the latter by:

$$\boxed{T_p = (\gamma - \gamma')mc^2} \quad -(14)$$

Thus:

$$T_p = \hbar(\omega - \omega') \quad -(15)$$

The correct theory of the photoelectric effect is:

$$\boxed{T = \hbar(\omega - \omega') - \Phi} \quad -(16)$$

The usual theory assumes:

$$\omega' = ?\ 0 \quad -(17)$$

which means:

$$\gamma' mc^2 = ?\ 0 \quad -(18)$$

The usual theory assumes that the photon is not scattered in the photoelectric effect.

This is incorrect, because if the photon is not scattered, it is stopped, so its velocity and momentum reduce to zero. In that case:

$$\gamma' \rightarrow 1 \quad -(19)$$

so

$$m \rightarrow ?\ 0. \quad -(20)$$

This is incorrect, because the photon cannot lose its mass m.

4) Note carefully that T_p is a kinetic energy, while γmc^2 and $\gamma' mc^2$ are total energies.

The consequences for absorption theory are profound, because the theory is always proposed in the same way as the photoelectric effect. The quantity being absorbed is always stated to be $\hbar\omega$, but it should be $(\gamma - \gamma')mc^2$. In atomic H for example, the usual theory is:

$$\Delta E = \hbar\omega = hcR_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad -(21)$$

where R_H is the Rydberg constant and n_1 and n_2 the initial and final quantum numbers.

This should be:

$$\boxed{\Delta E = (\gamma - \gamma')mc^2 = hcR_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \quad -(22)$$

The photon mass can be determined from the photoelectric effect using the equations of UFT 158 and note 159(1), with:

$$\underline{M}c^2 \rightarrow (\underline{M}c^2 - \Phi) \quad -(23)$$
