1) 159(11): Compton Scattering at 180° - the Compton Edge.

Compton scattering at

$$\theta = \pi = 180° \quad -(1)$$

is called Compton edge scattering. Therefore eq. (1) of note 158(10) becomes:

$$(\omega v + \omega' v')^2 = A \quad -(2)$$

$$A = \Omega^2 \omega^2 c^2 \left(1 + \frac{2Mc^2}{\hbar \omega \Omega} \right) \quad -(3)$$

so

$$v' = \frac{1}{\omega'} \left(A^{1/2} - \omega v \right) \quad -(4)$$

From eq. (9) of note 158(10):

$$\frac{v^2}{c^2} = 1 - \left(\frac{\omega'}{\omega}\right)^2 \left(1 - \frac{v'^2}{c^2}\right) \quad -(5)$$

$$= 1 - \left(\frac{\omega'}{\omega}\right)^2 + \left(\frac{\omega' v'}{\omega c}\right)^2$$

$$= 1 - \left(\frac{\omega'}{\omega}\right)^2 + \frac{1}{\omega^2 c^2}\left(A - 2\omega v A^{1/2} + \omega^2 v^2\right)$$

$$= 1 - \left(\frac{\omega'}{\omega}\right)^2 + \frac{A}{\omega^2 c^2} - \frac{2 v A^{1/2}}{\omega c^2} + \frac{v^2}{c^2}$$

So:

$$\boxed{v = \frac{\omega c^2}{2A^{1/2}} \left(1 - \left(\frac{\omega'}{\omega}\right)^2 + \frac{A}{\omega^2 c^2} \right)} \quad -(6)$$

$$m = \frac{\hbar \omega}{c^2} \left(1 - \frac{v^2}{c^2} \right)^{1/2} \quad -(7)$$