158(8): Reduction to Quadratic

The equations of note 158(7) are:

$$v^2 + \left(\frac{\omega'}{\omega}\right)\left(xv - \frac{y}{v}\right)^2 - 2v\left(xv - \frac{y}{v}\right)\frac{\omega'}{\omega}\cos\theta = \frac{B}{A^2} \quad -(1)$$

where $$\hbar\omega = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} mc^2. \quad -(2)$$

Eq. (1) reduces to a quadratic as follows:

$$v^2 + \left(\frac{\omega'}{\omega}\right)\left(x^2v^2 - 2xy + \frac{y^2}{v^2}\right) - 2\left(xv^2 - y\right)\frac{\omega'}{\omega}\cos\theta = \frac{B}{A^2},$$

$$v^2\left(1 + x^2\frac{\omega'}{\omega} - 2x\frac{\omega'}{\omega}\cos\theta\right) + y^2\frac{\omega'}{\omega}\frac{1}{v^2} + 2y\frac{\omega'}{\omega}\cos\theta - 2xy\frac{\omega'}{\omega} = \frac{B}{A^2}$$

i.e. $$\alpha v^2 + \frac{\beta}{v^2} + \gamma = \frac{B}{A^2} \quad -(2)$$

where:

$$\alpha = 1 + \frac{\omega'}{\omega}\left(x^2 - 2x\cos\theta\right) \quad -(3)$$

$$\beta = y^2\frac{\omega'}{\omega} \quad -(4)$$

$$\gamma = 2y\frac{\omega'}{\omega}\left(\cos\theta - x\right) \quad -(5)$$

Eq. (2) is the quadratic:

$$\boxed{\alpha v^4 + \left(\gamma - \frac{B}{A^2}\right)v^2 + \beta = 0} \quad -(6)$$

2) so there are two roots for v^2. Numerically,
v is very close to c, so use eqns. (2) and (6)
to find a quadratic in m^2 as follows. From eq. (2):

$$1 - \frac{v^2}{c^2} = \left(\frac{mc^2}{\hbar\omega}\right)^2 \quad -(7)$$

so

$$v^2 = c^2\left(1 - \left(\frac{mc^2}{\hbar\omega}\right)^2\right) \quad -(8)$$

Denote:

$$G = \left(\frac{mc^2}{\hbar\omega}\right)^2 \quad -(9)$$

$$-(10)$$

Then in eq. (6):

$$\alpha c^4 (1 - G)^2 + \left(\gamma - \frac{B}{A^2}\right)(1 - G)c^2 + \beta = 0$$

Denote

$$F = \gamma - \frac{B}{A^2} \quad -(11)$$

then

$$\alpha c^4 (1-G)^2 + c^2 F (1 - G) + \beta = 0 \quad -(12)$$

$$\alpha c^4 (1 - 2G + G^2) + c^2 F - c^2 F G + \beta = 0 \quad -(13)$$

i.e.

$$\alpha c^4 G^2 - (2\alpha c^4 + c^2 F) G + \alpha c^4 + c^2 F + \beta = 0 \quad -(14)$$

This is a quadratic:

$$a G^2 + b G + c = 0 \quad -(15)$$

3) where:

$$\left.\begin{aligned} a &= \alpha c^4, \\ b &= -\left(2\alpha c^4 + c^2 F\right) \\ c &= \alpha c^4 + c^2 F + \beta \end{aligned}\right\} \quad -(16)$$

So
$$\boxed{G = \left(\frac{mc^2}{\hbar\omega}\right)^2 = \frac{1}{2a}\left(-b \pm \left(b^2 - 4ac\right)^{1/2}\right)} \quad -(17)$$

This is a formula direct for m^2. Here:

$$\alpha = 1 + \frac{\omega'}{\omega}\left(x^2 - 2x\cos\theta\right) \quad -(18)$$

$$\beta = y^2\frac{\omega'}{\omega} \quad -(19)$$

$$\gamma = 2y\frac{\omega'}{\omega}\left(\cos\theta - x\right) \quad -(20)$$

$$x = \frac{\omega}{\omega'\cos\theta}, \quad y = \left(\frac{\omega}{2\omega'\cos\theta}\right)E$$

$$F = \gamma - \frac{B}{A^2}$$

$$E = \frac{B}{A^2} - D$$

$$D = c^2\left(\left(\frac{\omega'}{\omega}\right)^2 - 1\right)$$

$$B = \gamma_1^2 \underline{M}^2 v_1^2 = v_1^2\left(1 - \frac{v_1^2}{c^2}\right)^{-1}\underline{M}^2,$$

$$A = \left(\gamma_1 - 1\right)\underline{M}\left(1 - \frac{\omega'}{\omega}\right)^{-1}$$