1) 158(1): Some Remarks Concerning the Heisenberg Method

The Heisenberg "uncertainty principle" is inherent in the theory of superposition of waves in one dimension, (J.D. Jackson, "Classical Electrodynamics", 3rd ed., pp. 322 ff., eq. (7.82)). If the wave superposition propagates in Z, and the wavenumber is κ, then:

$$\Delta Z \Delta \kappa \geq \frac{1}{2} \quad (1)$$

where ΔZ and $\Delta \kappa$ are root mean square deviations. This is a property of waves, including matter waves of de Broglie. The latter proposed that:

$$p = \hbar \kappa \quad (2)$$

From eq. (1) in eq. (2):

$$\Delta Z \Delta p \geq \frac{\hbar}{2}. \quad (3)$$

This is elevated to the "Heisenberg Uncertainty Principle" in the Copenhagen interpretation of quantum mechanics. In the interpretation of the causal realist school, eq. (3) has no more information than eq. (1), which is a property of waves.

The wave property can be illustrated as in P. W. Atkins, "Molecular Quantum Mechanics" (Oxford Univ. Press, 2nd. ed., 1983), page 95. Consider a wavefunction of the Gaussian type:

2) $$\psi(x) = N \exp\left(-\frac{x^2}{2\Gamma}\right) \quad -(4)$$

In this case

$$\Delta x \Delta p = \langle x^2 \rangle^{1/2} \langle p^2 \rangle^{1/2} \quad -(5)$$

where

$$\langle x^2 \rangle = N^2 \int_{-\infty}^{\infty} x^2 \exp\left(-\frac{x^2}{\Gamma}\right) dx = \frac{\Gamma}{2} \quad -(6)$$

$$\langle p^2 \rangle = N^2 \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2\Gamma}\right)\left(-\hbar^2 \frac{d^2}{dx^2}\right) \exp\left(-\frac{x^2}{2\Gamma}\right) dx \quad -(7)$$

$$= \frac{\hbar^2}{2\Gamma}$$

We have $\langle x \rangle = 0, \ \langle p \rangle = 0. \quad -(8)$

So $$\delta x \, \delta p = \frac{\hbar}{2} \quad (9)$$

These equations come from the definition:

$$\langle x \rangle = \int \psi^* \hat{x} \psi \, d\tau \quad -(10)$$

$$\langle p \rangle = \int \psi^* \hat{p} \psi \, d\tau \quad -(11)$$

$$\langle x^2 \rangle = \int \psi^* \hat{x}^2 \psi \, d\tau \quad -(12)$$

$$\langle p^2 \rangle = \int \psi^* \hat{p}^2 \psi \, d\tau \quad -(13)$$

of expectation values. The <u>operators</u> are defined as follows.

3)

$$\hat{x}\phi = x\phi \quad -(14)$$

$$\hat{p}\phi = i\hbar\frac{\partial}{\partial x}\phi \quad -(15)$$

$$\hat{x}^2\phi = x^2\phi \quad -(16)$$

$$\hat{p}^2\phi = -\hbar^2\frac{\partial^2\phi}{\partial x^2} \quad -(17)$$

The causal realist school asserts that in these equations there is nothing that is "unknowable". The Copenhagen school asserts that the system contains "indeterminacy". The latter is interpreted to mean that if x and p are not known precisely, they commute:

$$[x, p]\phi = 0 \quad -(18)$$

$$= (xp - px)\phi$$

in which case x and p are simple multiples of ϕ.

However, in quantum mechanics:

$$p^\mu = i\hbar\partial^\mu \quad -(19)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right) \quad -(20)$$

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) \quad -(21)$$

so

$$\hat{E}\phi = i\hbar\frac{\partial}{\partial t}\phi \quad -(22)$$

$$\hat{p}\phi = -i\hbar\underline{\nabla}\phi. \quad -(23)$$

4) It follows that the classical:

$$T = \frac{p^2}{2m} \quad -(24)$$

becomes

$$-\frac{\hbar^2}{2m} \psi = T\psi \quad -(25)$$

i.e.

$$\hat{H}\psi = T\psi \quad -(26)$$

In this case, only the kinetic energy is considered,

so

$$H = T = E. \quad -(27)$$

This gives the usual Schrodinger equation:

$$\hat{H}\psi = E\psi \quad -(28)$$

It is also possible to write:

$$-i\hbar \frac{\partial \psi}{\partial x} = p\psi \quad -(29)$$

and

$$-i\hbar \psi^* \frac{\partial \psi}{\partial x} = \psi^* p \psi. \quad -(30)$$

A solution of eqns. (29) and (30) is:

$$p = \hbar \kappa, \quad \psi = \exp(i\kappa x) \quad -(31)$$

This is also a solution of eq. (25). It is seen that the de Broglie postulate:

$$p = \hbar \kappa \quad -(32)$$

is equivalent to

$$p = -i\hbar \frac{\partial}{\partial x} \quad -(33)$$

Essentially, the Schroedinger equation is:

$$\hat{x}\phi = x\phi\,, \quad — (34)$$

$$\hat{p}\phi = -i\hbar\frac{\partial\phi}{\partial x}\,. \quad — (35)$$

$$E\phi = i\hbar\frac{\partial\phi}{\partial t} \quad — (36)$$

The Heisenberg equation is:

$$[\hat{x}, \hat{p}]\phi = i\hbar\phi \quad — (37)$$

and contains no more information than the Schroedinger equation. Eq. (37) is:

$$\hat{x}(\hat{p}\phi) - \hat{p}(\hat{x}\phi)$$

$$= -i\hbar\, x\frac{\partial\phi}{\partial x} + i\hbar\frac{\partial}{\partial x}(x\phi)$$

$$= -i\hbar\, x\frac{\partial\phi}{\partial x} + i\hbar\, x\frac{\partial\phi}{\partial x} + i\hbar\phi$$

$$= i\hbar\phi \quad — (38)$$

Q.E.D.

If it were possible to solve the Schroedinger equation there would be no need of the Heisenberg equation.

As described by J. R. Croca in "Towards a Nonlinear Quantum Physics" (WS 2001), pp. 94 ff.

6) Eq (37) is simply a consequence of:

$$p^\mu = i\hbar \partial^\mu \quad -(39)$$

and has no deeper meaning. In Croca's work the uncertainty principle is shown to be incorrect by nine orders of magnitude. For the wavenumber to be correctly normalized to a value of unity, it must be:

$$\psi = \frac{1}{L^{1/2}} e^{i\kappa x} \quad -(40)$$

with

$$\int_{-L/2}^{L/2} \psi^* \psi \, dx = 1 \quad -(41)$$

Then:

$$\langle \hat{x} \rangle = \frac{1}{L} \int_{-L/2}^{L/2} \psi^* x \psi \, dx = \frac{1}{L} \left. \frac{x^2}{2} \right|_{-L/2}^{L/2} = 0, \quad -(42)$$

$$\langle \hat{p} \rangle = -\frac{i\hbar}{L} \int_{-L/2}^{L/2} e^{-i\kappa x} \frac{\partial}{\partial x} e^{i\kappa x} \, dx = \hbar \kappa \quad -(43)$$

$$\langle \hat{x}^2 \rangle = \frac{1}{L} \int_{-L/2}^{L/2} \psi^* x^2 \psi \, dx = \frac{1}{L} \left. \frac{x^3}{3} \right|_{-L/2}^{L/2} = \frac{L^2}{12} \quad -(44)$$

$$\langle \hat{p}^2 \rangle = -\frac{\hbar^2}{L} \int_{-L/2}^{L/2} e^{-i\kappa x} \frac{\partial^2}{\partial x^2} e^{i\kappa x} \, dx = \hbar^2 \kappa^2 \quad -(45)$$

So:

$$\left.\begin{array}{l} \langle \hat{x} \rangle = 0, \quad \langle \hat{x}^2 \rangle = \dfrac{L^2}{12}, \\ \langle \hat{p} \rangle = \hbar\kappa, \quad \langle \hat{p}^2 \rangle = \hbar^2 \kappa^2 . \\ : \psi = \dfrac{1}{L^{1/2}} e^{i\kappa x} \end{array}\right\} \quad -(46)$$

Therefore:

$$\delta \hat{x} = \left(\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2 \right)^{1/2} = \frac{L}{\sqrt{12}} \quad -(47)$$

$$\delta \hat{p} = 0$$

and

$$\boxed{\delta \hat{x} \, \delta \hat{p} = 0} \quad -(48)$$

The Heisenberg "uncertainty principle" is:

$$\delta \hat{x} \, \delta \hat{p} \geq \frac{\hbar}{2} \quad -(49)$$

<u>Copenhagen Interpretation</u>

This asserts (incorrectly) that eq. (49) must be true. This is possible if and only if L becomes infinite. Copenhagen then asserts that:

$$\delta \hat{p} = 0, \quad \delta \hat{x} = \infty \quad -(50)$$

and that x is "unknowable". All kinds of

8) complications arise from the fact that:

$$L \rightarrow \infty. \quad -(51)$$

The root cause of the assertion (51) is that the Born normalization is asserted to extend from $-\infty$ to ∞ along x.

Causal and Local Paradigm of de Broglie

A quantum particle is described by a pilot wave which moves with the particle. The solutions of the Schroedinger equation are real waves, not probability waves. So in the causal interpretation the Born probability interpretation and normalization does not apply. The real quantum particle is described by a finite, localized wavelet.

In an experiment on pp. 112 ff of his book, Croca shows that

$$\boxed{\Delta x \, \Delta p = 10^{-9} \frac{\hbar}{2} \quad \text{experimentally}} \quad -(52)$$

This violates eq. (49).

Conclusion

If L is finite in the Born interpretation, then

$$\Delta x \, \Delta p = 0 \quad -(53)$$

and experiments indicate

$$\Delta x \, \Delta p \sim 0 \quad -(54)$$