1) 158(3): Theory of the Photoelectric Effect for Finite Photon Mass m.

This was first developed in note 157(10) using the theory of Compton scattering with the work function defined by:

$$\Phi = \alpha M c^2 \quad -(1)$$

where α is characteristic of a material of the photoelectric effect. Therefore it is assumed ~~that~~:

$$M c^2 = \Phi / \alpha \quad -(2)$$

For one electron, $\alpha = 1. \quad -(3)$

The kinetic energy of the emitted electron is given by

$$T = E - \alpha M c^2 = \hbar(\omega_1 - \omega_2) \quad -(4)$$

The theory of Compton scattering is the special case $-(5)$

(3) gives:

$$\omega_1 - \omega_2 = \frac{\hbar}{Mc^2}\left[\omega_1\omega_2 - (\omega_1^2 - \omega_0^2)^{1/2}(\omega_2^2 - \omega_0^2)^{1/2}\cos\theta\right] + \frac{m^2c^2}{\hbar M}$$

so

$$T = \frac{\hbar^2}{Mc^2}\left[\omega_1\omega_2 - (\omega_1^2 - \omega_0^2)^{1/2}(\omega_2^2 - \omega_0^2)^{1/2}\cos\theta\right] + \frac{m^2c^2}{M} \quad -(6)$$

2) In the more general case (2):

$$T = \frac{\hbar^2 \alpha}{\Phi}\left[\omega_1\omega_2 - \left(\omega_1^2-\omega_0^2\right)^{1/2}\left(\omega_2^2-\omega_0^2\right)^{1/2}\cos\theta\right] + m^2c^4\frac{\alpha}{\Phi} \quad -(7)$$

So the kinetic energy of the emitted electron is:

$$\boxed{T = \frac{\alpha}{\Phi}\left[\hbar^2\left(\omega_1\omega_2 - \left(\omega_1^2-\omega_0^2\right)^{1/2}\left(\omega_2^2-\omega_0^2\right)^{1/2}\cos\theta\right) + m^2c^4\right]} \quad -(8)$$

Therefore, knowing α, the photon mass can be found in theory from the photoelectric effect.

The usual theory of Compton scattering assumes:

$$\left.\begin{array}{l} m = ?\ 0 \\ \omega_0 = ?\ 0 \end{array}\right\} \quad -(9)$$

in which case:

$$T = \frac{\alpha}{\Phi}\left(\hbar^2\omega_1\omega_2\left(1-\cos\theta\right)\right) \quad -(10)$$

and if the work function is known, α can be found from eq. (1).

3) The usual theory of the Compton effect can be used for the photoelectric effect if eq. (1) is assumed. Therefore the relativistic kinetic energy of the electron after collision with the photon is

$$T = \left(\frac{\hbar^2}{Mc^2}\right) \omega_1 \omega_2 \left(1 - \cos\theta\right) \quad - (11)$$

So the Compton effect and photoelectric effect: the relativistic kinetic energy is

$$T = (\gamma - 1) Mc^2 \quad - (12)$$

So
$$\gamma - 1 = \left(\frac{\hbar}{Mc^2}\right)^2 \omega_1 \omega_2 \left(1 - \cos\theta\right) \quad - (13)$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \left(\frac{\hbar}{Mc^2}\right)^2 \omega_1 \omega_2 \left(1 - \cos\theta\right) \quad - (14)$$

If $v \ll c$ — (15)

$$\gamma \sim 1 + \frac{1}{2}\frac{v^2}{c^2} \quad - (16)$$

So the non-relativistic electron velocity after

4) collision is given by:

$$v^2 = 2c^2\left(\frac{\hbar}{mc^2}\right)^2 \omega_1\omega_2\left(1-\cos\theta\right) \quad -(17)$$

$$\boxed{v^2 = 2\left(\frac{\hbar}{mc}\right)^2 \omega_1\omega_2\left(1-\cos\theta\right)} \quad -(18)$$

The Compton wavelength of the electron is:

$$\frac{\hbar}{mc} = 2.426309 \times 10^{-12}\ m \quad -(19)$$

In the original experiment, Compton used X ray frequencies. If it is assumed that ω_1 and ω_2 are in the radio frequency range then:

$$\omega_1 \sim \omega_2 \sim 10^{10}\ rad\ s^{-1} \quad -(20)$$

so:

$$v^2 \sim 2 \times 5.886975 \times 10^{-4}\left(1-\cos\theta\right) \quad -(21)$$

$$v \sim 0.0343132\left(1-\cos\theta\right)^{1/2} \quad -(22)$$

$$m\,s^{-1}$$

It is seen that this result for the electron velocity after collision is modified by photon mass to:

5)

$$T = (\gamma - 1) M c^2 \qquad -(23)$$

$$= \frac{\hbar^2}{Mc^2}\left(\omega_1\omega_2 - (\omega_1^2-\omega_0^2)^{1/2}(\omega_2^2-\omega_0^2)^{1/2}\cos\theta\right) + \frac{m^2c^4}{Mc^2}$$

where $\omega_0 = mc^2/\hbar$. $-(24)$

If it were possible to measure the electron velocity v with sufficient accuracy, the photon mass m could be determined.

Estimate of Accuracy Needed

If it is assumed that

$$m \sim 10^{-52}\ \text{kg} \qquad -(25)$$

then

$$\omega_0 \sim 0.1\ \text{rad s}^{-1} \qquad -(26)$$

and

$$\frac{m^2c^4}{Mc^2} \sim 10^{-57}\ \text{J}, \qquad -(27)$$

$$\frac{\hbar^2\omega_1\omega_2}{Mc^2} \sim 10^{-34}\ \text{J} \qquad -(28)$$

Since:

$$\left.\begin{aligned}\omega_1 \sim \omega_2 &\sim 10^{10}\ \text{rad s}^{-1}\\ \omega_0 &\sim 0.1\ \text{rad s}^{-1}\end{aligned}\right\} \qquad -(29)$$

eq. (23) is well approximated by:

6)

$$T = (\gamma - 1) M c^2 \sim \frac{\hbar^2}{M c^2} \omega_1 \omega_2 (1 - \cos\theta) + \frac{m^2 c^2}{M} \quad -(30)$$

So

$$\gamma = 1 + \left(\frac{\hbar}{M c^2}\right)^2 \omega_1 \omega_2 (1 - \cos\theta) + \frac{m^2 c^2}{M^2 c^2}$$

$$= \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (31)$$

If

$$v \ll c \quad -(32)$$

$$\frac{v^2}{2c^2} \sim \left(\frac{\hbar}{M c^2}\right)^2 \omega_1 \omega_2 (1 - \cos\theta) + \left(\frac{m}{M}\right)^2$$

$$\boxed{v^2 \sim 2\left(\left(\frac{\hbar}{M c}\right)^2 \omega_1 \omega_2 (1 - \cos\theta) + \left(\frac{m}{M}\right)^2 c^2\right)} \quad -(33)$$

The correction due to photon mass is:

$$\boxed{\Delta v \sim \frac{m}{M} c} \quad -(34)$$

which is a simple result that can be tested experimentally given enough precision. For a mass m of $\sim 10^{-52}$ kgm, $M = 9.11 \times 10^{-31}$ kgm:

$$\boxed{\Delta v \sim 3.3 \times 10^{-14} \ m s^{-1}} \quad -(35)$$