158(6): A Simple Equation for Photon Mass from the Compton Effect.

In note 158(5) it was shown that the Compton effect for the interaction of a photon with an electron with initial momentum p is:

$$\omega - \omega' = \frac{\hbar}{E}\omega\omega'(1-\cos\theta) + c^2\frac{p}{E}k\cos\phi \quad -(1)$$

Consider the case of the usual Compton effect:

$$p = 0 \quad -(2)$$

then $$\omega - \omega' = \frac{\hbar}{Mc^2}\omega\omega'(1-\cos\theta) \quad -(3)$$

where M is the electron mass. This is the usual formula for the Compton effect.

Now consider the photon to have mass m.

Then:

$$\hbar\omega = \gamma mc^2 \quad -(4)$$

$$\hbar\omega' = \gamma' mc^2 \quad -(5)$$

where:

$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2}, \quad -(6)$$

$$\gamma' = \left(1-\frac{v'^2}{c^2}\right)^{-1/2} \quad -(7)$$

2) Therefore:

$$\boxed{\frac{1}{\gamma'} - \frac{1}{\gamma} = \frac{m}{M}\left(1 - \cos\theta\right)} \quad -(8)$$

From conservation of energy:

$$\gamma mc^2 + Mc^2 = \gamma' mc^2 + E' \quad -(9)$$

where E' is the final energy of the electron:

$$E' = \left(p'^2 c^2 + M^2 c^4\right)^{1/2} \quad -(10)$$

so

$$\boxed{\gamma - \gamma' = \frac{1}{mc^2}\left(E' - Mc^2\right)} \quad -(11)$$

From conservation of momentum:

$$\hbar \underline{k} = \hbar \underline{k}' + \underline{p}' \quad -(12)$$

where

$$\hbar \underline{k} = \gamma m \underline{v} \quad -(13)$$

$$\hbar \underline{k}' = \gamma' m \underline{v}' \quad -(14)$$

$$\underline{p}' = \hbar\left(\underline{k} - \underline{k}'\right) \quad -(15)$$

so:

$$p'^2 = \hbar^2\left(k^2 + k'^2 - 2kk'\cos\theta\right) \quad -(15a)$$

$$\text{i.e.} \quad p'^2 = m^2\left(\gamma^2 v^2 + \gamma'^2 v'^2 - 2\gamma\gamma' vv'\cos\theta\right)$$

i.e.

3)

$$m^2\left(\gamma^2 v^2 + \gamma'^2 v'^2 - 2\gamma\gamma' v v' \cos\theta\right) = p'^2$$

— (16)

Equations (8), (11) and (16) are three eqns in three unknowns, v, v' and m. The other quantities are known experimentally, p' is the final momentum of the electron. To an excellent approximation, p' may be found from eq. (15) and p' from eq. (15a)

Computer Algebra

Find m in terms of p' and E', the scattering angle θ and the electron mass M.

Experiment

Measure p' experimentally, also giving E'.