153(4): Equation of Motion and Orbital Theorem from Metric.

This note gives complete details of the calculation for convenience of reference, and to show that the equation of motion is spacetime. The calculation is given in cylindrical polar coordinates and in the XY plane ($dZ = 0$), but can be generalized to any metric that obeys the Orbital Theorem of UFT 111. The calculation can also be generalized to other symmetries of spacetime.

Consider therefore the metric:

$$ds^2 = c^2 d\tau^2 = e^{-r_0/r} c^2 dt^2 - e^{r_0/r} dr^2 - r^2 d\phi^2 \quad -(1)$$

where τ is the proper time. The hamiltonian is defined in general relativity as the invariant:

$$H = \mathcal{L} = T = \frac{1}{2} mc^2 = \frac{1}{2} m \left(e^{-r_0/r} c^2 \left(\frac{dt}{d\tau}\right)^2 - e^{r_0/r} \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right) \quad -(2)$$

The hamiltonian is defined as half the rest energy mc^2:

$$H = \frac{1}{2} m \left(\frac{ds}{d\tau}\right)^2 \quad -(3)$$

Here m is the mass of an object in orbit around another object. It is seen that H/m is pure geometry. This is the geometry of spacetime, so energy is available in spacetime.

With the lagrangian $\mathcal{L}$ of eq. (2), the Euler Lagrange equation gives constants of motion. These are unchanging or conserved quantities. They are total energy E, linear momentum p, and angular momentum L, and are defined as follows:

$$E = mc^2 e^{-r_0/r} \frac{dt}{d\tau} \quad -(4)$$

$$p = m e^{r_0/r} \frac{dr}{d\tau} \quad -(5)$$

$$L = mr^2 \frac{d\phi}{d\tau} \quad -(6)$$

Now multiply both sides of eq. (2) by $e^{-r_0/r}$:

$$\frac{1}{2} mc^2 e^{-r_0/r} = \frac{1}{2} m \left(\left(e^{-r_0/r} c \left(\frac{dt}{d\tau}\right) \right)^2 - \left(\frac{dr}{d\tau}\right)^2 - e^{-r_0/r} r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right) \quad -(7)$$

Rearrange terms to give:

$$\frac{1}{2} m \left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2} m \left(\left(e^{-r_0/r} c \left(\frac{dt}{d\tau}\right)\right)^2 - c^2 e^{-r_0/r} - e^{-r_0/r} r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right)$$

$$= \frac{1}{2} \frac{E^2}{mc^2} - \frac{1}{2} e^{-r_0/r} \left(mc^2 + \frac{L^2}{mr^2} \right) \quad -(8)$$

The equation of motion is:

$$\boxed{\frac{1}{2} m \left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2} \left(\frac{E^2}{mc^2} - e^{-r_0/r} \left(mc^2 + \frac{L^2}{mr^2} \right) \right)} \quad -(9)$$

To obtain the orbital equation, the dependence on proper time τ is eliminated using:

$$\frac{dr}{d\tau} = \frac{d\phi}{d\tau} \frac{dr}{d\phi} = \left(\frac{L}{mr^2} \right) \frac{dr}{d\phi} \quad -(10)$$

where eq. (6) has been used. So from (10) in (9):

5)

$$\frac{1}{m}\left(\frac{L}{r^2}\right)^2\left(\frac{dr}{d\phi}\right)^2 = \frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right) \quad -(11)$$

$$\left(\frac{dr}{d\phi}\right)^2 = m\left(\frac{r^2}{L}\right)^2\left(\frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right)\right) \quad -(12)$$

$$= r^4\left(\left(\frac{E}{Lc}\right)^2 - e^{-r_0/r}\left(\left(\frac{mc}{L}\right)^2 + \frac{1}{r^2}\right)\right) \quad -(13)$$

$$= r^4\left(\frac{1}{b^2} - e^{-r_0/r}\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right) \quad -(14)$$

where

$$b = \frac{Lc}{E}, \quad a = \frac{L}{mc} \quad -(15)$$

are <u>constants of motion.</u>

The orbital equation is obtained by inverting eq. (14) and taking the square root:

$$\boxed{\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-r_0/r}\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2}} \quad -(16)$$

In gravitational theory, the distance r_0 is:

$$r_0 = \frac{2MG}{c^2} \quad -(17)$$

where G is Newton's constant, c the vacuum speed of light, and M the mass of an object that influences the mass m. In light deflection by the sun, m is the photon mass, and M is the mass of the sun. The light deflection is:

$$\Delta \phi = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left(\frac{1}{b^2} - e^{-r_0/r} \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-1/2} dr \qquad -(18)$$

where R_0 is the distance of closest approach.

In UFT 150 this important calculation was carried out correctly for the first time, giving the first estimate of the mass of the photon from light deflection due to gravitation. The approximation:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} \qquad -(19)$$

was used. This gave a plausible result of about 10^{-41} kilograms for photon mass. More accurately, eq. (18) should be used when:

$$r_0 \sim r \qquad -(20)$$

or when $\underline{M}$ is much heavier than the solar mass.

UFT 150 showed that Einstein's calculation was wildly incorrect, and could not possibly have been verified by Eddington. The correct method opens up the route to many advances in cosmology.