1)

153(7): Generalization of the Orbital Theorem

The most general spherical spacetime is:

$$ds^2 = \exp(-2\alpha) c^2 dt^2 + \exp(2\beta) dr^2 + r^2 d\phi^2 \quad -(1)$$

as given by Carroll, eq. (7.13). In general:

$$\alpha = \alpha(r, t), \quad \beta = \beta(r, t) \quad -(2)$$

i.e. α and β are functions of r and t. Now that it is known that the Einstein field equation is incorrect, a plausible method is needed to find α and β. A guide as to how to proceed is given by the fact that the relativistic Kepler problem is described and solved by:

$$\exp(-\alpha(r)) = 1 - \frac{r_0}{r} \quad -(3)$$

$$\exp(\beta(r)) = \left(1 - \frac{r_0}{r}\right)^{-1} \quad -(4)$$

This gives processing elliptical orbits.

It is also known that the orbits of binary pulsars are inward moving processing ellipses described by

$$\exp(-\alpha(r)) = 1 - \frac{r_0}{r} - \frac{r_1}{r^2} \quad -(5)$$

$$\exp(\beta(r)) = \left(1 - \frac{r_0}{r} - \frac{r_1}{r^2}\right)^{-1} \quad -(6)$$

so it is reasonable to assume that in general:

$$\exp(-\alpha(r)) = 1 - \frac{r_0}{r} - \dots - \frac{r_n}{r^n} \quad -(7)$$

$$\exp(\beta(r)) = \left(1 - \frac{r_0}{r} - \dots - \frac{r_n}{r^n}\right)^{-1} \quad -(8)$$

2)

These series expansions can be built up by an extension of the Orbital Theorem of UFT III:

$$m r = \frac{r}{n} = \int dr \quad -(9)$$

to:

$$m_1 r = \int dr = r - \beta_1 \quad -(10)$$

$$m_2 \frac{r^2}{2} = \int r\,dr = \frac{r^2}{2} - \frac{\beta_2}{2} \quad -(11)$$

$$m_3 \frac{r^3}{3} = \int r^2 dr = \frac{r^3}{3} - \frac{\beta_3}{3} \quad -(12)$$

$$\vdots$$

$$m_n \frac{r^n}{n} = \int r^{n-1} dr = \frac{r^n}{n} - \frac{\beta_n}{n} \quad -(13)$$

where $\beta_1, \dots, \beta_n$ are constants of integration. So:

$$m = \frac{1}{n}\left(m_1 + m_2 + \dots + m_n\right) = 1 - \frac{\beta_1}{nr} - \frac{\beta_2}{nr^2} - \dots - \frac{\beta_n}{nr^n}$$

$$\boxed{\begin{aligned} m &= 1 - \frac{r_0}{r} - \frac{r_1}{r^2} - \dots - \frac{r_n}{r^n} \quad -(14) \\ n &= m^{-1} \quad -(15) \end{aligned}}$$

In chapter 15 of GCUFT6, r_1 was found by comparison with the orbit of a binary pulsar, which is an inward moving and precessing ellipse.

The Maclaurin expansion:

$$e^{-\alpha} = 1 - \alpha + \frac{\alpha^2}{2!} - \dots \quad -(16)$$

can be written up to the first two terms as:

3)

$$\alpha(r) = \frac{r_0}{r} \quad -(17)$$

So $$\exp\left(-\frac{r_0}{r}\right) = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \ldots \quad (18)$$

but this function gives rise to a metric that gives an <u>expanding</u> precessing ellipse.

It may be ~~that~~ some solar system anomalies are described by eq. (18), which comes from the Maclaurin expansion:

$$f(\alpha) = f(0) + \alpha\left(\frac{df}{d\alpha}\right)_{\alpha=0} + \frac{\alpha^2}{2!}\left(\frac{d^2 f}{d\alpha^2}\right)_{\alpha=0} + \ldots \quad -(19)$$

The equation of motion for a metric of type (18) is:

$$\frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right)\right) - (20)$$

i.e.:

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) = \frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 - \frac{r_0}{r}mc^2 + \frac{L^2}{mr^2} - \frac{r_0}{r}\frac{L^2}{mr^2} + \frac{1}{2}\left(\frac{r_0}{r}\right)^2\left(mc^2 + \frac{L^2}{mr^2}\right) - (21)$$

The effective potential is:

$$V = -\frac{mMG}{r} - \frac{MGL^2}{mc^2r^3} + \frac{L^2}{2mr^2}\left(1 + \left(\frac{2MG}{c^2 r}\right)^2\right) + 2m\left(\frac{MG}{cr}\right)^2 \quad -(22)$$

This contains two new repulsive terms in addition to the centrifugal force. These terms force the orbit outwards by a small amount.

The orbital equation is:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-r_0/r}\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2}$$

$$= \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r} + \frac{1}{2}\left(\frac{r_0}{r}\right)^2 - \ldots\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} \qquad -(23)$$

and the light deflection is:

$$\Delta\phi = \int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-r_0/r}\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr \qquad -(24)$$

This equation is also true for the deflection of any object of mass m, such as a meteorite, whose orbit can be observed accurately and any anomaly determined.