153(13): Maximization of the Effect of Electromagnetism on Gravitation.

The following is a straightforward calculation based on the minimal prescription. In this first calculation the spin connection is not yet used.

Define the gravitational four potential:

$$\Phi^\mu = \left(\frac{\Phi}{c}, \underline{\Phi} \right) \quad -(1)$$

and the electromagnetic four potential:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A} \right). \quad -(2)$$

Define the four momentum:

$$p^\mu = \left(\frac{E}{c}, \underline{p} \right) \quad -(3)$$

The minimal prescription means that:

$$\boxed{p^\mu \rightarrow p^\mu + e_1 A^\mu + m \Phi^\mu} \quad -(4)$$

for a particle of mass m and charge e_1 in the potentials A^μ and Φ^μ.

The hamiltonian is:

$$H = \frac{1}{2m} p^\mu p_\mu \quad -(5)$$

and is quadratic in p^μ so contains a cross-term between electromagnetism and gravitation. For example if we consider the scalar potentials, the hamiltonian is:

$$H_1 = \frac{1}{2mc^2}\left(E_0 + e_1\phi + m\Phi\right)^2 \quad - (6)$$

If for simplicity:

$$E_0 = 0 \quad - (7)$$

then

$$H_1 = \frac{1}{2mc^2}\left(e_1^2\phi^2 + m^2\Phi^2 + 2me_1\phi\Phi\right) \quad - (8)$$

and

$$\boxed{H_1(\text{cross term}) = \frac{e_1\phi\Phi}{c^2}} \quad - (9)$$

Similarly, from the momentum part of p^μ:

$$H_2 = \frac{1}{2m}\left(e_1^2\underline{A}\cdot\underline{A} + m^2\underline{\Phi}\cdot\underline{\Phi} + 2me_1\underline{A}\cdot\underline{\Phi}\right) - (10)$$

So:

$$\boxed{H(\text{cross term}) = e_1\left(\frac{\phi\Phi}{c^2} + \underline{A}\cdot\underline{\Phi}\right)} \quad - (11)$$

The cross term depends only on e_1 and not on m. If resonance is introduced into the problem then H(cross term) is maximized. The electric field is much stronger than the magnetic field because in S. I. units:

$$E^{(0)} = cB^{(0)} \quad - (12)$$

so it is more practical to use the electric field to

3)

charge gravitation.

The charge in force from eq. (4) is:

$$\underline{F} = e_1 \underline{E} + m\underline{g} \quad -(12)$$

Define this to be in the Z axis, so:

$$\underline{F} = F\underline{k} \quad -(13)$$

and

$$F = e_1 E + mg \quad -(14)$$

where

$$g = \dot{v} = \ddot{r} \quad -(15)$$

so:

$$\boxed{F = e_1 E + m\ddot{r}} \quad -(16)$$

<u>Euler Bernoulli Resonance</u>

The simplest type of Euler Bernoulli resonance is described by:

$$\boxed{F + kr = F_0 \cos \omega t} \quad -(17)$$

where k is the Hooke constant and $F_0 \cos \omega t$ a driving term. So from eqs. (16) and (17):

$$m\ddot{r} + e_1 E + kr = F_0 \cos \omega t \quad -(18)$$

so:

$$\boxed{\begin{array}{c} m\ddot{r} + kr = F_0 \cos \omega t - e_1 E, \\ g = m\ddot{r} \end{array}} \quad -(19)$$

Now assume that

$$\boxed{E = E_0 \cos \omega t} \quad -(20)$$

4) Therefore eq. (19) is:

$$m\ddot{r} + kr = 2e_1E_0\cos\omega t \quad -(21)$$

which is rewritten as:

$$\ddot{r} + \omega_0^2 r = A\cos\omega t \quad -(22)$$

with

$$\omega_0^2 = \frac{k}{m}, \quad A = \frac{2e_1}{m}E_0. \quad -(23)$$

Amplitude resonance occurs at

$$\boxed{r(t) = \left(\frac{A}{\omega_0^2-\omega^2}\right)\cos\omega t} \quad -(24)$$

Energy resonance also occurs. If the kinetic energy is defined as:

$$T = \frac{1}{2}m\dot{r}^2 \quad -(25)$$

From eqns. (24) and (25), kinetic energy resonance occurs at:

$$T = \frac{mA^2}{2}\left(\frac{\omega^2}{\omega_0^2-\omega^2}\right)\sin^2\omega t \quad -(26)$$

so

$$\boxed{\langle T\rangle = \frac{mA^2}{4}\cdot\frac{\omega^2}{\omega_0^2-\omega^2}} \quad -(27)$$

because:

$$\langle\sin^2\omega t\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\sin^2\omega t\,dt = \frac{1}{2} \quad -(28)$$

So kinetic energy resonance occurs at the

5)

Same frequency as amplitude resonance:

$$\boxed{\omega = \omega_0} \quad -(29)$$

Potential energy is proportional to the square of the amplitude and in this system potential energy resonance also occurs at the frequency (29).

Resonance in H (cross term) of eq. (11) occurs at the frequency (29).

At resonance gravitational kinetic energy is maximized by the alternating electric field (20), and the mixed potential energy (11) is also maximized by the alternating electric field (20). The gravitational field:

$$\underline{g} = m\ddot{\underline{r}} \quad -(30)$$

is maximized by the alternating electric field (20).

Practical Devices in Aircraft and Spacecraft.

The device is carried on board the craft and is designed to maximize $\underline{g}$ in a direction opposite to the earth's $\underline{g}$.