153(1): Euler-Bernoulli Resonance in a Spherically Symmetric Spacetime.

In general, the well known equation for Euler-Bernoulli resonance is the linear oscillator:

$$m\ddot{r} + 2\beta\dot{r} + \omega_0^2 r = F_0 \cos(\omega t) \quad -(1)$$

in the non-relativistic, linear, limit. The left hand side is Hooke's law plus friction, and the right hand side is the driving term. This is a mechanism that supplies the system with energy from an external source at a rate equal to that absorbed by the damping medium. Energy is transferred and total energy is conserved. As is well known (Marion and Thornton chapter 3) eq. (1) produces amplitude and energy resonance. At resonance, the kinetic energy:

$$T = \frac{1}{2} m \dot{r}^2 \quad -(2)$$

becomes very large, but total energy is conserved.

If spacetime is the mechanism that gives the driving term, then the origin of $F_0 \cos(\omega t)$ is a metric. At resonance, the kinetic energy T supplied by spacetime is very large, and total energy is conserved. This spacetime energy is transferred to the unforced oscillator:

$$m\ddot{r} + 2\beta\dot{r} + \omega_0^2 r = 0 \quad -(3)$$

which is any kind of naturally occurring oscillator. The naturally occurring oscillator may be an electric circuit, as is well known.

2) Consider the metric of spherical spacetime:

$$H = \mathcal{L} = T = \frac{1}{2}mc^2 = \frac{1}{2}m\left(e^{-r_0/r}c^2\left(\frac{dt}{d\tau}\right)^2 - e^{r_0/r}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2\right) \quad -(4)$$

which is:

$$\frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right)\right) \quad -(5)$$

in the notation of previous notes. In the approximation:

$$e^{-r_0/r} = 1 - \frac{r_0}{r} \quad -(6)$$

$$\frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - \left(1-\frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right)\right). \quad -(7)$$

If there is no angular momentum:

$$L = 0 \quad -(8)$$

then

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) = \frac{p^2}{2m} - \frac{1}{2}mc^2\frac{r_0}{r} \quad -(9)$$

If

$$r_0 << r, \quad -(10)$$

$$E = \gamma mc^2, \quad p = \gamma m\frac{dr}{dt} \quad -(11)$$

In the limit

$$v << c \quad -(12)$$

eq. (9) is:

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} - \frac{1}{2}mc^2\frac{r_0}{r} \quad -(13)$$

where:

$$v = \frac{dr}{dt}, \quad p = mv \quad -(14)$$

The effective potential energy in eq. (13) is

3)

$$V = -\frac{1}{2}mc^2\frac{r_0}{r} \quad -(15)$$

and the force is:

$$F = -\frac{\partial V}{\partial r} = -\frac{1}{2}mc^2\frac{r_0}{r^2} \quad -(16)$$

For gravitation: $$r_0 = \frac{2MG}{c^2} \quad -(17)$$

so $$F = -mMG/r^2 \quad -(18)$$

which is the Newton inverse square law.

For electrostatics $$r_0 = \frac{e_1}{m}\frac{e_2}{4\pi\epsilon_0 c^2} \quad -(19)$$

so $$F = -\frac{e_1 e_2}{4\pi\epsilon_0 r^2} \quad -(20)$$

which is the Coulomb inverse square law.

The Newton and Coulomb force laws been derived from spacetime.

The driving force of spacetime is therefore:

$$F = -\frac{1}{2}mc^2\frac{r_0}{r^2} = F_0\cos(\omega t) \quad -(21)$$

so $$\boxed{r_0 = -\left(\frac{2r^2}{mc^2}\right)F_0\cos(\omega t)} \quad -(22)$$

and the metric is:

4)

$$ds'^2 = c^2 d\tau^2 = c^2\left(1 - \frac{r_0}{r}\right)dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(23)$$

Writing:

$$\frac{\partial V_0}{\partial r} = -F_0 \cos(\omega t) \quad -(24)$$

then:

$$r_0 = \left(\frac{2r^2}{mc^2}\right)\frac{\partial V_0}{\partial r} \quad -(25)$$

The potential energy V_0 of spacetime produces a driving force that drives any naturally occurring unforced oscillator into <u>resonance</u>. At resonance the kinetic energy (T) is maximized. For the frictionless system:

$$m\ddot{r} + \omega_0^2 r = F_0 \cos(\omega t) \quad -(26)$$

<u>the kinetic energy T goes to infinity at resonance.</u>

<u>Spacetime is able to produce infinite energy at the resonance point:</u>

$$\boxed{r(t) = \frac{F_0}{m}\left(\frac{\cos(\omega t)}{\omega_0^2 - \omega^2}\right), \quad -(27) \qquad \omega_0 = \omega.}$$