153(15): Fundamental Definitions of the Gravitational and Electromagnetic Fields.

These are based on the fundamental geometrical structure of spacetime as represented in the Cartan torsion:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(1)$$

$$= \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \Omega^a_{\mu\nu}$$

where

$$\Omega^a_{\mu\nu} = \omega^a_{\mu\nu} - \omega^a_{\nu\mu} \quad -(2)$$

and

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu . \quad -(3)$$

Therefore:

$$\boxed{\tau^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu} \quad -(4)$$

where:

$$\tau^a_{\mu\nu} = T^a_{\mu\nu} - \Omega^a_{\mu\nu} . \quad -(5)$$

Summing over the a indices:

$$\tau_{\mu\nu} = \partial_\mu q_\nu - \partial_\nu q_\mu \quad -(6)$$

The electromagnetic field is:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(7)$$

where

$$F_{\mu\nu} = A^{(0)} \tau_{\mu\nu} , \quad -(8)$$

$$A_\mu = A^{(0)} q_\mu \quad -(9)$$

The gravitational field is:

$$g_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu \quad -(10)$$

where

$$\Phi_\mu = \Phi^{(0)} q_\mu \quad -(11)$$

By antisymmetry:

$$\partial_\mu A_\nu = -\partial_\nu A_\mu \quad -(12)$$

$$\partial_\mu \Phi_\nu = -\partial_\nu \Phi_\mu \quad -(13)$$

In these definitions:

$$F_{\mu\nu} = F_{\mu\nu}^{(1)} + F_{\mu\nu}^{(2)} + F_{\mu\nu}^{(3)} \quad -(14)$$

$$g_{\mu\nu} = g_{\mu\nu}^{(1)} + g_{\mu\nu}^{(2)} + g_{\mu\nu}^{(3)} \quad -(15)$$

$$A_\mu = A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} \quad -(16)$$

$$\Phi_\mu = \Phi_\mu^{(1)} + \Phi_\mu^{(2)} + \Phi_\mu^{(3)} \quad -(17)$$

Furthermore:

$$A_\mu = A_{\mu I} + A_{\mu R} \quad -(18)$$

$$\Phi_\mu = \Phi_{\mu I} + \Phi_{\mu R} \quad -(19)$$

where I denotes irrotational and R denotes rotational.

For example, consider what is usually known as "the static electric field". This is defined by:

3)

$$\underline{E}_{I} = -\underline{\nabla}\phi_{I} - \frac{\partial \underline{A}_{I}}{\partial t}, \quad -(20)$$

with:

$$\underline{\nabla} \times \underline{E}_{I} = 0, \quad -(21)$$

$$\underline{\nabla}\phi_{I} = \frac{\partial \underline{A}_{I}}{\partial t} \quad -(22)$$

The "static" magnetic field is:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{R} \quad -(23)$$

Here

$$\underline{A} = \underline{A}_{I} + \underline{A}_{R} \quad -(24).$$

So

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}, \quad -(25)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(26)$$

where

$$\underline{A} = \underline{A}^{(1)} + \underline{A}^{(2)} + \underline{A}^{(3)} \quad -(27)$$

and

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i A^{(0)} \underline{A}^{(3)*}, \quad -(28)$$

so

$$\boxed{\underline{A} = \underline{A}_{I}^{(1)} + \underline{A}_{I}^{(2)} + \underline{A}_{I}^{(3)} + \underline{A}_{R}^{(1)} + \underline{A}_{R}^{(2)} + \underline{A}_{R}^{(3)}} \quad -(29)$$

$$A_{\mu} = (A_{0}, -\underline{A}) \quad -(30)$$

4) If we consider the Coulombic electric field strength in the Z axis:

$$\underline{E} = -\left(\frac{e_1 e_2}{4\pi \epsilon_0 r^2}\right)\underline{k} \quad -(31)$$

$$r = Z\underline{k} \quad -(32)$$

then:

$$\underline{E} = -\frac{\partial \underline{A}_I^{(3)}}{\partial t} \quad -(33)$$

$$\underline{A}_I^{(3)} = \left(\int \frac{e_1 e_2}{8\pi \epsilon_0 r^2}\, dt\right)\underline{k} \quad -(34)$$

$$= A^{(0)}\underline{k}$$

$$\boxed{A^{(0)} = \int \frac{e_1 e_2}{8\pi \epsilon_0 r^2}\, dt} \quad -(35)$$

If we consider the plane wave:

$$\underline{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right) e^{i(\omega t - kz)} \quad -(36)$$

then

$$\underline{B}_R^{(1)} = \underline{\nabla} \times \underline{A}_R^{(1)} = \frac{B^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right) e^{i\phi} \quad -(37)$$

$$\underline{E}_R^{(1)} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) e^{i\phi} \quad -(38)$$

Similarly:

5) Similarly:

$$\underline{A}_R^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{-i(\omega t - \kappa Z)} \quad -(39)$$

$$\underline{B}_R^{(2)} = \frac{B^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{-i(\omega t - \kappa Z)} \quad -(40)$$

$$\underline{E}_R^{(2)} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{-i(\omega t - \kappa Z)} \quad -(41)$$

$$\underline{A}_R^{(3)} = 0, \quad \underline{A}_I^{(3)} = A^{(0)}\underline{k} \quad -(42)$$

and

$$\underline{A}_R^{(1)} \times \underline{A}_R^{(2)} = iA^{(0)}\underline{A}_I^{(3)} \quad -(43)$$

$$\underline{B}_R^{(1)} \times \underline{B}_R^{(2)} = iB^{(0)}\underline{B}_I^{(3)} \quad -(44)$$

$$\underline{E}_R^{(1)} \times \underline{E}_R^{(2)} = iE^{(0)}\underline{E}_I^{(3)} \quad -(45)$$

$$B^{(0)} = \kappa A^{(0)}, \quad E^{(0)} = cB^{(0)} \quad -(46)$$

$$A^{(0)} = \int \frac{e_1 e_2}{8\pi\epsilon_0 r^2}\, dt \quad -(47)$$

$$\underline{B}_I^{(3)} = B^{(0)}\underline{k} \quad -(48)$$

$$\underline{E}_I^{(3)} = E^{(0)}\underline{k} \quad -(49)$$

Here $\underline{B}_I^{(3)}$ is the radiated magnetic field and the inverse Faraday effect observed in the "static electric field". For $\underline{E}_I^{(3)}$ is the accelerating charges producing radiation, all these fields are present.

6) Resonance Structures for Charge.

These are obtained from an equation of the type:

$$\ddot{q} + \omega_0^2 q = A\cos\omega t \quad -(50)$$

where

$$\boxed{q(t) = \left(\frac{A}{\omega_0^2 - \omega^2}\right)\cos\omega t.} \quad -(51)$$

At resonance:

$$\omega_0 = \omega \quad -(52)$$

and

$$\boxed{q \to \infty.} \quad -(53)$$

If we consider eq. (50):

$$\omega_0^2 = \frac{1}{CL} \quad -(54)$$

where C is the capacitance and L is the inductance,

$$A = \frac{E}{L} \quad -(55)$$

where E is the electromotive force. So eq. (50) is

$$L\ddot{q} + \frac{1}{C} q = E\cos\omega t. \quad -(56)$$

Its mechanical equivalent is:

$$m\ddot{r} + kr = F\cos\omega t, \quad -(57)$$

where m is mass, k is Hooke's constant, r is displacement and F is force.