52(3): Metric with Cylindrical Symmetry.

Assume a metric of the following type:

$$ds^2 = e^{-z_0/z} c^2 dt^2 - dr^2 - r^2 d\phi^2 - e^{z_0/z} dZ^2 \quad -(1)$$

In a cylinder of fixed radius:

$$dr = 0 \quad -(2)$$

so

$$ds^2 = e^{-z_0/z} c^2 dt^2 - e^{z_0/z} dz^2 - r^2 d\phi^2 \quad -(3)$$

The lagrangian is:

$$\mathcal{L} = T = \frac{1}{2} mc^2 = \frac{m}{2}\left(e^{-z_0/z} c^2 \left(\frac{dt}{d\tau}\right)^2 - e^{z_0/z}\left(\frac{dz}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2\right) \quad -(4)$$

$$E = mc^2 e^{-z_0/z} \frac{dt}{d\tau}, \quad L = mr^2 \frac{d\phi}{d\tau}, \quad p = m e^{z_0/z} \frac{dr}{d\tau} \quad -(5)$$

The equation of motion is:

$$m\left(\frac{dz}{d\tau}\right)^2 = \frac{E^2}{mc^2} - e^{-z_0/z}\left(mc^2 + \frac{L^2}{mr^2}\right) \quad -(6)$$

and

$$\frac{d\phi}{dz} = \frac{1}{z^2}\left(\frac{1}{b^2} - e^{-z_0/z}\left(\frac{1}{a^2} + \frac{1}{z^2}\right)\right)^{-1/2} \quad -(7)$$

In the case where dr is not zero, the calculation is more complicated, and will be the subject of the next note.