152(2) : Development of the Spherically Symmetric Metric.

: The metric may be written as:
$$ds^2 = c^2 d\tau^2 = e^{-r_0/r} c^2 dt^2 - e^{r_0/r} dr^2 - r^2 d\phi^2 \qquad -(1)$$

L Cylindrical polar coordinates in XY. Here:
$$r_0 = \frac{2MG}{c^2} \qquad -(2)$$

The Lagrangian is:
$$\mathcal{L} = T = \frac{1}{2} mc^2 = \frac{m}{2}\left(e^{-r_0/r} c^2 \left(\frac{dt}{d\tau}\right)^2 - e^{r_0/r}\left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right) \qquad -(3)$$

The constants of motion are:
$$E = mc^2 e^{-r_0/r} \frac{dt}{d\tau} \quad , \quad L = mr^2 \frac{d\phi}{d\tau} \quad , \quad p = me^{r_0/r}\frac{dr}{d\tau} \qquad -(4)$$

So the equation of motion is:
$$m\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right) \qquad -(5)$$

Limits
1) In the limit:
$$e^{-r_0/r} \rightarrow 1 \qquad -(6)$$

i.e
$$r \rightarrow \infty \quad \text{for a given } r_0 \qquad -(7)$$

$$m\left(\frac{dr}{d\tau}\right)^2 \rightarrow \frac{E^2}{mc^2} - mc^2 - \frac{L^2}{mr^2} \qquad -(8)$$

i.e.
$$\frac{E^2}{mc^2} - mc^2 = m\left(\frac{dr}{d\tau}\right)^2 + \frac{L^2}{mr^2} \qquad -(9)$$

eq. (9) becomes:
$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2. \quad -(11)$$

Therefore eq. (5) may be rewritten as:
$$\frac{E^2}{mc^2} - e^{-r_0/r} mc^2 = m\left(\frac{dr}{d\tau}\right)^2 + e^{-r_0/r}\frac{L^2}{mr^2} \quad -(12)$$

From eq. (9):
$$\frac{d\phi}{dr} = \frac{1}{r}\left(\left(\frac{v}{\omega r}\right)^2 - 1\right)^{-1/2} \quad -(13)$$

In a spiral galaxy it is observed that as:
$$r \to \infty \quad -(14)$$

Then
$$v = \text{constant.} \quad -(15)$$

It is also known from 15/(2) that:
$$\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{b^2}\left(1 - \frac{1}{\gamma^2}\right) = \left(\frac{v}{c}\right)^2 \frac{1}{b^2} = \left(\frac{v}{r^2\omega}\right)^2, \quad -(16)$$

where
$$b = c\frac{L}{E} = \frac{r^2\omega}{c}, \quad \omega = \frac{d\phi}{dt}. \quad -(17)$$

Therefore
$$\frac{v}{r^2\omega} = \text{constant} \quad -(18)$$

and if
$$\boxed{\begin{array}{c} L = mr^2\omega = \text{constant,} \\ v = \text{constant} \end{array}} \quad -(19)$$

i.e. the non-relativistic limit ($\gamma \to 1$), the velocity curve of a galaxy is explained by constant angular momentum, as in previous work. Thus in eq. (13):
$$\frac{v}{\omega r} = \text{constant.} \quad r = \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{1/2} r \quad -(20)$$

Define:
$$\beta := \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{1/2} \quad -(21)$$

$\frac{13}{4a}$) then:

$$\frac{d\phi}{dr} = \frac{1}{r}\left(r^2\beta^2 - 1\right)^{-1/2} \quad -(22)$$

$$\rightarrow \frac{1}{r^2\beta} \quad \text{as} \quad r \rightarrow \infty$$

So

$$\boxed{\phi = -\frac{1}{r\beta}} \quad -(23)$$

This is the hyperbolic spiral, and is fitted to the whirlpool galaxy in Fig. (6.7) of GCUFT 5.

So metric (1) explains the main characteristics of a whirlpool galaxy in the limit $r \rightarrow \infty$, $v \ll c$.

2) In the limit

$$e^{-r_0/r} \rightarrow 0 \quad -(24)$$

$$\frac{E^2}{mc^2} = m\left(\frac{dr}{d\tau}\right)^2 = \text{constant} \quad -(24a)$$

This is the limit: $r_0 \gg r \quad -(24b)$

If there is a very heavy mass $\underline{M}$ at the centre of the galaxy then condition (24) may apply, as it may apply as r becomes small. In eq. (24a), $dr/d\tau$ is the rate of change of r with respect to proper time τ. The latter is the time in the frame of the star, i.e. in a frame of reference fixed on the star. In this frame the velocity of the star is a constant of motion because E

4)

, is a constant of motion. If the metric (1) is written as:

$$ds^2 = c^2 d\tau^2 = -e^{-r_0/r} c^2 dt^2 + e^{r_0/r} dr^2 + r^2 d\phi^2 \quad -(25)$$

then in the limit (24):

$$c^2 d\tau^2 \longrightarrow e^{r_0/r} dr^2 \quad -(26)$$

and

$$\frac{dr}{d\tau} = C \exp\left(-\frac{r_0}{2r}\right) \quad -(27)$$

i.e

$$\left(\frac{dr}{d\tau}\right)^2 = c^2 \exp\left(-\frac{r_0}{r}\right) = \left(\frac{E}{mc}\right)^2 \quad -(28)$$

or

$$\boxed{e^{-r_0/r} \longrightarrow \left(\frac{E}{mc^2}\right)^2} \quad -(29)$$

$$= \text{constant}.$$

If we now define:

$$dr \cdot dr = v^2 dt^2 = e^{r_0/r} dr^2 \quad -(30)$$

the

$$d\tau = \frac{v}{c} dt \quad -(31)$$

and

$$\frac{dr}{dt} = \frac{v}{c} \frac{dr}{d\tau} = \left(\frac{E}{mc^2}\right) v \quad -(32)$$

i.e

$$\frac{dr}{dt} = \left(\frac{E}{mc^2}\right) v = \exp\left(-\frac{r_0}{2r}\right) v \quad -(33)$$

$$\longrightarrow 0$$

The orbital characteristics in this limit are

5)

given by:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-r/r_0}\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} \quad -(34)$$

$$\rightarrow \frac{b}{r^2}$$

i.e the hyperbolic spiral $\phi = b\int\frac{dr}{r^2} = -\frac{b}{r} \quad -(35)$

Angle of Deflection of a Sun in the Whirlpool Galaxy

This is given by:
$$\qquad -(36)$$

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-r/r_0}\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr$$

where R_0 is the distance of closest approach, and where:

$$\left. a = \frac{L}{mc} = \text{constant}, \quad b = \frac{cL}{E} = \text{constant}, \atop a = \left(\frac{E}{mc^2}\right)b, \quad \frac{a}{b} = \frac{E}{mc^2} \right\} \quad -(37)$$

At some point the sun will escape the central part of the galaxy.

The Tetrads and Torsion

In previous work the whirlpool galaxy was described as spacetime torsion. So the metrical description (1) must be compatible with previous work. This means

b) t Cartan tetrads of eq.(1) must be evaluated, and the torsion worked out for the tetrads, with a model for the spin connection. The metric elements of eq.(1) may be written as:

$$g_{\mu\nu} = \begin{bmatrix} e^{-r_0/r} & 0 & 0 \\ 0 & -e^{r_0/r} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad - (38)$$

So

$$g_{00} = e^{-r_0/r}, \quad g_{11} = -e^{r_0/r}, \quad g_{22} = 1$$

$$- (39)$$

The tetrads are defined by:

$$g_{\mu\nu} = V_{\mu}^{a} V_{\nu}^{b} \eta_{ab} \quad - (40)$$

where η_{ab} is the Minkowski metric, so

$$g_{00} = (V_0^0)^2, \quad g_{11} = -V_1^1 V_1^1 \quad - (41)$$

and the relevant tetrads are

$$\boxed{V_0^0 = \exp\left(-\frac{r_0}{2r}\right) = -V_1^1} \quad - (42)$$

The torsion is worked out from these tetrads as usual.

Issue of Singularity.

Eq. (29) shows that as:

$$r \to 0 \quad - (43)$$

7)

then:

$$e^{-r_0/r} \rightarrow \left(\frac{E}{mc^2}\right)^2 \quad -(44)$$

$$= \text{constant} \neq 0$$

so

$$\boxed{r \neq 0 \quad \text{identically}} \quad -(45)$$

and in this theory there is no singularity. There is no big bang and no black holes. The inference of these incorrect assertions is due to the use of an incorrect Einstein field equation and of the approximation:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r}. \quad -(46)$$

Clearly, under the condition (43), eq. (46) is not valid, it is valid if and only if:

$$\frac{r_0}{r} \ll 1 \quad -(47)$$

Orbital Theorem of UFT III

Eq (1) is a solution of the orbital Theorem, but is only a solution of the incorrect Einstein field eq. in the approximation (46).