152(1): The Meaning of β in the General Orbital Equation.

The general orbital equation is:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - e^{-2\beta}\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} \quad -(1)$$

in the notation of paper 151. Now expand the exponential:

$$e^{-2\beta} = 1 - 2\beta + \frac{4\beta^2}{2!} - \frac{8\beta^3}{3!} + \ldots \quad (2)$$

The first level of approximation is:

$$e^{-2\beta} \sim 1 \quad -(3)$$

and this gives the Minkowski metric. The second level of approximation is:

$$e^{-2\beta} \sim 1 - 2\beta = 1 - \frac{r_0}{r} \quad -(4)$$

and this gives the gravitational metric, with:

$$2\beta = \frac{r_0}{r} = \frac{2mG}{c^2 r} \quad -(5)$$

so

$$\boxed{\beta = \frac{mG}{c^2 r}} \quad -(6)$$

The next level of approximation is the type of metric that produces a precessing ellipse spiralling inwards, as in a binary pulsar:

$$e^{-2\beta} \sim 1 - 2\beta + 2\beta^2 \quad -(7)$$

$$= 1 - \frac{r_0}{r} + \frac{1}{2}\left(\frac{r_0}{r}\right)^2 - \ldots \quad (8)$$

2)

This suggests that the most general metric is:

$$\boxed{\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \exp\left(-\frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2}}. \quad -(9)$$

For a binary pulsar:

$$e^{-r_0/r} = \left(\frac{1}{b^2} - \frac{\xi^2}{r^2}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)^{-1} \quad -(10)$$

so $$\frac{d\phi}{dr} = \xi r \quad , \quad r = r_0 \exp\left(\xi \phi\right). \quad -(11)$$

This general metric is constructed for spherically symmetric spacetime. Other metrics may be constructed for electrodynamics or any type of spacetime. <u>The symmetry of the spacetime determines the symmetry of the metric.</u>

<u>Numerical Calculation</u>

1) For a given ξ it is possible to determine a and b by solving eq. (10).

2) It would be interesting to evaluate the photon mass from the metric (9) using:

3)

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \exp\left(-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(12)$$

using successive terms in:

$$e^{-r_0/r} = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \frac{1}{3!}\left(\frac{r_0}{r}\right)^3 + \dots \quad -(13)$$

and for eq. (12) directly.

In the photon sun system:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} \quad -(14)$$

to an excellent approximation, and:

$$b = R_0 \quad -(15)$$

as found in paper 150. We have:

$$a = \left(\frac{E}{mc^2}\right) b = \left(\frac{\hbar\omega}{mc^2}\right) R_0 \quad -(16)$$

3) Repeat (2) to find the effect of successive terms on a precessing ellipse spiralling inwards (the binary pulsar).