

152(6): Electrodynamic and Mixed Metrics for a Spherically Symmetric Spacetime.

The metric is defined by:

$$ds^2 = c^2 dt^2 = e^{-\alpha(r)} c^2 dt^2 - e^{\alpha(r)} dr^2 - r^2 d\phi^2 - (1)$$

Let for electrostatics and gravitation.

Consider the H atom with one proton of charge e_2 and one electron of charge $-e_1$ and mass m . Define:

$$r_1 = 2 \left(\frac{e_1}{m} \right) \left(\frac{e_2}{4\pi \epsilon_0 c^2} \right) - (2)$$

and $r_2 = \frac{2MG}{c^2} - (3)$

where M is the proton mass.

For the H atom:

$$r_1 = 5.636 \times 10^{-15} \text{ m} - (4)$$

$$r_2 = 2.262 \times 10^{-84} \text{ m} - (5)$$

Electromagnetism and gravitation can be described in terms of the radii r_1 and r_2 . The combined effect is described by:

$$r_0 := r_1 + r_2 - (6)$$

For the H atom the radius r_0 is dominated entirely by r_1 . However, in other circumstances r_0 may be influenced by r_2 .

For comparison, the proton radius by electron scattering experiments is:

$$r(\text{proton}) = (0.8 - 0.86) \times 10^{-15} \text{ m} \quad - (7)$$

and the Bohr radius of the electron is:

$$r(\text{Bohr}) = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 5.29177 \times 10^{-11} \text{ m}, \quad - (8)$$

so r_1 is situated close to the proton radius, and r_2 is well inside the proton radius.

This is a thought experiment based on a classical theory, and no quantum effects are considered. On a macroscopic or cosmological scale, the radius r_1 and r_2 are well defined classically.

The equation of motion of the system is:

$$m \left(\frac{dr}{d\tau} \right)^2 = \frac{E^2}{mc^2} - e^{-r_0/r} \left(mc^2 + \frac{L^2}{mr^2} \right) \quad - (9)$$

where the following are constants of motion:

$$E = mc^2 e^{-r_0/r} \frac{dt}{d\tau}, \quad L = mr^2 \frac{d\phi}{d\tau}, \quad - (10)$$

$$p = me^{-r_0/r} \frac{dr}{d\tau} \quad - (11)$$

Here E is the total energy, L is the angular momentum, p is the linear momentum.

Eq. (9) is, in the approximation:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} \quad - (12)$$

$$m \left(\frac{dr}{d\tau} \right)^2 = \frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r} \right) \left(mc^2 + \frac{L^2}{mr^2} \right) \quad - (13)$$

i.e.

$$\begin{aligned} \frac{E^2}{mc^2} - mc^2 &= m \left(\frac{dr}{d\tau} \right)^2 - \frac{r_0}{r} mc^2 - \frac{r_0}{r} \cdot \frac{L^2}{mr^2} + \frac{L^2}{mr^2} \\ &= m \left(\frac{dr}{d\tau} \right)^2 - \frac{(r_1 + r_2)}{r} \left(mc^2 + \frac{L^2}{mr^2} \right) + \frac{L^2}{mr^2} \quad - (14) \end{aligned}$$

The "effective potential" of this system is:

$$V = - \frac{(r_1 + r_2)}{r} \left(mc^2 + \frac{L^2}{mr^2} \right) + \frac{L^2}{mr^2} \quad - (15)$$

The "inverse square" part of it is:

$$V(\text{inv. sq.}) = - \frac{(r_1 + r_2) mc^2}{2r} \quad - (16)$$

Now a factor $1/2$ multiplies both side of eq. (14) to define the Lagrangian in order to obtain the observed classical limit. So

$$\boxed{V(\text{inv. sq.}) = - \frac{e_1 e_2}{4\pi \epsilon_0 r} - \frac{mg}{r}} \quad - (17)$$

+) This is a linear combination of the classical e/m and gravitational potentials. So the metric gives the Coulomb and Newton laws, plus relativistic and centrifugal corrections. The model is not changed
mass orbiting matter.

In the approximation (12) there are no cross terms in eq. (17). So electromagnetism and gravitation are rigorously independent in this approximation. This is what is desired in the laboratory. The two component laws of eq. (17) are each true to instrumental precision, one law does not affect the other.

However, in more accurate approximations such as:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} + \frac{1}{2} \left(\frac{r_0}{r} \right)^2 - (18)$$

electromagnetism influences gravitation. The effective potential is changed, and its inverse square part is:

$$V(\text{inv sq.}) = -\frac{mc^2}{2} \left(\frac{(r_1 + r_2)}{r} - \frac{1}{2} \frac{(r_1 + r_2)^2}{r^2} \right) - (19)$$

$$= \frac{-e_1 e_2}{4\pi \epsilon_0 r} - \frac{mG}{r} + \frac{mc^2}{4r^2} (r_1^2 + 2r_1 r_2 + r_2^2)$$

$$= -\frac{e_1 e_2}{4\pi \epsilon_0 r} - \frac{mG}{r} + \frac{1}{mc} \left(\left(\frac{e_1 e_2}{4\pi \epsilon_0 r} \right)^2 + \left(\frac{mG}{r} \right)^2 + 2 \left(\frac{e_1 e_2}{4\pi \epsilon_0} \right) \frac{mG}{r^2} \right)$$

5) The cross term is :

$$V(\text{cross term}) = \left(\frac{e_1 e_2 M G}{2\pi \epsilon_0 m c^2} \right) \frac{1}{r^2} \quad - (20)$$

This is repulsive, so opposes the gravitational attraction.

Therefore this is the counter-gravitational force in a spherical spacetime. For :

$$e_1 = e_2 = r = m = 1 \quad - (21)$$

the Coulombic attraction is $-1/(4\pi\epsilon_0)$ and the cross term is $+G/(2\pi\epsilon_0 c^2)$, i.e. of order G/c^2 smaller, i.e. $\sim 10^{-27}$ times smaller.

This explains why the Coulomb law is so accurate in the laboratory. In order to see any effect of electromagnetism in gravitation the condition is :

$$\frac{e_1 e_2 M G}{2\pi m \epsilon_0 c^2} \frac{1}{r^2} \sim \frac{M G}{r} \quad - (22)$$

i.e.

$$\boxed{\frac{e_1 e_2}{2\pi m \epsilon_0 c^2 r} \sim 1} \quad - (23)$$

$$6) \text{ i.e. } \frac{L_1 L_2}{m r} \sim 2\pi f \cdot C^2 = 5 \times 10^6 - (24)$$

In order to test this theory, great care has to be taken to remove artifacts, by using a high vacuum.
