1) Equation of Orbit

Gravitational Metric

This is regarded in ECE theory as a solution of the Orbital Theorem:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2\left(1-\frac{r_0}{r}\right) - \left(1-\frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(1)$$

in the plane

$$dZ = 0, \quad -(2)$$

where

$$r_0 = \frac{2mG}{c^2}. \quad -(3)$$

From eq. (1), the Lagrangian is:

$$\mathcal{L} = T = \frac{1}{2}mc^2 = \frac{m}{2}\left(c^2\left(\frac{dt}{d\tau}\right)^2\left(1-\frac{r_0}{r}\right) - \left(1-\frac{r_0}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2\right) \quad -(4)$$

so the Lagrange equation gives the constants of motion:

$$E = mc^2\left(1-\frac{r_0}{r}\right)\frac{dt}{d\tau}\,; \quad L = mr^2\frac{d\phi}{d\tau}\,, \quad p_r = m\left(1-\frac{r_0}{r}\right)^{-1}\frac{dr}{d\tau} \quad -(5)$$

The equation of orbits is obtained by multiplying both sides of eq. (4) by $\left(1-\frac{r_0}{r}\right)$:

$$\left(1-\frac{r_0}{r}\right)mc^2 = mc^2\left(1-\frac{r_0}{r}\right)^2\left(\frac{dt}{d\tau}\right)^2 - m\left(\frac{dr}{d\tau}\right)^2 - mr^2\left(1-\frac{r_0}{r}\right)\left(\frac{d\phi}{d\tau}\right)^2 \quad -(6)$$

so:

$$m\left(\frac{dr}{d\tau}\right)^2 = mc^2\left(1-\frac{r_0}{r}\right)^2\left(\frac{dt}{d\tau}\right)^2 - \left(1-\frac{r_0}{r}\right)mc^2 - \left(1-\frac{r_0}{r}\right)mr^2\left(\frac{d\phi}{d\tau}\right)^2$$

$$= \frac{E^2}{mc^2} - \left(1-\frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right). \quad -(7)$$

Now use:

$$\frac{dr}{d\tau} = \frac{d\phi}{d\tau}\frac{dr}{d\phi} \quad -(8)$$

$$= \left(\frac{L}{mr^2}\right)\frac{dr}{d\phi} \quad -(9)$$

so $$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{r^4}{a^2} + r^2\right)$$

$$\boxed{\left(\frac{dr}{d\phi}\right)^2 = r^4 \left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)} \quad -(10)$$

where $$a = \frac{L}{mc}, \quad b = c\frac{L}{E} \quad -(11)$$

are constants.

This equation is fairly successful in solar system orbits, but fails in whirlpool galaxies.

The solution of equation (10) is:

$$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr. \quad -(12)$$

The orbit of a photon is given by:

$$a \to \infty, \quad m \to 0 \quad -(13)$$

i.e. $$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\frac{1}{r^2}\right)^{-1/2} dr \quad -(14)$$

Using $$u = \frac{1}{r}, \quad du = -\frac{1}{r^2} dr \quad -(15)$$

eq. (12) is $$\phi = -\int \frac{du}{\left(\left(\frac{1}{b^2} - \frac{1}{a^2}\right) + \left(\frac{r_0}{a^2}\right)u - u^2 + r_0 u^3\right)^{1/2}} \quad -(16)$$

The integral is exact in the limit $r_0 u^3$ smaller than the other terms:

$$\phi \to -\int \frac{du}{\left(\left(\frac{1}{b^2} - \frac{1}{a^2}\right) + \left(\frac{r_0}{a^2}\right)u - u^2\right)^{1/2}} \quad -(17)$$

and may be solved with:

$$\int \frac{dx}{(ax^2+bx+c)^{1/2}} = \frac{1}{\sqrt{-a}} \sin^{-1}\left(\frac{2ax+b}{(b^2-4ac)^{1/2}}\right) \quad -(18)$$

with, in this formula:

$$a=-1,\quad b=\frac{2m^2MG}{L^2},\quad c=\frac{m}{L^2}\left(\frac{E^2}{mc^2}-mc^2\right) \quad -(19)$$

So:

$$\sin\phi = \left(\frac{2}{r}-\frac{2m^2MG}{L^2}\right)\left(\frac{4m^4M^2G^2}{L^4}+\frac{4m}{L^2}\left(\frac{E^2}{mc^2}-mc^2\right)\right)^{-1/2}$$

$$= \left(\frac{L^2}{kr}-1\right)\left(1+\frac{L^2}{mk^2}\left(\frac{E^2}{mc^2}-mc^2\right)\right)^{-1/2} \quad -(20)$$

In the non-relativistic limit:

$$\frac{E^2}{mc^2}-mc^2 \rightarrow \text{Newtonian limit.} \quad -(21)$$

therefore in eq. (17):

$$\phi = \sin^{-1} x - \gamma \quad -(22)$$

So where γ is a constant of integration.

$$x = \sin(\phi+\gamma) = \sin\phi\cos\gamma + \cos\phi\sin\gamma \quad -(23)$$

Now choose: $\gamma = \frac{\pi}{2}$ $-(24)$

to obtain the ellipse:

4)

$$\cos\phi = \frac{\left(\frac{L^2}{kr} - 1\right)}{\left(1 + \frac{L^2}{mk^2}\left(\frac{E^2}{mc^2} - mc^2\right)\right)^{1/2}} \quad -(25)$$

i.e.

$$\boxed{\frac{\alpha}{r} = 1 + \epsilon\cos\phi} \quad -(26)$$

where:

$$\alpha = \frac{L^2}{mk} \quad -(27)$$

$$\epsilon = \left(1 + \frac{L^2}{mk^2}\left(\frac{E^2}{mc^2} - mc^2\right)\right)^{1/2} \quad -(28)$$

Comparison with the Newtonian Result

The Newtonian result is an ellipse, but one in which:

$$\epsilon = \left(1 + \frac{2E_N L^2}{mk^2}\right)^{1/2} \quad -(29)$$

Here:

$$E_N = \frac{1}{2}mv^2 - \frac{k}{r} \quad -(30)$$

where

$$v^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(31)$$

In eq. (28), the energy is defined by:

$$E = mc^2\left(1 - \frac{r_0}{r}\right)\left(\frac{dt}{d\tau}\right) \quad -(32)$$

5)

in which the proper time is defined by:

$$c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(33)$$

If the particle is at rest in both frames, then:

$$c^2 d\tau^2 = c^2 \left(1 - \frac{r_0}{r}\right) dt^2 \quad -(34)$$

in which case:

$$\left(\frac{dt}{d\tau}\right)^2 = \left(1 - \frac{r_0}{r}\right)^{-1} \quad -(35)$$

It is possible to write eq. (33) as:

$$c^2 d\tau^2 = c^2 dt'^2 - d\underline{r}' \cdot d\underline{r}' \quad -(36)$$

where

$$d\underline{r}' \cdot d\underline{r}' = v^2 dt'^2 \quad -(37)$$

$$= \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\phi^2$$

and

$$dt'^2 = \left(1 - \frac{r_0}{r}\right) dt^2 \quad -(38)$$

so

$$c^2 d\tau^2 = (c^2 - v^2)\left(1 - \frac{r_0}{r}\right) dt^2$$

$$\boxed{d\tau^2 = \left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{r_0}{r}\right) dt^2} \quad -(39)$$

so

$$\frac{E^2}{mc^2} - mc^2 = mc^2 \left(1 - \frac{v^2}{c^2}\right)^{-1} \left(1 - \frac{r_0}{r}\right) - mc^2 \quad -(40)$$

$$\sim mv^2 - 2\frac{k}{r}$$

$$= 2E_N$$

Q.E.D.

6)

Therefore the Newtonian limit is obtained by neglecting the r_0/r^3 term in eq. (10). If this term is reinstated the ellipse becomes a precessing ellipse. This happens to fit data in the solar system only because r_0/r is very small there.

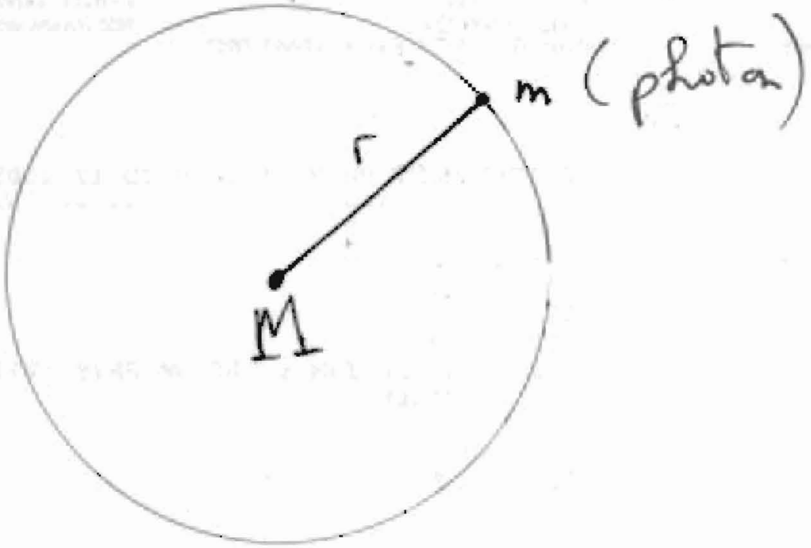
Other New Metrics

The metric (1) happens to be one out of an infinite possible solutions of the Einstein field equation, but it cannot be claimed to be more than that, because the Einstein field equation is incorrect. Metric (1) fails completely to describe whirlpool galaxies, so there is no purpose in claiming that it is a generally valid metric. The task now is to look for a metric that describes all data. In paper 149 one such metric was tried:

$$
\begin{aligned}
ds^2 &= c^2 dt^2 - d\underline{r} \cdot d\underline{r} - \frac{2k}{r} dt^2 \\
&= c^2 \left(1 - \frac{r_0}{r}\right) dt^2 - d\underline{r} \cdot d\underline{r} \qquad (41) \\
&= c^2 d\tau^2
\end{aligned}
$$

and in the next note the orbital equation will be explored for this, with the purpose of evaluating firstly the difference to the orbital equation (10).

150(2): Photon Mass Experiment based on the Sagnac Effect.



Experimental Method

A mass M is placed in the centre of a static Sagnac platform, and the Sagnac frequency shift due to the mass M is measured.

Theory

The photon mass is m, so light no longer travels on a null geodesic. In the absence of the mass M its metric is

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad -(1)$$

where

$$d\underline{r} \cdot d\underline{r} = dr^2 + r^2 d\phi^2 + dZ^2 \quad -(2)$$

in cylindrical polar coordinates. The radius r is constant in the XY plane of the paper, so:

$$dr = 0 \quad -(3)$$

$$dZ = 0 \quad -(4)$$

and

2)
$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - r^2 d\phi^2 \quad -(5)$$

By definition:
$$\underline{dr} \cdot \underline{dr} = r^2 d\phi^2 = v^2 dt^2 \quad -(6)$$

so
$$\omega = \frac{d\phi}{dt} = \frac{v}{r} \quad -(7)$$

$$d\tau = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt \quad -(8)$$

Here ω is the angular frequency measured by the Sagnac interferometer, v is the velocity of light of photon mass m.

In the presence of mass M:
$$ds^2 = c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - \underline{dr} \cdot \underline{dr} \quad -(9)$$

Define
$$\boxed{dt'^2 = \left(1 - \frac{r_0}{r}\right) dt^2} \quad -(10)$$

$$\underline{dr} \cdot \underline{dr} = v^2 dt'^2 \quad -(11)$$

$$= \left(1 - \frac{r_0}{r}\right) v^2 dt^2 \quad -(12)$$

Therefore:
$$r^2 d\phi^2 = \left(1 - \frac{r_0}{r}\right) v^2 dt^2 \quad -(13)$$

and
$$\omega_1 = \frac{d\phi}{dt} = \left(1 - \frac{r_0}{r}\right)^{1/2} \frac{v}{r}$$

$$\boxed{\omega_1 = \omega \left(1 - \frac{r_0}{r}\right)^{1/2}} \quad -(14)$$

and
$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{r_0}{r}\right) dt^2 \quad -(15)$$

3) In this notation:

$$r_0 = \frac{2MG}{c^2} \quad -(16)$$

If

$$r_0 << r \quad -(17)$$

$$\omega_1 \sim \omega\left(1 - \frac{1}{2}\frac{r_0}{r}\right)$$

$$= \omega\left(1 - \frac{MG}{c^2 r}\right) \quad -(18)$$

So:

$$\boxed{\Delta\omega = \omega - \omega_1 = \left(\frac{MG}{c^2 r}\right)\omega} \quad -(19)$$

This is the shift to lower frequency of ω due to gravitation. The photon of mass m is constrained to a circular orbit around M.

In the rest frame of the photon of mass m, the energy of attraction to M is:

$$U = -\frac{mMG}{r} \quad -(20)$$

i.e.

$$|U| = mMG/r.$$

In the rest frame of the photon:

$$\hbar\omega_0 = mc^2 \quad -(21)$$

where ω_0 is its rest angular frequency by the de Broglie equation.

4) Therefore:

$$\Delta\omega = \omega_0 = \frac{mc^2}{\hbar} = \frac{MG}{c^2 r}\,\omega \quad -(22)$$

and:

$$m = \left(\frac{\hbar G}{c^4}\right)\left(\frac{M\omega}{r}\right) \quad -(23)$$

$$\boxed{m = 8.711 \times 10^{-79}\left(\frac{M\omega}{r}\right)} \quad -(24)$$

For visible light of $\sim 10^{16}$ radians per second, for M = one kilogram, r = one metre:

$$m \sim 10^{-63} \text{ kilograms} \quad -(25)$$

Key Assumption

This is: $\omega_0 = \omega - \omega_1 \quad -(26)$

which means that the gravitational red shift is due to the rest frequency of the photon of mass m. The reason for this is that in its rest frame, the photon sees M as static. So the energy of attraction between m and M is eq. (20). In the rest frame of the photon, eq. (21) is always true. The result (25) is in good agreement with lower bounds on photon mass. In the standard model, the photon has no mass, and has no rest frame. This contradicts light bending by gravitation.

150(3): Orbit of the Photon in General.

In order that the photon be attracted by a mass M it must be assumed that the photon has a mass m. Its orbit is then

$$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(1)$$

where the constants of motion a and b are defined by:

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E} \quad -(2)$$

and where

$$r_0 = \frac{2MG}{c^2} \quad -(3)$$

The angular momentum L and the energy E are constants of motion. If it were possible to observe the orbit of a photon around a massive object M, L and E could be determined experimentally. By Kepler's second law:

$$L = 2m \frac{dA}{dt} \quad -(4)$$

Various limits of eq. (1) are usually used in the usual literature. Examples are given below.

1) Limit of $m \to 0$

In this limit, $a \to \infty$ $-(5)$

so

$$\phi \to \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\frac{1}{r^2}\right)^{-1/2} dr \quad -(6)$$

In the limit:

$$r \to \infty \quad -(7)$$

$$\phi \to \int \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{r^2}\right)^{-1/2} dr \quad -(8)$$

The integral in eq. (8) can be evaluated using:

$$u = \frac{1}{r}, \quad du = -\frac{1}{r^2}\,dr \qquad -(9)$$

so

$$\phi = -\int \frac{du}{\left(\frac{1}{b^2} - u^2\right)^{1/2}} \qquad -(10)$$

This integral is an example of:

$$\int \frac{dx}{(Ax^2 + Bx + C)^{1/2}} = \frac{1}{\sqrt{-A}} \sin^{-1}\left(\frac{2Ax + B}{(B^2 - 4AC)^{1/2}}\right) \qquad -(11)$$

with:

$$A = -1, \quad B = 0, \quad C = 1/b^2 \qquad -(12)$$

so

$$\phi = -\sin^{-1}\left(\frac{-2/r}{(4/b^2)^{1/2}}\right) \qquad -(13)$$

$$= -\sin^{-1}\left(-\frac{b}{r}\right) \qquad -(14)$$

More generally

$$\phi = -\sin^{-1}\left(-\frac{b}{r}\right) + y \qquad -(15)$$

where y is the constant of integration. So

$$\sin^{-1}\left(-\frac{b}{r}\right) = y - \phi \qquad -(16)$$

and

$$-\frac{b}{r} = \sin(y - \phi) \qquad -(17)$$

$$= \sin y \cos\phi - \cos y \sin\phi$$

If

$$y = \frac{n\pi}{2} \qquad -(18)$$

$$n = 0, 1, 2, \ldots$$

3) then

$$\boxed{\sin\phi = \frac{b}{r} = \frac{cL}{Er}} \quad -(19)$$

For small angles:

$$\sin\phi \sim \phi = \frac{cL}{Er} \quad -(20)$$

in which:

$$L = mr^2\frac{d\phi}{d\tau}, \quad E = mc^2\left(1-\frac{r_0}{r}\right)\frac{dt}{d\tau} \quad -(21)$$

so

$$\phi \sim \frac{r}{c}\left(1-\frac{r_0}{r}\right)\frac{d\phi}{dt} \quad -(22)$$

If the photon is constrained to a circular orbit:

$$\omega = \frac{d\phi}{dt} = \frac{v}{r} \quad -(23)$$

so in this case:

$$\phi = \frac{v}{c}\left(1-\frac{2MG}{c^2 r}\right) \quad -(24)$$

The deflection is

$$\boxed{\Delta\phi = \frac{2MG}{c^2 r}} \quad -(25)$$

This is the Newtonian result because an approximation has been used which neglects the non-Newtonian term in eq. (6). This is the r_0/r^3 term inside the brackets in the denominator. It is seen that the photon mass m does not appear in the formula for the deflection. This is too rough an approximation therefore.

4)

2) <u>Limit of Finite a</u>

If the r_0/r^3 term is neglected in eq. (1):

$$\phi \to \int \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{a^2} + \frac{r_0}{a^2}\frac{1}{r} - \frac{1}{r^2}\right)^{-1/2} dr \quad -(26)$$

As shown in note 150(1) the solution of eq. (26) is the ellipse:

$$\frac{d}{r} = 1 + \epsilon \cos\phi \quad -(27)$$

where

$$d = \frac{L^2}{m^2 M G} \quad -(28)$$

$$\epsilon = \left(1 + \frac{L^2}{m M^2 G^2}\left(\frac{E^2}{mc^2} - mc^2\right)\right)^{1/2} \quad -(29).$$

In the non-relativistic limit, eq. (29) becomes

$$\epsilon = \left(1 + \frac{2E_N L^2}{m^2 M G}\right)^{1/2} \quad -(30)$$

where:

$$E_N = \frac{1}{2}mv^2 - \frac{mMG}{r} \quad -(31)$$

The quantities $2d$ and ϵ are respectively the latus rectum and eccentricity of the orbit. <u>These can be found by</u> <u>observation</u>. So:

$$d = A/m^2 \quad -(32)$$

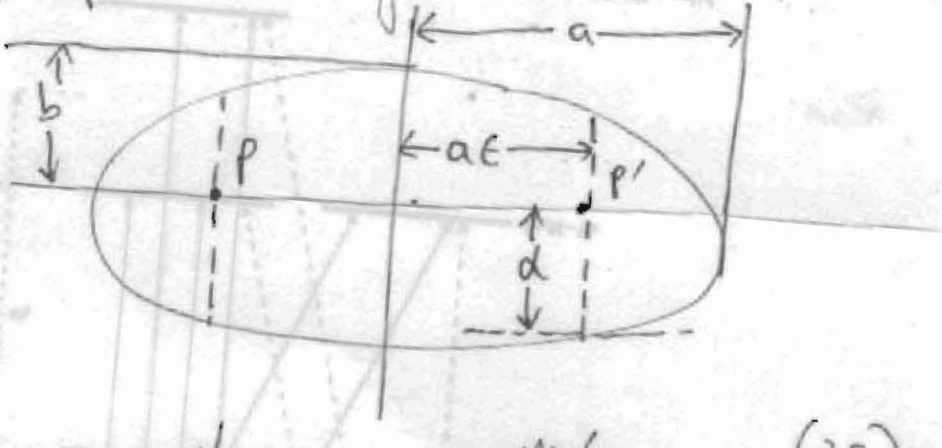
$$\epsilon = (1 + B/m^2)^{1/2} \quad -(33)$$

where $A = \frac{L^2}{MG}$, $B = \frac{2E_N L^2}{MG} = 2E_N A$ $-(34)$

) Therefore if A and B are known independently, the mass m can be found by observation of the orbit. In theory, the photon mass can be found in this way.

The ellipse is as follows:

Fig. (1)



in which

$$a = \frac{d}{1-\epsilon^2} = \frac{mMG}{2|E_N|} \quad -(35)$$

$$b = \frac{d}{(1-\epsilon^2)^{1/2}} = \frac{L}{(2m|E_N|)^{1/2}} \quad -(36)$$

From eq. (4):

$$dt = \frac{2m}{L}\, dA \quad -(37)$$

The area A of the ellipse is

$$A = \pi a b \quad -(38)$$

and is covered in the interval τ. So:

$$\int_0^\tau dt = \frac{2m}{L} \int_0^A dA \quad -(39)$$

$$\boxed{\tau = \frac{2m}{L} A} \quad -(40)$$

using

$$b = (da)^{1/2} \quad -(41)$$

$$\tau^2 = \frac{4\pi^2 m}{mMG}\, a^3 \quad -(42)$$

6)

i.e.

$$\boxed{\tau^2 = \left(\frac{4\pi^2}{mG}\right) a^3} \quad -(43)$$

This is Kepler's Third Law of 1619.

It is seen that the mass m cancels out from eqs. (40) and (43). However, a more accurate calculation shows that m should be the reduced mass:

$$\mu = \frac{mM}{m+M} \quad -(44)$$

so:

$$\tau^2 = \frac{4\pi^2 a^3}{G(m+M)} \quad -(45)$$

and

$$\boxed{m = \frac{4\pi^2 a^3}{\tau^2 G} - M} \quad -(46)$$

This is a formula for the photon mass, m, or the mass of any object orbiting a mass M in the Newtonian limit. The experimental problem of measuring photon mass is therefore reduced to measuring or estimating a and τ for a photon, or laser beam.

For accurate calculation, m should be replaced by μ wherever it occurs, and k defined by mMG.

Therefore the accurate formula for α is:

$$\alpha = \frac{L^2}{\mu m M G} \quad -(47).$$

7) In an approximately circular orbit:

$$\alpha = r \quad -(48)$$

and

$$L = \mu r^2 \omega \quad -(49)$$

$$\omega = \frac{v}{r} \quad -(50)$$

so with

so

$$v^2 = (m + M)\frac{G}{r} \quad -(51)$$

This is a special case of Kepler's equation:

$$v^2 = G(m + M)\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(52)$$

for the elliptical orbit, when a is the same as r.

In a circular orbit of the photon as it moves 150(2), the photon mass is:

$$\boxed{m = \frac{v^2 r}{G} - M} \quad -(53)$$

so

$$\boxed{m = \frac{\omega^2 r^3}{G} - M} \quad -(54)$$

This is an exact formula for the photon mass, or the mass of a planet or any other object in orbit.

Eqns. (46) and (53) give all details of the orbit.

If $m \to 0$, then:

$$\frac{MG}{r} \to v^2 \quad -(55)$$

This equation means that the velocity of the photon

8) is changed from c by the amount MG/r. If M is one kilogram and r is one metre,

$$v = G^{1/2} = 8.17 \times 10^{-6}\ m s^{-1} \quad -(56)$$

which compares with the speed of light

$$c = 2.998 \times 10^{8}\ m s^{-1} \quad -(57)$$

in the absence of the mass M.

A more accurate calculation must be based on the ellipse:

$$\frac{\alpha}{r} = 1 + \epsilon \cos\phi \quad -(58)$$

with

$$\alpha = \frac{L^2}{\mu k} \quad -(59)$$

and

$$\epsilon = \left(1 + \frac{L^2}{\mu k^2}\left(\frac{E^2}{\mu c^2} - \mu c^2\right)\right)^{1/2} \quad -(60)$$

and it becomes easier to use the method of note 50(2).

The next note will evaluate the effect of the r_0/r^3 term in eq. (1) a close calculation.

50(4): Deflection of Light by Gravitation, a Critical Appraisal of the Einstein Calculation

It is well known that Einstein based his calculation on the metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - dr^2 \left(1 - \frac{r_0}{r}\right)^{-1} - r^2 d\phi^2 \quad -(1)$$

in cylindrical polar coordinates. Here:

$$r_0 = \frac{2MG}{c^2} \quad -(2)$$

where M is the mass of the attracting object, G is Newton's constant and c is the vacuum velocity of light. The first thing to note is that in order for the Newtonian dynamics to be regained correctly, the metric (1) must be rewritten as:

$$\frac{E^2}{2\mu c^2} = \frac{1}{2}\mu \left(\frac{dr}{d\tau}\right)^2 + \frac{1}{2}\left(1 - \frac{r_0}{r}\right)\left(\mu c^2 + \frac{L^2}{\mu r^2}\right) \quad -(3)$$

where

$$\mu = \frac{mM}{m+M} \quad -(4)$$

is the reduced mass of the two particle problem. The constants of motion in eq. (3) are:

$$E = \mu c^2 \left(1 - \frac{r_0}{r}\right)\frac{dt}{d\tau} \quad -(5)$$

$$L = \mu r^2 \frac{d\phi}{d\tau}. \quad -(6)$$

Einstein assumed the null geodesic condition:

$$ds^2 = 0 \quad -(7)$$

but this means that the attracted object of mass m must be massless and propagate at c identically in vacuo. This is a self contradiction at the

2)

beginning of the Einstein calculation. The correct calculation must be based on:

$$\frac{1}{2}\left(\frac{E^2}{\mu c^2}-\mu c^2\right)=\frac{1}{2}\mu\left(\frac{dr}{d\tau}\right)^2-\mu\frac{MG}{r}+\frac{L^2}{2\mu r^2}-\frac{MGL^2}{\mu c^2 r^3} \quad -(8)$$

The left hand side is the total kinetic energy and in the Newtonian limit reduces to:

$$T=\frac{1}{2}\mu v^2-\mu\frac{MG}{r} \quad -(9)$$

where

$$v^2=\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(10)$$

<u>Note carefully</u> <u>that</u> in <u>general</u> <u>relativity</u> <u>there</u> is <u>no</u> <u>force</u> or <u>potential energy.</u>

The term:

$$V=-\mu\frac{MG}{r}+\frac{L^2}{2\mu r^2}-\frac{MGL^2}{\mu c^2 r^3} \quad -(11)$$

is misleadingly known as "the effective potential energy", but it is pure kinetic in nature. The Newtonian limit of eq. (8) is therefore:

$$\frac{1}{2}\mu v^2-\mu\frac{MG}{r}=\frac{1}{2}\mu\left(\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2\right)-\mu\frac{MG}{r} \quad -(12)$$

i.e.

$$\frac{1}{2}\left(\frac{E^2}{\mu c^2}-\mu c^2\right)\rightarrow\frac{1}{2}\mu v^2-\mu\frac{MG}{r} \quad -(13)$$

$$\left(\frac{dr}{d\tau}\right)^2\rightarrow\left(\frac{dr}{dt}\right)^2 \quad -(14)$$

$$\frac{L^2}{2\mu r^2}\rightarrow\frac{1}{2}\mu r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(15)$$

$$\frac{MGL^2}{\mu c^2 r^3}\rightarrow 0 \quad -(16)$$

3) If a null geodesic is assumed as in eq. (7), the left hand side of eq. (8) is:

$$\text{LHS(null geodesic)} = \frac{1}{2}\frac{E^2}{\mu c^2} \quad -(17)$$

and this never reduces to the Newtonian dynamics. This is self contradictory because the photon must have mass m if it is to be attracted by mass M. The Einsteinian calculation should be completely revised to make it self-consistent. However in this note we criticise the original calculation. So the calculation is based on:

$$\frac{1}{2}\frac{E^2}{\mu c^2} = \frac{1}{2}\mu\left(\frac{dr}{d\tau}\right)^2 - \frac{\mu M G}{r} + \frac{L^2}{2\mu r^2} - \frac{MGL^2}{\mu c^2 r^3} \quad -(18)$$

Einstein made a further assumption:

$$\frac{dr}{d\tau} = 0 \quad -(19)$$

i.e. assumed that the photon is in a circular orbit. There is no a priori basis for this assumption. So:

$$\frac{1}{2}\frac{E^2}{\mu c^2} = -\frac{\mu M G}{r} + \frac{L^2}{2\mu r^2} - \frac{MGL^2}{\mu c^2 r^3} \quad -(20)$$

Einstein also assumes that:

$$V = \frac{L^2}{2\mu r^2} - \frac{MGL^2}{\mu c^2 r^3} \quad -(21)$$

which is equivalent to

$$r \rightarrow \infty. \quad -(22)$$

This can be seen from the fact that:

4)

$$V = \mu\left(-\frac{mG}{r} + \frac{1}{2}r^2\left(\frac{d\phi}{d\tau}\right)^2 - \frac{mGr}{c^2}\left(\frac{d\phi}{d\tau}\right)^2\right) \quad -(23)$$

However, from eq. (11) it is seen that in the limit (22):

$$V \rightarrow 0 \quad -(24)$$

because L is a constant of motion and the second two terms of the right hand side of eq. (11) go to zero more rapidly than the first term.

<u>In my opinion</u>, the <u>use of eq. (21)</u> is <u>a basic error that invalidates the Einstein method</u>.

The rest of this note therefore illustrates the original method for the sake of making a "baseline calculation" for much needed improvement. Therefore the "potential" used by Einstein is:

$$V = \frac{L^2}{2\mu r^3}\left(r - \frac{2mG}{c^2}\right) \quad -(25)$$

It is seen clearly that <u>this is self contradictory</u>, there is no "Newtonian" or "Coulombic" attraction, i.e. inverse square attraction, between m and M.

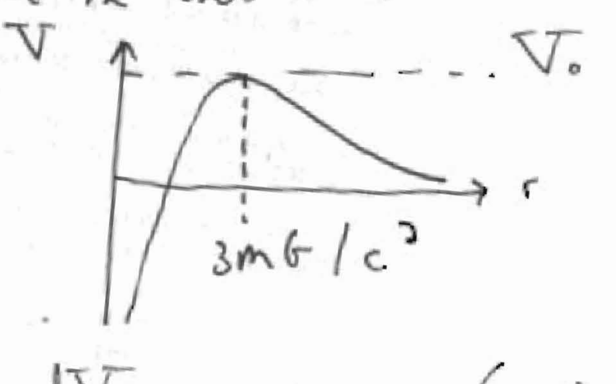


Fig. (1)

Using

$$\frac{dV}{dr} = 0 \quad -(26)$$

the maximum of V occurs at

5)

$$r_0 = \frac{3MG}{c^2} \quad -(27)$$

$$V_0 = \frac{L^2 c^4}{54 \mu m^2 G^2} \quad -(28)$$

The minimum energy needed to overcome the barrier represented by V_0 is:

$$\frac{1}{2} \frac{E_0^2}{\mu m c^2} = V_0 \quad -(29)$$

The next stage of the calculation is to define the impact parameter:

$$b = \frac{cL}{E_0} = \sqrt{27}\left(\frac{mG}{c^2}\right) \quad -(30)$$

The capture cross section is πb^2. The calculation asserts that a photon will be captured at an impact parameter less than b.

However, this conclusion does not have any meaning for reasons stated already. In other words the original equation of motion (3) has seen incorrectly approximated.

Einstein's light deflection calculation proceeds using the equation of motion in the approximation (25):

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\frac{1}{r^2}\right)^{-1/2} \quad -(31)$$

The basic idea is that the orbit of the photon has a turning point at the largest radius R_0 for which:

6)

$$V(R_0) = E^2/2 \quad -(32)$$

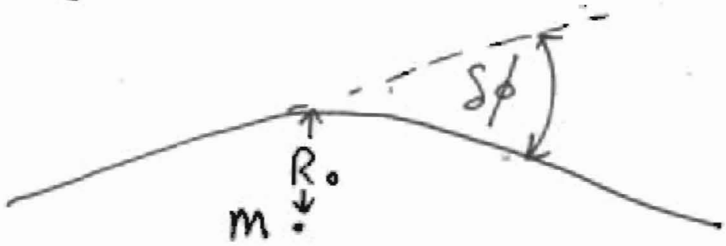


Fig (2)

The turning point is calculated using:

$$\frac{E^2}{2\mu c^2} = \frac{L^2}{2\mu r^3}\left(r - \frac{2mG}{c^2}\right) \quad -(33)$$

i.e. $$r^3 - b^2\left(r - \frac{2mG}{c^2}\right) = 0 \quad -(34)$$

for which $$R_0 = \frac{2}{\sqrt{3}}\frac{cL}{E}\cos\left(\frac{1}{3}\cos^{-1}\left(-3^{3/2}\frac{mG}{c^2 b}\right)\right) \quad -(35)$$

The total deflection, by symmetry, is:

$$\Delta\phi = 2\int_{R_0}^{\infty}\left(\frac{r^4}{b^2} - r(r - r_0)\right)^{-1/2} dr \quad -(36)$$

If $$u = 1/r \quad -(37)$$

then $$\boxed{\Delta\phi = 2\int_0^{1/R_0}\frac{du}{\left(\frac{1}{b^2} - u^2 + r_0 u^3\right)^{1/2}}} \quad -(38)$$

in which $$R_0^3 = b^2(R_0 - r_0) \quad -(39)$$

so $$\frac{1}{b^2} = \frac{R_0 - r_0}{R_0^3}. \quad -(40)$$

Einstein's final step was as follows:

7)

$$\left.\frac{\partial(\Delta\phi)}{\partial m}\right|_{m=0} = 2\int_0^{1/R_0} \left.\left(\frac{(R_0^{-3} - u^3)\,du}{(R_0^{-2} - r_0 R_0^{-3} - u^2 + r_0 u^3)^{3/2}}\right)\right|_{m=0}$$

$$= 2\int_0^{1/b} \left(\frac{b^{-3} - u^3}{(b^{-2} - u^2)^{3/2}}\right) du$$

$$= \frac{4}{b} \quad -(41)$$

So to first order in m the deflection is:

$$\Delta\phi = \frac{4m}{b} = \frac{4mG}{c^2 b} \quad -(42)$$

Finally it is assumed that:

$$b = R_0 \quad -(43)$$

so

$$\boxed{\Delta\phi = \frac{4mG}{c^2 R_0}} \quad -(44)$$

This is a convoluted calculation based on several assumptions that are untenable. If the integral (38) is evaluated numerically, a different result to (44) is obtained in general.

If the cubic term in the denominator of eq. (38) is neglected the integral can be evaluated analytically:

$$\Delta\phi = 2\int_0^{1/R_0} \left(\frac{1}{R_0^2} - \frac{r_0}{R_0^3} - u^2\right)^{-1/2} du \quad -(45)$$

$$= 2\left(\sin^{-1}\left(-\left(1 - \frac{r_0}{R_0}\right)^{-1/2}\right) - \sin^{-1} 0\right)$$

i.e. from note 150(3), if

$$\theta = -\int\left(\frac{1}{b^2}-u^2\right)^{-1/2} du \quad -(46)$$

then for a circular orbit:

$$\theta = \frac{v}{c}\left(1+\frac{2mG}{c^2 r}\right) \quad -(47)$$

$$= \frac{v}{c}\left(1+\frac{2mG u}{c^2}\right) \quad -(48)$$

This result uses the definition of b:

$$b = c\frac{L}{E} \quad -(49)$$

The integral (45) is:

$$\Delta\phi = -2\int_{1/R_0}^{0}\left(\frac{1}{b^2}-u^2\right)^{-1/2} du. \quad -(50)$$

$$= 2\left(\frac{v}{c}-\frac{v}{c}-\frac{2mG}{c^2 R_0}\right) \quad -(51)$$

$$\Delta\phi = -\frac{4mG}{c^2 R_0} \quad -(52)$$

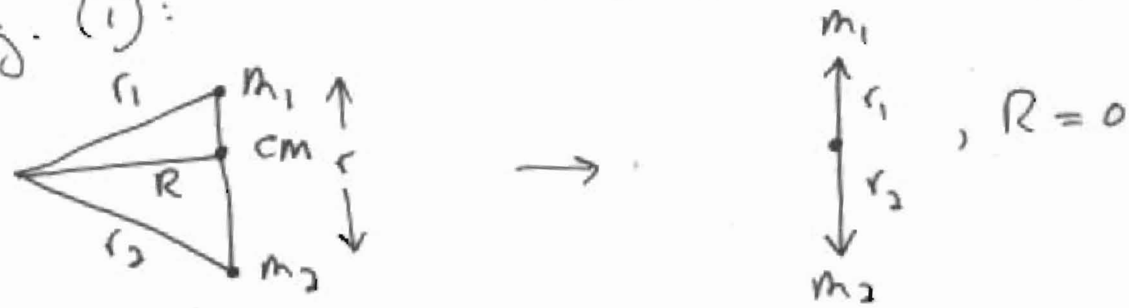
i.e.

$$|\Delta\phi| = \frac{4mG}{c^2 R_0} \quad -(53)$$

This result happens to agree with accurate NASA Cassini measurements, but is not meaningful because it relies on the same approximations as used by Einstein. The latter's method used unjustifiable approximations and a completely new approach is needed.

1) 150(5): Calculation of Light Deflection for a Finite Photon Mass.

Start with the basics of the two particle problem as in Fig. (1):



Here:
$$r = |\underline{r}_1 - \underline{r}_2| . \quad — (1)$$

The lagrangian is:
$$\mathcal{L} = \frac{1}{2} m_1 |\dot{\underline{r}}_1|^2 + \frac{1}{2} m_2 |\dot{\underline{r}}_2|^2 - U(r) \quad — (2)$$

The centre of mass (CM) is defined by:
$$m_1 \underline{r}_1 + m_2 \underline{r}_2 = \underline{0}, \quad — (3)$$

with
$$\underline{r} = \underline{r}_1 - \underline{r}_2 \quad — (4)$$

so
$$\mathcal{L} = \frac{1}{2} \mu |\dot{\underline{r}}|^2 - U(\underline{r}) \quad — (5)$$

where the reduced mass is:
$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad — (6)$$

The interaction energy is:
$$U = - \frac{m_1 m_2}{r} G \quad — (7)$$
$$\boxed{U = - \mu (m_1 + m_2) \frac{G}{r}} \quad — (8)$$

Therefore the reduced mass μ interacts with the

2)

sum of masses $(m_1 + m_2)$.

If the metric is assumed to be the gravitational metric of the Orbital Theorem (UFT 111) then:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2\left(1 - \frac{r_0}{r}\right) - dr^2\left(1 - \frac{r_0}{r}\right)^{-1} - r^2 d\phi^2 \quad -(9)$$

The equation of motion is obtained by multiplying this by $\mu / 2$. Only in this way is the Newtonian limit obtained correctly. So:

$$\frac{1}{2}\mu\, ds^2 = \frac{1}{2}\mu c^2 d\tau^2 = \frac{1}{2}\mu c^2 dt^2\left(1 - \frac{r_0}{r}\right) - \frac{1}{2}\mu\, dr^2\left(1 - \frac{r_0}{r}\right)^{-1} - \frac{1}{2}\mu r^2 d\phi^2 \quad -(10)$$

In order to obtain eq. (7) correctly:

$$r_0 = 2(m_1 + m_2)\frac{G}{c^2} \quad -(11)$$

The constants of motion are:

$$E = \mu c^2\left(1 - \frac{r_0}{r}\right)\frac{dt}{d\tau}, \quad L = \mu r^2 \frac{d\phi}{d\tau} \quad -(12)$$

so the equation of motion is:

$$\frac{1}{2}\mu\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{2\mu c^2} - \frac{1}{2}\mu c^2\left(1 - \frac{r_0}{r}\right) - \frac{1}{2}\frac{L^2}{\mu r^2}\left(1 - \frac{r_0}{r}\right) \quad -(13)$$

which is:

$$\left(\frac{E^2}{2\mu c^2} - \frac{1}{2}\mu c^2\right) = \frac{1}{2}\mu\left(\frac{dr}{d\tau}\right)^2 - \frac{m_1 m_2 G}{r} + \frac{L^2}{2\mu r^2} - \frac{(m_1 + m_2)L^2 G}{c^2 r^3} \quad -(14)$$

The orbital equation is obtained from eq. (13) using

$$\frac{dr}{d\tau} = \frac{d\phi}{d\tau}\frac{dr}{d\phi} = \left(\frac{L}{\mu r^2}\right)\frac{dr}{d\phi} \quad -(15)$$

So:

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right) \quad -(16)$$

where

$$a = \frac{L}{\mu c}, \quad b = \frac{cL}{E} \quad -(17)$$

are constants of motion.

From eq. (16):

$$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(18)$$

As in note 150(4) the deflection of light due to gravitation is:

$$\boxed{\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr} \quad -(19)$$

where R_0 is the distance of closest approach.

Numerical Integration

Eq. (19) can be integrated numerically with a and b regarded as parameters. For light deflection by sun:

$$G = 6.6726 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$$

$$m_2 = \text{mass of sun} = 1.989 \times 10^{30}\, kg$$

$$R_0 = \text{radius of sun} = 6.955 \times 10^{8}\, m$$

$$\Delta\phi = 1.75 \text{ arcseconds} = 8.484 \times 10^{-6} \text{ radians}$$

The photon mass m_1 is unknown but is thought

4)

to be less than about 10^{-63} kilograms. The reduced mass μ does not appear in a and b because of eqns. (12)

Eq. (19) is therefore:

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{2(m_1+m_2)G}{c^2 r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr$$

— (20)

For all practical purposes this is:

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{2m_2 G}{c^2 r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr$$

— (21)

$= 8.484 \times 10^{-6}$ radians

There are two constants of motion, a and b. So more information is needed to evaluate them. However, the definite integral in eq. (21) can be used to express $\Delta\phi$ in terms of a and b, using numerical integration.

— (22)

Newtonian Approximation

This is obtained by assuming:

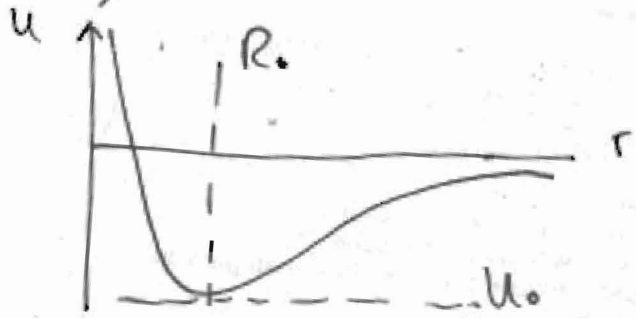
$$\Delta\phi \to 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2} + \frac{2m_2 G}{c^2 a^2 r}\right)^{-1/2} dr$$

In this limit, the "effective potential" is:

5)

$$U = -\frac{k}{r} + \frac{L^2}{2\mu r^2} \quad -(23)$$

Fig (2)



It has a minimum at

$$r = \frac{L^2}{\mu k}, \quad -(24)$$

$$\frac{dU}{dr} = 0 \quad -(25)$$

at which

$$\frac{k}{r} = \frac{L^2}{\mu r^2} \quad -(26)$$

However, the complete potential is:

$$U = -\frac{k}{r} + \frac{L^2}{2\mu r^2} - \frac{(m_1+m_2)L^2 G}{c^2 r^3} \quad -(27)$$

$$U = -\frac{m_1 m_2 G}{r} + \frac{L^2}{2 m_2 r^2} - \frac{m_2 L^2 G}{c^2 r^3} \quad -(28)$$

for all practical purposes.

The next note will use eq. (28) to investigate the ~~x~~ constants a and b in the integral (21).

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1) 150(6): Integral to be Evaluated Numerically.

: The integral used by Albert Einstein was:

$$\Delta\phi = 2\int_0^{1/R_0} \left(\frac{1}{b^2} - u^2 + r_0 u^3 \right)^{-1/2} du \quad - (1)$$

where

$$\frac{1}{b^2} = \frac{1}{R_0^2} - \frac{r_0}{R_0^3} \quad - (2)$$

with

$$r_0 = \frac{2mG}{c^2} \quad - (3)$$

Here R_0 = distance of closest approach
m = mass of the sun
G = Newton's constant
c = vacuum speed of light.

One can use:

R_0 = radius of sun = 6.955×10^8 metres

$m = 1.989 \times 10^{30}$ kilograms
$G = 6.6726 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$
$c = 2.998 \times 10^8\ m\,s^{-1}$

Experimentally (NASA Cassini):

$\Delta\phi = 1.75$ arcseconds $= 8.484 \times 10^{-6}$ radians.

Albert Einstein evaluated eq. (1) in an approximation to give $\Delta\phi = \dfrac{4mG}{c^2 R_0}$ $- (4)$

as is well known.

2)

However, it is not well known that several ad hoc approximations are used to obtain (1). It is not even clear that eq. (1) actually gives eq. (4). So the first thing to do is to check that a contemporary numerical integration of eq. (1) gives the experimental result.

Of course it is known that eq. (4) <u>happens</u> to give the experimental result using R_0 as the radius of the sun. However, does eq. (1) give the experimental result? <u>This is an important question.</u> Einstein's assumptions are as follows.

1) In the <u>correct</u> equation:

$$\Delta \phi = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r} \right) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-1/2} dr$$

$$= 2 \int_0^{1/R_0} \left(\frac{1}{b^2} - \left(1 - r_0 u \right) \left(\frac{1}{a^2} + u^2 \right) \right)^{-1/2} du \quad -(5)$$

it is assumed by Einstein that

$$a \rightarrow \infty, \quad -(6)$$

so eq. (5) reduces to eq. (1).

In my opinion it is better not to make the assumption (6) for reasons given in the next note, and to evaluate eq. (5) correctly to give $\Delta \phi$ as a function of a and b, using the sun's radius for R_0.

150(7): Assumptions made by Albert Einstein in the Calculation of Light Deflection due to Gravitation.

The main assumption is:

$$a \rightarrow \infty. \quad -(1)$$

where

$$a = \frac{L}{\mu c}. \quad -(2)$$

Here L is the angular momentum, a constant of motion, and

$$\mu = \frac{mM}{m+M} \quad -(3)$$

is the reduced mass. If m is the photon mass and M is the mass of the sun, then

$$m << M \quad -(4)$$

and

$$\mu = m \quad -(5)$$

for all practical purposes. Note carefully that m must be identically non-zero, so a cannot be infinite. So Einstein introduced a singularity. In consequence the photon mass m disappeared from his calculation.

The angular momentum is a constant of motion and is defined by:

$$L = mr^2 \frac{d\phi}{d\tau} \quad -(6)$$

in cylindrical polar coordinates. Here τ is the proper time. Since L is a constant of motion (i.e. first integral of motion), Einstein's assumption (1) means:

$$m \rightarrow 0, \quad \frac{d\phi}{d\tau} \rightarrow \infty. \quad -(7)$$

In eq. (6)

$$\frac{d\phi}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \left(1 - \frac{r_0}{r}\right)^{-1/2} \frac{d\phi}{dt} \quad -(8)$$

where t is the time in the laboratory frame, τ is the proper time, the time in the rest frame of the photon of mass m. Here v is the magnitude of the total velocity of the photon, and $r_0 = 2mG/c^2$.

So the assumption (1) means:

$$d\tau \to 0 \quad -(9)$$

and

$$ds^2 = c^2 d\tau^2 \to 0. \quad -(10)$$

This means that one cannot divide by $d\tau$ in the equation of motion if eq. (1) is used.

It is seen from eq. (8) that as:

$$v \to c \quad -(11)$$

then

$$\frac{d\phi}{d\tau} \to \infty. \quad -(12)$$

Eq. (11) is known as the "ultra-relativistic limit," but v cannot be identically the same as c.

The so-called "effective potential" of the calculation is:

$$V(r) = \frac{mc^2}{2}\left(-\frac{r_0}{r} + \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3}\right) \quad -(13)$$

$$= -\frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2 r^3}. \quad -(14)$$

By using eq (1), Einstein assumed:

$$V(r) = \frac{mc^2}{2}\left(\frac{a^2}{r^2} - \frac{r_0 a^2}{r^3}\right) \quad -(15)$$

with

$$m \to 0, \quad a \to \infty \quad -(16)$$

He also assumed circular orbits, i.e.:

3)

$$F(r) = -\frac{dV(r)}{dr} = 0 \quad -(17)$$

i.e.

$$r_0 r^2 - 2a^2 r + 3 r_0 a^2 = 0. \quad -(18)$$

This means:

$$r = \frac{a^2}{r_0}\left(1 \pm \left(1 - \frac{3r_0^2}{a^2}\right)^{1/2}\right). \quad -(19)$$

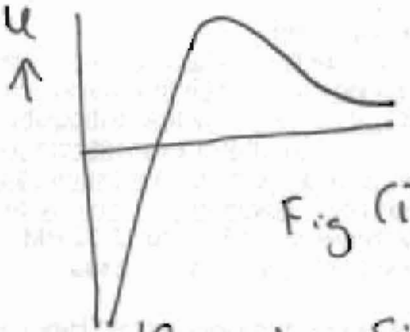

Fig (1).

In retrospect, one can see that this was done just for ease of calculation.

The standard school associates the + sign in eqn (19) with "a stable outer radius," and the − sign with an unstable inner radius." It is then claimed that the condition (1) means:

$$a \gg r_0. \quad -(20)$$

However, this must mean that a is not infinite, and that the photon mass m is identically non-zero. In his calculation of light deflection, Einstein used:

$$r_{inner} = \frac{a^2}{r_0}\left(1 - \left(1 - \frac{3r_0^2}{a^2}\right)^{1/2}\right)$$

$$\sim \frac{a^2}{r_0}\left(1 - 1 + \frac{3}{2}\frac{r_0^2}{a^2}\right) \quad -(21)$$

$$\rightarrow \frac{3}{2} r_0 \quad \text{as} \quad a \rightarrow \infty$$

This r_{inner} radius is always assumed to be

4)

physically meaningful is the standard school. It is eq. (27) of note 150(4):

$$r_0 = \frac{3mG}{c^2} \quad -(22)$$

at which V_0 is a maximum:

$$V_0 = \frac{L^2 c^4}{54 m M^2 G} \quad -(23)$$

It is then assumed that:

$$\frac{1}{2}\frac{E_0^2}{mc^2} = V_0 \quad -(24)$$

where E_0 is the minimum energy needed to overcome the barrier represented by V_0. The impact parameter is defined by:

$$b = \frac{cL}{E_0} = \sqrt{27}\left(\frac{mG}{c^2}\right) \quad -(25).$$

and the capture cross section by πb^2.

This is claimed that a photon will be captured at an impact parameter less than b.

For the sun:

$$b = 7.672 \times 10^3 \text{ metres}$$

which is much less than the radius of the sun:

$$R_0 = 6.955 \times 10^8 \text{ metres.}$$

So it is claimed that a photon will never be captured by the sun, just deflected by the sun at a distance of closest approach R_0. From the claim:

$$\Delta\phi = \frac{4mG}{c^2 R_0} \quad -(26)$$

it is seen that R_0 must be the sun's radius.

5) This is because:

$$\Delta\phi = 1.75 \text{ arcseconds} = 8.484 \times 10^{-6} \text{ radians,}$$
$$M = 1.989 \times 10^{30} \text{ kilograms}$$
$$G = 6.6726 \times 10^{-11} \ m^3 \, kg^{-1} \, s^{-2}$$
$$c = 2.998 \times 10^{8} \ m \, sec^{-1}$$

so:

$$R_0 = \frac{4MG}{c^2 \Delta\phi} = 6.962 \times 10^{8} \text{ metres}$$

which compares with the sun's radius of 6.955×10^{8} metres.

In Einstein's calculation, the distance of closest approach is obtained from:

$$\frac{E^2}{2mc^2} = \frac{L^2}{2mr^3}\left(r - \frac{2MG}{c^2}\right) \quad -(27)$$

and

$$b = \frac{cL}{E} \quad -(28)$$

i.e.

$$r^3 = b^2\left(r - \frac{2MG}{c}\right) \quad -(29)$$

At closest approach:

$$R_0^3 = b_0^2\left(R_0 - r_0\right) \quad -(30)$$

Therefore:

$$\Delta\phi = 2\int_0^{1/R_0}\left(\frac{1}{b_0^2} - u^2 + r_0 u^3\right)^{-1/2} du \quad -(31)$$

$$\frac{1}{b_0^2} = \frac{1}{R_0^2} - \frac{r_0}{R_0^3} \quad -(32)$$

Using an obscure method, Einstein claimed

) that the integral (31) produces the result (26). Finally it was incorrectly claimed that the Eddington experiment produced the result (26).

If one were looking at this calculation objectively and uninfluenced by dogma, it would seem to be based on shaky assumptions. The main assumption is that eq. (13) or (14) can be replaced by eq. (15). This means:

$$V(r) = -\frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \rightarrow \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \quad -(33)$$

This can only be true if:

$$\infty. \quad -(34)$$

$$V(r) = m\left(-\frac{MG}{r} + r\left(\frac{d\phi}{d\tau}\right)^2\left(\frac{1}{2}r - \frac{MG}{c^2}\right)\right) \quad -(35)$$

$$\rightarrow mr\left(\frac{d\phi}{d\tau}\right)^2\left(\frac{1}{2}r - \frac{MG}{c^2}\right) \quad -(36)$$

with $m \rightarrow 0, \; \frac{d\phi}{d\tau} \rightarrow \infty.$

These assumptions mean:

$$r \rightarrow \infty \quad -(37)$$

because from eq. (33):

$$\frac{mMG}{r} \rightarrow 0. \quad -(38)$$

In eq. (33), m, M, G, c and L are all constants, so the only possibility is eq. (37). However, Einstein's calculation was

1) a finite r throughout, and this is a self-contradiction. For example the r_0 of eq. (22) and the R_0 of eq. (32).

Looked at in another way, Einstein used eq. (1) in eq. (13), i.e.

$$V(r) = \frac{mc^2}{2}\left(-\frac{r_0}{r} + \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3}\right) \quad -(39)$$

with

$$a \rightarrow \infty. \quad -(40)$$

He assumed $$V(r) \xrightarrow[a \rightarrow \infty]{} \frac{mc^2}{2} a^2 \left(\frac{1}{r^2} - \frac{r_0}{r^3}\right) \quad -(40)$$

with:

$$m \rightarrow 0, \; a \rightarrow \infty \quad -(41)$$

Therefore $V(r)$ is defined only as a mathematical limit. With the additional limit (37) we have:

$$V(r) \xrightarrow[\substack{a \rightarrow \infty \\ m \rightarrow 0 \\ r \rightarrow \infty}]{} \frac{mc^2}{2} a^2 \left(\frac{1}{r^2} - \frac{r_0}{r^3}\right) \quad -(41)$$

so $V(r)$ is mathematically indeterminate. It is:

$$V(r) \xrightarrow[\substack{m \rightarrow \infty \\ a \rightarrow \infty \\ r \rightarrow \infty}]{} \frac{mc^2}{2}\left(\frac{a}{r}\right)^2\left(1 - \frac{r_0}{r}\right) \quad -(42)$$

$$\boxed{V \xrightarrow[\substack{m \rightarrow \infty \\ a \rightarrow \infty \\ r \rightarrow \infty}]{} \frac{mc^2}{2}\left(\frac{a}{r}\right)^2} \quad -(43)$$

In this limit it cannot be determined whether

8) the potential $V(r)$ has any finite value. If the photon mass is identically zero, as in Einstein's calculation, $V(r)$ vanishes.

The correct method of carrying out the calculation is to replace eq. (27) by:

$$\frac{E^2}{2mc^2} - \frac{1}{2}mc^2 = -\frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \quad -(44)$$

with

$$\frac{cL}{E} = b \quad -(45)$$

in order to find b_0 at $r = R_0$. Finally replace eq. (31) by:

$$\Delta\phi = 2\int_0^{1/R_0}\left(\frac{1}{b_0^2} - (1 - r_0 u)\left(\frac{1}{a^2} + u^2\right)\right)^{-1/2} du$$

to find $\Delta\phi$ as a function of a.

This calculation determines a from the experimentally measured $\Delta\phi$, i.e. it finds $L/(mc)$ from $\Delta\phi$. If L can be estimated independently, the photon mass m can be found.

5(8): Computation of Light Deflection in Terms of Photon Mass.

The correct integral to use is:

$$\Delta \phi = 2 \int_0^{1/R_0} \left(\frac{1}{b^2} - (1 - r_0 u)\left(\frac{1}{a^2} + u^2\right)\right)^{-1/2} du \qquad -(1)$$

In order to find a relation between a and b, the correct equation of motion must be used:

$$\frac{E^2}{mc^2} = \left(1 - \frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right) + m\left(\frac{dr}{d\tau}\right)^2, \qquad -(2)$$

where E is the total energy, L is the total angular momentum, m is the mass of the photon and:

$$r_0 = \frac{2MG}{c^2} \qquad -(3)$$

here M is the mass of the sun.

Einstein's assumptions were:

) Circular orbit, i.e.:

$$\frac{dr}{d\tau} = 0 \qquad -(4)$$

Neglect of mc^2, so eq.(2) became

$$\frac{E^2}{mc^2} = \left(1 - \frac{r_0}{r}\right)\frac{L^2}{mr^2} \qquad -(5)$$

In terms of a and b, eq.(2) is: -(6)

$$\frac{1}{b^2} = \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) + \frac{1}{a^2c^2}\left(\frac{dr}{d\tau}\right)^2$$

2)

The first thing to notice is that the assumption of a circular orbit introduces a severe self-contradiction because eq. (6) becomes:

$$\frac{1}{b^2} = \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) \quad -(7)$$

and the integral (1) is:

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(8)$$

So because of eq. (7), the denominator in the integrand of eq. (8) goes to zero.

<u>This</u> severe self <u>inconsistency</u> appears to <u>have been overlooked</u> for <u>more</u> than <u>ninety years</u>.

Einstein's method was to use:

$$\frac{1}{a^2} = 0 \quad -(9)$$

in eqns. (7) and (8). The parameter b was calculated at the distance of closest approach, denoted R_0, so with eq. (9):

$$\frac{1}{b_0^2} = \left(1 - \frac{r_0}{R_0}\right)\frac{1}{R_0^2} \quad -(10)$$

<u>It</u> is <u>impossible</u> to <u>avoid</u> the <u>conclusion that</u> <u>Einstein's</u> <u>method</u> is <u>meaningless</u>.

3)

The correct method is to use eq. (8) with a non circular orbit:

$$\frac{1}{a^2c^2}\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right) \neq 0 \quad -(7)$$

in which:

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{dr}{d\phi}\right)^2\left(\frac{d\phi}{d\tau}\right)^2 \quad -(8)$$

$$= \frac{L^2}{m^2r^4}\left(\frac{dr}{d\phi}\right)^2 = \frac{a^2c^2}{r^4}\left(\frac{dr}{d\phi}\right)^2 \quad -(9)$$

so

$$\left(\frac{dr}{d\phi}\right)^2 = r^2\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right) \neq 0 \quad -(10)$$

and

$$\boxed{\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} \neq 0} \quad -(11)$$

Einstein assumed a circular orbit, so:

$$\frac{dr}{d\phi} = 0 \quad -(12)$$

$$\frac{d\phi}{dr} = \infty \quad -(13)$$

The only correct method is to compute the integral (8) for a non-circular orbit.

4) This means that the experimentally measured deflection $\Delta'\phi$ must be expressed in terms of a and b:

$$a = \frac{L}{mc}, \quad b = c\frac{L}{E} \quad -(14)$$

These are related by:

$$a = \left(\frac{E}{mc^2}\right) b \quad -(15)$$

$$-(16)$$

so:

$$\Delta\phi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\left(\frac{E}{mc^2}\right)^2\frac{1}{a^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr$$

$$= 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{a^2}\left(\left(\frac{E}{mc^2}\right)^2 - 1 + \frac{r_0}{r}\right) - \left(1-\frac{r_0}{r}\right)\frac{1}{r^2}\right)^{-1/2} dr \quad -(17)$$

$$\Delta\phi = 2\int_{0}^{1/R_0}\left(\frac{1}{b^2} - \left(1-r_0u\right)\left(\frac{1}{a^2}+u^2\right)\right)^{-1/2} du \quad -(18)$$

where

$$\frac{1}{b^2} = \left(\frac{E}{mc^2}\right)^2\frac{1}{a^2} \quad -(19)$$

$$= \frac{1}{c^2}\left(\frac{E}{L}\right)^2$$

Therefore $\Delta\phi$ can be worked out in terms

5) of the distance of closest approach R_0, the photon mass m, and the constants of motion, E and L.

It is known that the photon mass m is very small, but must be identically non-zero. In the first approximation, it is assumed that the spacetime at distance of closest approach R_0 is approximately a Minkowski spacetime. This is a good approximation because:

$$\frac{r_0}{R_0} = \frac{2MG}{c^2 R_0} = 4.242 \times 10^{-6} \quad -(20)$$

so

$$\frac{r_0}{R_0} << 1. \quad -(21)$$

The photon mass is very small, so to a very good approximation:

$$E = \hbar\omega, \; L = \hbar \quad -(22)$$

for one photon.

Therefore

$$\boxed{a = \frac{\hbar}{mc}, \; b = \frac{c}{\omega}} \quad -(23)$$

From eqs. (18) and (23), the photon mass m can be worked out from the measured $\Delta\phi$:

$$\boxed{\Delta\phi = 2\int_0^{1/R_0} \left(\left(\frac{c}{\omega}\right)^2 - (1 - r_0 u)\left(\left(\frac{mc}{\hbar}\right)^2 + u^2\right)\right)^{-1/2} du}$$

$$-(24)$$

6)

For visible frequency light:

$$\omega \sim 10^{16} \text{ radians per second} \quad -(25)$$

So

$$\boxed{\begin{aligned} a &= \frac{2.8427}{m} \times 10^{-42} \text{ metres} \\ b &= 2.998 \times 10^{-8} \text{ metres} \end{aligned}} \quad -(26)$$

using

$$\hbar = 1.05459 \times 10^{-34} \text{ Js}$$

$$c = 2.997925 \times 10^{8} \text{ m s}^{-1}$$

It is seen that the two lengths a and b are about the same if m is very small. The only thing that is claimed from this method is that it is plausible in a first approximation. In the last analysis, the only things that can be obtained from $\Delta \phi$ are a and b.

1) 150(a): Precession of Orbits in Einstein's Theory.

The theory relies on the "effective potential":

$$V(r) = \frac{mc^2}{2}\left(-\frac{r_0}{r} + \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3}\right) \quad -(1)$$

It is again assumed that orbits are circular, so:

$$F = -\frac{dV(r)}{dr} = 0 \quad -(2)$$

$$= -\frac{mc^2}{2r^4}\left(r_0 r^2 - 2a^2 r + 3r_0 a^2\right)$$

However, orbits are not circular.

The solutions of eq. (2) are:

$$r_{outer} = \frac{a^2}{r_0}\left(1 + \left(1 - \frac{3r_0^2}{a^2}\right)^{1/2}\right) \quad -(3)$$

$$r_{inner} = \frac{a^2}{r_0}\left(1 - \left(1 - \frac{3r_0^2}{a^2}\right)^{1/2}\right) \quad -(4)$$

Having assumed that the orbits are circular, the precession of the ellipse is calculated by assuming that there is a small radial deviation from the circular orbit. This is calculated from the angular frequency:

$$\omega_r^2 = \frac{1}{m}\left(\frac{d^2V}{dr^2}\right)_{r = r_{outer}} \quad -(5)$$

It is claimed that:

$$\omega_r^2 = \left(\frac{c^2 r_0}{2r_{outer}^4}\right)\left(r_{outer} - r_{inner}\right)$$

2)

$$:= \omega_\phi^2 \left(1 - \frac{3r_0^2}{a^2}\right) \quad -(6)$$

so
$$\omega_r \sim \omega_\phi \left(1 - \frac{3r_0^2}{4a^2} + \cdots\right) \quad -(7)$$

If the time for one revolution is τ then the orbital precession for one revolution is:

$$\delta\phi = \frac{3\pi m^2 c^2 r_0^2}{2L^2} \quad -(8)$$

using
$$\omega_\phi \tau = 2\pi. \quad -(9)$$

Checks Using Computer Algebra

It should be checked that differentiation of eq. (1) actually produces eq. (6) as claimed in the standard physics.

Self Inconsistency of the Method

It is first assumed that:

$$\frac{d^2V}{dr^2} = 0 \quad -(10)$$

and then assumed that:

$$\frac{d^2V}{dr^2} \neq 0 \quad -(11)$$

The basic assumption is that:

$$\delta\phi = \tau\left(\omega_\phi - \omega_r\right) \quad -(12)$$

3) but the initial assumption (2) means:

$$\delta\phi = 0. \quad -(13)$$

and also means $\frac{dr}{d\phi} = 0. \quad -(14)$

The Correct Method

The correct method is to evaluate the integral

$$\phi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr$$

$$-(15)$$

to give ϕ as a function of r. This method gives a precessing ellipse. In the Newtonian limit:

$$\phi(\text{Newton}) = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\frac{1}{a^2} + \frac{1}{r^2}\right)^{-1/2} dr \quad -(16)$$

and gives a static ellipse.

This method makes no assumption about a circular orbit, because in a circular orbit:

$$\frac{1}{b^2} = \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) \quad -(17)$$

and in eq (15) $\phi \to \infty \quad -(18)$

for all r.