44(1): Analysis of New Accelerations.

The a Type Accelerations

This is given by the orbital type of acceleration, $\underline{a}^a{}_o$, and the spin type of acceleration, $\underline{a}^a{}_s$. These are SOF relativistic accelerations defined by:

$$\underline{a}^a{}_o = \frac{\partial \underline{v}^a}{\partial t} + c\,\underline{\nabla}\, v^a{}_o + c\,\omega^a{}_{ob}\,\underline{v}^b - c\, v^b{}_o\, \underline{\omega}^a{}_b \qquad (1)$$

and

$$\frac{\underline{a}^a{}_s}{c} = \underline{\nabla} \times \underline{v}^a - \underline{\omega}^a{}_b \times \underline{v}^b . \qquad (2)$$

The spin connection terms in eq. (1) represent differences between a Newtonian orbit and a relativistic orbit. The terms in eq. (2) are:

i) The relativistic vorticity, $\underline{\nabla} \times \underline{v}^a$;

ii) The relativistic Coriolis acceleration $c\,\omega^a{}_b \times \underline{v}^b$.

Now define the forces:

$$\underline{F}^a{}_o = m\,\underline{a}^a{}_o \qquad (3)$$

$$\frac{1}{c}\,\underline{F}^a{}_s = \frac{m}{c}\,\underline{a}^a{}_s \qquad (4)$$

The force $\underline{F}^a{}_s / c$ is c times smaller than the force $\underline{F}^a{}_o$. They are SOF relativistic forces

2) and are analogous respectively to the electric field strength $\underline{E}^a$ and magnetic flux density $\underline{B}^a$ in generally covariant electrodynamics (ECE theory):

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c\nabla A_0^a - c\,\omega^a_{0b}\underline{A}^b + c\,A^b_0\,\underline{\omega}^a_b \quad (5)$$

$$\underline{B}^a = \nabla \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad (6)$$

The sign change between eq. (1) and eq. (5) is a matter of convention. In electrodynamics $\underline{B}^a$ is c times smaller than $\underline{E}^a$ in S.I. units.

The acceleration in eq. (3) are the gravitomagnetic type. In paper 117, the gravitomagnetic acceleration was used to explain the precession of the equinox. It may be denoted:

$$\underline{\Omega}^a = \underline{a}^a_S / c \quad (7)$$

and

$$\boxed{\underline{\Omega}^a = \nabla \times \underline{v}^a - \underline{\omega}^a{}_b \times \underline{v}^b} \quad (8)$$

3) In non relativistic dynamics the vorticity is defined by:

$$\underline{\Omega} = \underline{\nabla} \times \underline{v} \quad -(9)$$

but this is not generally covariant by definition.

Limit of Zero Spin Connection

In this limit, eqs. (1) and (2) reduce to Newtonian dynamics and classical vorticity respectively:

$$\underline{a}^a_0 \rightarrow \frac{\partial \underline{v}^a}{\partial t} + c\underline{\nabla} v^a_0, \quad -(10)$$

$$\frac{\underline{a}^a_s}{c} = \underline{\Omega}^a \rightarrow \underline{\nabla} \times \underline{v}^a \quad -(11)$$

Now sum up over:

$$a = (1), (2), (3) \quad -(12)$$

so:

$$\underline{a}_0 = \underline{a}^{(1)}_0 + \underline{a}^{(2)}_0 + \underline{a}^{(3)}_0 \quad -(13)$$

$$\underline{\Omega} = \underline{\Omega}^{(1)} + \underline{\Omega}^{(2)} + \underline{\Omega}^{(3)} \quad -(14)$$

and

$$\underline{a}_0 = \frac{\partial \underline{v}}{\partial t} + c\underline{\nabla} v_0 \quad (15)$$

$$\underline{\Omega} = \underline{\nabla} \times \underline{v} \quad -(16)$$

4) Define the gravitational potential by:

$$\Phi = c V_0 \quad -(17)$$

then:

$$\underline{a}_0 = \frac{\partial \underline{v}}{\partial t} + \underline{\nabla} \Phi . \quad -(18)$$

The antisymmetry law of ECE means that:

$$\frac{\partial \underline{v}}{\partial t} = \underline{\nabla} \Phi \quad -(19)$$

and this is the weak equivalence principle. So

$$\underline{F} = m \frac{\partial \underline{v}}{\partial t} = m \underline{\nabla} \Phi \quad -(20)$$

This gives Newtonian dynamics and orbits.

As in previous work we may write

$$\underline{\Omega}^a_0 = c\, \omega^a_{0b} v^b - c V_0\, \underline{\omega}^a_b \quad -(21)$$

$$\underline{\Omega}^a_s = - \underline{\omega}^a_b \times \underline{V}^b \quad -(22)$$

so $\underline{\Omega}^a_0$ gives the relativistic correction of Newton, and $\underline{\Omega}^a_s$ the relativistic correction of vorticity.

144(2): The $\underline{w}$ Type Velocities and α Accelerations

The spin velocity is defined by:

$$\underline{w}^a = c\left(\underline{\nabla} \times \underline{r}^a - \underline{\omega}^a{}_b \times \underline{r}^b\right) \quad -(1)$$

and the α acceleration by:

$$\underline{\alpha}^a{}_{orb} = \frac{\partial \underline{w}^a}{\partial t} + c\,\underline{\nabla}\,\omega^a_0 + c\,\omega^a{}_{0b}\,\underline{w}^b - c\,\omega^b{}_0\,\underline{\omega}^a{}_b \quad -(2)$$

and

$$\underline{\alpha}^a_{spin} = c\left(\underline{\nabla} \times \underline{w}^a - \underline{\omega}^a{}_b \times \underline{w}^b\right). \quad -(3)$$

In the limit of vanishing spin connection

$$\underline{w}^a \rightarrow c\,\underline{\nabla} \times \underline{r}^a \quad -(4)$$

$$\underline{\alpha}^a_{spin} \rightarrow c\,\underline{\nabla} \times \underline{w}^a \quad -(5)$$

Summing up over the a indices:

$$\underline{w} \rightarrow c\,\underline{\nabla} \times \underline{r} \quad -(6)$$

$$\underline{\alpha}_{spin} \rightarrow c\,\underline{\nabla} \times \underline{w} \quad -(7)$$

So:

$$\boxed{\underline{\alpha}_{spin} = c^2\,\underline{\nabla} \times (\underline{\nabla} \times \underline{r})} \quad -(8)$$

Now use the vector identity ("Vector Analysis Problem Solver", problem 10.22, page 442):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = \underline{\nabla}(\underline{\nabla} \cdot \underline{r}) - \nabla^2 \underline{r} \quad -(9)$$

2) to find that:

$$\boxed{\underline{\alpha}_{spin} = c^2\left(\underline{\nabla}\left(\underline{\nabla}\cdot\underline{r}\right) - \nabla^2\underline{r}\right)} \quad -(10)$$

It is also useful to use the result (VAPS problem 11-8) that if

$$\underline{v} = \underline{\omega}_{av}\times\underline{r} \quad -(11)$$

then

$$\underline{\omega}_{av} = \frac{1}{2}\underline{\nabla}\times\underline{v} \quad -(12)$$

The spin ~~a~~ acceleration is:

$$\underline{a}^a_{spin} = c\left(\underline{\nabla}\times\underline{v}^a - \underline{\omega}^a{}_b\times\underline{v}^b\right) \quad -(13)$$

and in the limit of vanishing ~~acceleration~~ spin connection:

$$\underline{a}^a_{spin}\rightarrow c\left(\underline{\nabla}\times\underline{v}^a\right) \quad -(14)$$

Summing over a:

$$\underline{a}_{spin} = c\,\underline{\nabla}\times\underline{v}$$

$$\boxed{\underline{a}_{spin} = 2c\,\underline{\omega}_{av}} \quad -(15)$$

In summary:

$$\boxed{\begin{aligned}\underline{a}_{spin} &= 2c\,\underline{\omega}_{av}\\ \underline{\alpha}_{spin} &= c^2\left(\underline{\nabla}\left(\underline{\nabla}\cdot\underline{r}\right) - \nabla^2\underline{r}\right)\end{aligned}} \quad -(16)$$

In the limit of vanishing spin connection, Eqs. (16) are the result of spacetime torsion.

3) Applications

1) Whirlpool Galaxy

The whirlpool galaxy is characterised by a constant $\underline{v}$ in eq. (12), so $\underline{\omega}$ and $\underline{a}_{spin}$ are constant. The angular velocity of the whirlpool galaxy is:

$$\underline{\omega}_{av} = \frac{1}{2c} \underline{a}_{spin} = \text{constant} \quad -(17)$$

2) Viscous Fluid

As in VAPS p. 478, the most general form of second derivatives that can occur in a vector equation is a linear combination of the terms $\nabla^2 \underline{r}$ and $\underline{\nabla} (\underline{\nabla} \cdot \underline{r})$. So $\underline{a}_{spin}$ is the most general second derivative of $\underline{r}$. Analogously, the $\underline{a}_{spin}$ acceleration can be used to build up a viscous force; the latter is usually expressed as

$$\underline{f}_v = \mu \nabla^2 \underline{v} + (\mu + \mu') \underline{\nabla} (\underline{\nabla} \cdot \underline{v}) \quad -(18)$$

where μ and μ' are coefficients.

From eq. (15):

$$\frac{1}{c} \underline{\nabla} \times \underline{a}_{spin} = \underline{\nabla} \times (\underline{\nabla} \times \underline{v})$$

4)

$$= \underline{\nabla}(\underline{\nabla}\cdot\underline{v}) - \nabla^2 \underline{v} \quad -(19)$$

This has the structure of viscous force. The vorticity is:

$$\underline{\Omega} = \underline{a}_{spin}/c = \underline{\nabla}\times\underline{v} \quad -(20)$$

The Effect of Spin Connection

This is to introduce non-inertial acceleration such as centripetal and Coriolis. For example:

$$\underline{w}^a = c\left(\underline{\nabla}\times\underline{r}^a - \underline{\omega}^a{}_b\times\underline{r}^b\right). \quad -(21)$$

Summing over a:

$$\underline{w} = c\left(\underline{\nabla}\times\underline{r} - \underline{\omega}_b\times\underline{r}^b\right). \quad -(22)$$

Defining the spin connection vector

$$\underline{\omega} = \omega_{23}\underline{i} + \omega_{31}\underline{j} + \omega_{12}\underline{k} \quad -(23)$$

then

$$r\underline{\omega} = -\underline{\omega}_b\times\underline{r}^b \quad -(24)$$

and

$$\boxed{\underline{w} = c\left(\underline{\nabla}\times\underline{r} + r\underline{\omega}\right)} \quad -(25)$$

The term $r\underline{\omega}$ comes from a spinning frame of reference. This is used in the analysis of a gyroscope and in the levitron.

5) Note that the units of $\underline{\omega}$ are inverse metres, and it should not be confused with the angular velocity $\underline{\omega}_{av}$.

Similarly:

$$\underline{a}_{spin} = c\left(\underline{\nabla} \times \underline{v} + v\underline{\omega}\right) \quad -(26)$$

$$\underline{\alpha}_{spin} = c\left(\underline{\nabla} \times \underline{W} + W\underline{\omega}\right). \quad -(27)$$

From eq. (12), the angular velocity in general is:

$$\boxed{\underline{\omega}_{av} = \frac{1}{2}\left(\underline{\nabla} \times \underline{v} + v\underline{\omega}\right)} \quad -(28)$$

and consists of a vorticity $\underline{\nabla} \times \underline{v}$ and a contribution $v\underline{\omega}$ due to the spin connection. The spin connection vector is proportional to angular velocity:

$$\underline{\omega} = \frac{2}{v}\underline{\omega}_{av} \quad -(29)$$

If v is constant as in a whirlpool galaxy, then the angular velocity and spin connection are directly proportional:

$$\underline{\omega} = \alpha\,\underline{\omega}_{av} \quad -(30)$$

where α is a constant.

144(3): Plane Wave Solution for Spin Velocity and Acceleration in the Limit of Vanishing Connection.

In the limit of vanishing spin connection the spin velocity is:

$$\frac{\mathbf{w}}{c} = \nabla \times \mathbf{r} \qquad (1)$$

and the spin acceleration is:

$$\frac{\boldsymbol{\alpha}_{spin}}{c^2} = \nabla \times (\nabla \times \mathbf{r}) \qquad (2)$$

The plane wave solution to these equations is:

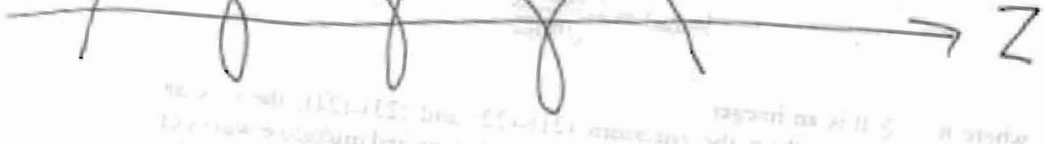
$$\mathbf{r} = \frac{r}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right) \exp\left(i(\omega t - \kappa Z)\right) \qquad (3)$$

where

$$r = |\mathbf{r}|. \qquad (4)$$

Eq (3) is the function describing a helix about Z:

Fig (1)



Therefore:

$$\frac{\mathbf{w}}{c} = i\kappa \mathbf{r}, \qquad (5)$$

$$\boxed{\mathbf{w} = i\omega \mathbf{r}} \qquad (6)$$

where:

$$\omega = \kappa c \qquad (7)$$

is the angular frequency of a particle travelling along the helix.

2) Similarly:

$$\boxed{\underline{\alpha}_{spin} = \omega^2 \underline{r}} \quad -(8)$$

Therefore $\underline{w}$ and $\underline{\alpha}_{spin}$ are the velocity and acceleration of the photon along the helical path in Fig (1), because for the photon, eq. (7) is true.

Therefore eqs. (1) and (2) are equations of electrodynamics as well as equations of dynamics. They are kinematic equations of the photon. This is clearly a limit of special relativity because it is the limit in which the spin connection vanishes. The spin connection goes to zero but is not identically zero, because for the latter case, the torsion vanishes, and $\underline{w}$ and $\underline{\alpha}_{spin}$ are also identically zero.

The basic hypothesis of paper 143 is

$$V^a_{\mu\nu} = (D \wedge r^a)_{\mu\nu} \quad -(9)$$

and this leads to the kinematic equation of the photon. In this example, the spin velocity

3) is the velocity of the photon along a helix, and the spin acceleration $\underline{\alpha}$ spin is the acceleration of the photon along the helical trajectory.

Interestingly, a finite spin connection will produce generally covariant corrections to the trajectory of a photon in free space. These may occur in cosmological situation for example. One well known example is the light bending by gravitation. In this development it is seen from eq. (3) that the frame itself propagates along Z. This is the philosophy of general relativity, the spin velocity $\underline{w}$ and the spin acceleration $\underline{\alpha}$ spin are defined in terms of a propagating frame of reference.

In deriving eq. (8) we have used:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = \underline{\nabla} (\underline{\nabla} \cdot \underline{r}) - \nabla^2 \underline{r} \qquad -(10)$$

For eq. (3):

$$\underline{\nabla} \cdot \underline{r} = 0 \qquad -(11)$$

so

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = -\nabla^2 \underline{r} \qquad -(12)$$

144(4): Plane Wave Solution for Velocity and Acceleration In the Limit of Vanishing Spin Connection

In this case:

$$\frac{\underline{v}}{c} = \frac{1}{c}\frac{\partial \underline{r}}{\partial t} + \underline{\nabla}\, r_0 \quad -(1)$$

By antisymmetry:

$$\frac{1}{c}\frac{\partial \underline{r}}{\partial t} = \nabla r_0 \quad -(2)$$

The line function is:

$$\underset{\sim}{r} = \frac{r}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) e^{i(\omega t - \kappa Z)} \quad -(3)$$

which is a plane wave. The positive four vector is:

$$r^{\mu} = \left(r_0, \underset{\sim}{r}\right) \quad -(4)$$

Therefore:

$$\boxed{\underline{v} = \frac{\partial \underline{r}}{\partial t} = c\nabla r_0 = i\omega \underline{r}} \quad -(5)$$

and

$$\underline{\nabla}\, r_0 = i\frac{\omega}{c}\underset{\sim}{r} = i\kappa \underline{r} \quad -(6)$$

Therefore

$$\boxed{\underline{v} = 2 i\omega \underline{r}} \quad -(7)$$

The acceleration is:

$$\underline{a}_{\text{orbital}} = \frac{\partial \underline{v}}{\partial t} + c\underline{\nabla}\, v_0 \quad -(8)$$

2) where:

$$\frac{\partial \mathbf{v}}{\partial t} = c \nabla V_0 \quad -(9)$$

by antisymmetry. So:

$$\boxed{\mathbf{a}_{orbital} = -4\omega^2 \mathbf{r}} \quad -(10)$$

and

$$\nabla V_0 = -\frac{2\omega^2}{c}\mathbf{r} \quad -(11)$$

From eq. (6) there are equations for r_0 as follows:

$$\frac{\partial r_0}{\partial X} = -\frac{i\kappa r}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right) \quad -(12)$$

$$\frac{\partial r_0}{\partial Y} = \frac{\kappa r}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right) \quad -(13)$$

From eq. (11)

$$\frac{\partial V_0}{\partial X} = -\frac{2\omega^2}{\sqrt{2}c} \exp\left(i(\omega t - \kappa Z)\right) \quad -(14)$$

$$\frac{\partial V_0}{\partial Y} = -\frac{2i\omega^2}{\sqrt{2}c} \exp\left(i(\omega t - \kappa Z)\right) \quad -(15)$$

Here

$$r^\mu = (r_0, \mathbf{r}) \quad -(16)$$

$$V^\mu = (V_0, \mathbf{V}) \quad -(17)$$

Here r_0 and V_0 are integrated using the Stokes Theorem as follows:

3)

$$r_0 = \frac{1}{c}\oint \underline{v}\cdot d\underline{r} = \frac{1}{c}\int_S \underline{\nabla}\times\underline{v}\cdot d\underline{A} \qquad -(18)$$

and

$$v_0 = \frac{1}{c}\oint \underline{a}\cdot d\underline{r} = \frac{1}{c}\int_S \underline{\nabla}\times\underline{a}\cdot d\underline{A} \qquad -(19)$$

where

$$\underline{a} = 2\frac{\partial \underline{v}}{\partial t} \qquad -(20)$$

so

$$v_0 = 2\frac{\partial r_0}{\partial t} \qquad -(21)$$

If the four momentum is defined as

$$p^\mu = m v^\mu = \left(\frac{E}{c}, \underline{p}\right) \qquad -(22)$$

then

$$\boxed{E = mcv_0} \qquad -(23)$$

therefore

$$E = 2mc\frac{\partial r_0}{\partial t} \qquad -(24)$$

The rest energy for a photon is defined by de Broglie's Theorem:

$$E_0 = \hbar\omega = mc^2 \qquad -(25)$$

in which case:

$$\boxed{c = 2\frac{\partial r_0}{\partial t}} \qquad -(26)$$

so $r_0 \sim 1.5\times 10^8$ metres

and is a rest length.

144(5): New Fields.

These are found from the minimal prescription:

$$p_\mu^a = m v_\mu^a = e A_\mu^a \quad -(1)$$

$$\quad -(2)$$

so: $A_\mu^a = \frac{m}{e} v_\mu^a$.

Orbital Vector Potential

The space like part of this is defined by:

$$\underline{A}_{orb}^a = \frac{m}{e}\left(\frac{\partial \underline{r}^a}{\partial t} + c\nabla r_0^a + c\omega_{0b}^a \underline{r}^b - c r_0 \cdot \underline{\omega}^a{}_b\right) \quad -(3)$$

Spin Vector Potential

The space like part of this is defined by:

$$\frac{\underline{A}_{spin}^a}{c} = \frac{m}{e}\left(\nabla \times \underline{r}^a - \underline{\omega}^a{}_b \times \underline{r}^b\right) \quad -(4)$$

The spin vector potential is c times smaller in magnitude than the orbital vector potential.

The usual electric and magnetic fields are derived from the orbital part of the potential. By convention, the sign is changed for the electric field, so:

$$\underline{E}_{orb}^a = -\frac{\partial \underline{A}^a_{orb}}{\partial t} - c\nabla A_{0\,orb}^a - c\omega_{0b}^a \underline{A}_{orb}^b + c A_{0\,orb}^b \cdot \underline{\omega}^a{}_b \quad -(5)$$

and

2).

$$\underline{B}^a_{orb} = \underline{\nabla} \times \underline{A}^a_{orb} - \underline{\omega}^a{}_b \times \underline{A}^b_{orb} \quad - (6)$$

New Types of Electric and Magnetic Field Derived From the Spin Potential.

These are c times smaller in magnitude than the well known electric and magnetic fields, and are defined by:

$$\frac{\underline{E}^a_{spin}}{c} = -\frac{1}{c}\left(\frac{\partial \underline{A}^a_{spin}}{\partial t} + c\underline{\nabla} A^a_{0\,spin} + c\,\omega^a{}_{0b}\,\underline{A}^b_{spin} - c\,A^b_{0\,spin}\,\underline{\omega}^a{}_b \right) \quad - (7)$$

$$\frac{\underline{B}^a_{spin}}{c} = \frac{1}{c}\left(\underline{\nabla} \times \underline{A}^a_{spin} - \underline{\omega}^a{}_b \times \underline{A}^b_{spin} \right) \quad - (8)$$

It can be seen that the orbital and spin vector potentials each have an internal structure defined by eqns. (3) and (4). The new types of electric and magnetic fields are c times smaller than the well known types of electric and magnetic fields. This means they are ten million times smaller in magnitude.

3) The internal structure of the usual vector potential (orbital vector potential) as given in eq. (3), leads to the possibility of many new types of spin connection resonance.

Dynamics and Gravitational Theory.

In this theory there exist new types of ~~the~~ spin vector potentials and dynamics and theory of gravitation, and new types of spin connection resonances.

New Field Equations

There exist new field equations for the spin electric and magnetic fields.

144(6): New Spin Connection Resonance Structures

Define: $$R_\mu^a = \frac{m}{e} r_\mu^a \quad -(1)$$

Here:

$$\underline{A}^a_{orb} = \frac{\partial \underline{R}^a}{\partial t} + c\underline{\nabla} R_0^a + c\omega^a_{0b}\underline{R}^b - cR_0^b\underline{\omega}^a_b \quad -(2)$$

$$\frac{1}{c}\underline{A}^a_{spin} = \underline{\nabla}\times\underline{R}^a - \underline{\omega}^a_b\times\underline{R}^b \quad -(3)$$

From the Cartan and Evans identities:

$$\underline{\nabla}\cdot\underline{A}^a_{spin} = 0 \quad -(4)$$

$$\underline{\nabla}\times\underline{A}^a_{orb} + \frac{1}{c}\frac{\partial\underline{A}^a_{spin}}{\partial t} = \underline{0} \quad -(5)$$

$$\underline{\nabla}\cdot\underline{A}^a_{orb} = \xi^a \quad -(6)$$

$$\underline{\nabla}\times\underline{A}^a_{spin} - \frac{1}{c}\frac{\partial\underline{A}^a_{orb}}{\partial t} = \underline{\xi}^a \quad -(7)$$

Eqs. (2) and (3), when used in eqs. (4) to (7), produce various spin connection resonance in:

$$R_\mu^a = (R_0^a, -\underline{R}^a) \quad -(8)$$

$$= \frac{m}{e} r_\mu^a$$

This means that there is an internal resonance property of $\underline{A}^a_{spin}$ and $\underline{A}^a_{orb}$.

144(7): Interaction of Spin and Orbital E and B

The basic structure of the theory is:

$$A = \frac{m}{e} D \wedge r \quad - (1)$$

$$F = D \wedge A \quad - (2)$$

The two-form on the LHS of eq. (1) is re-arranged to give the one-form potential A^a_μ used in eq. (2). This gives:

$$\underline{A}^a_{orb} = \frac{m}{e}\left(\frac{\partial \underline{r}^a}{\partial t} + c\nabla r^a_0 + c\omega^a_{0b}\underline{r}^b - c r^b_0 \underline{\omega}^a_b\right) \quad - (3)$$

$$\underline{A}^a_{spin} = \frac{m}{e}\,c\left(\nabla \times \underline{r}^a - \underline{\omega}^a_b \times \underline{r}^b\right) \quad - (4)$$

$$\boxed{\text{It is seen that } \underline{A}^a_{spin} \text{ is } c \text{ times smaller than } \underline{A}^a_{orb}}$$

Analogously, the magnetic flux density $\underline{B}$ is c times smaller in magnitude than the electric field strength $\underline{E}$. Similarly, the gravitomagnetic field $\underline{\Omega}$ is c times smaller than the $\underline{g}$ gravitational quantity of Newtonian theory. Therefore $\underline{A}^a_{spin}$ is going to be a small correction to the usual (Heaviside) $\underline{A}^a_{orbital}$. We can write:

2)

$$|\underline{A}^a_{orb}| = c\,|\underline{A}^a_{spin}|. \quad -(5)$$

Analogously:

$$|\underline{E}| = c|\underline{B}| \quad -(6)$$

in S.I. units, and:

$$|\underline{g}| = c\,|\underline{\Omega}|. \quad -(7)$$

Now write:

$$\underline{B}^a(tot) = \underline{B}^a_{orb} + c\,\underline{B}^a_{spin} \quad -(8)$$

$$\underline{E}^a(tot) = \underline{E}^a_{orb} + \underline{E}^a_{spin} \quad -(9)$$

Note that:

$$|\underline{E}^a_{orb}| = c\,|\underline{E}^a_{spin}| \quad -(10)$$

$$|\underline{B}^a_{orb}| = c\,|\underline{B}^a_{spin}| \quad -(11)$$

Thus:

$$\underline{\nabla} \cdot \underline{B}^a(tot) = 0 \quad -(12)$$

$$\underline{\nabla} \times \underline{E}^a(tot) + \frac{\partial \underline{B}^a(tot)}{\partial t} = \underline{0} \quad -(13)$$

$$\underline{\nabla} \cdot \underline{E}^a(tot) = \rho^a(tot) / \epsilon_0 \quad -(14)$$

$$\underline{\nabla} \times \underline{B}^a(tot) - \frac{1}{c^2}\frac{\partial \underline{E}^a(tot)}{\partial t} = \mu_0 \underline{J}^a(tot) \quad -(15)$$

The total $\underline{E}^a$ and $\underline{B}^a$ fields in eqns (8) and (9) are dominated by the orbital fields, which have a magnitude c times greater than the spin fields. However, a possible solution of eq. (13) is:

$$\nabla \times \underline{E}^a(\text{orbital}) + \frac{\partial \underline{B}^a}{\partial t}(\text{spin}) = \underline{0} \quad -(16)$$

and another possible solution is:

$$\nabla \times \underline{E}^a(\text{spin}) + \frac{\partial \underline{B}^a}{\partial t}(\text{orbital}) = \underline{0} \quad -(17)$$

In certain circumstances the orbital electric field may induce a spin magnetic field, and vice versa. The other two possible Faraday laws of induction are:

$$\nabla \times \underline{E}^a(\text{orbital}) + \frac{\partial \underline{B}^a}{\partial t}(\text{orbital}) = \underline{0} \quad -(18)$$

$$\nabla \times \underline{E}^a(\text{spin}) + \frac{\partial \underline{B}^a}{\partial t}(\text{spin}) = \underline{0} \quad -(19)$$

The Coulomb Law

This is:

$$\nabla \cdot \underline{E}(\text{tot.}) = \rho(\text{total})/\epsilon_0 \quad -(20)$$

where

$$\underline{E}(\text{tot}) = \underline{E}(\text{orb}) + \underline{E}(\text{spin}) \quad -(21)$$

4)

Here:

$$\underline{E}(\text{orb}) = -\frac{\partial \underline{A}}{\partial t}(\text{orb}) - c\underline{\nabla} A_0(\text{orb}) - c\,\omega_{0b}\underline{A}^{b}_{\text{orb}} + cA^{b}_{0}(\text{orb})\,\underline{\omega}_{b} \qquad -(22)$$

$$\underline{E}(\text{spin}) = -\frac{\partial \underline{A}}{\partial t}(\text{spin}) - c\underline{\nabla} A_0(\text{spin}) - c\,\omega_{0b}\underline{A}^{b}_{\text{spin}} + cA^{b}_{0}(\text{spin})\,\underline{\omega}_{b}. \qquad -(23)$$

Here:

$$\underline{\nabla} \cdot \underline{A}(\text{spin}) = 0 \qquad -(24)$$

$$\underline{\nabla} \times \underline{A}(\text{orb}) + \frac{\partial \underline{A}}{\partial t}(\text{spin}) = \underline{0} \qquad -(25)$$

$$\underline{\nabla} \cdot \underline{A}(\text{orb}) = \xi \qquad -(26)$$

$$\underline{\nabla} \times \underline{A}(\text{spin}) - \frac{1}{c^2}\frac{\partial \underline{A}}{\partial t}(\text{orb}) = \underline{\xi} \qquad -(27)$$

$$-(28)$$

Here:

$$\underline{A}_{\text{orb}} = \frac{m}{e}\left(\frac{\partial \underline{r}}{\partial t} + c\underline{\nabla} r_0 + c\,\omega_{0b}\underline{r}^{b} - c\,r^{b}_{0}\,\underline{\omega}_{b}\right)$$

$$\frac{\underline{A}_{\text{spin}}}{c} = \frac{m}{e}\left(\underline{\nabla} \times \underline{r} - \underline{\omega}_{b} \times \underline{r}^{b}\right) \qquad -(29)$$

144(10): Experiment to Detect the E and B Spin Fields

Consider an electron trajectory:

$$\underset{\sim}{r} = \frac{r}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) \exp \left(i (\omega t - \kappa Z) \right) \quad -(1)$$

where

$$r = | \underline{r} | \quad -(2)$$

As in notes 144(3) this produces the spin velocity:

$$\frac{\underline{w}}{c} = \underline{\nabla} \times \underline{r} \quad -(3)$$

and spin acceleration:

$$\frac{\underline{\alpha}_{spin}}{c^2} = \underline{\nabla} \times (\underline{\nabla} \times \underline{r}) \quad -(4)$$

Thus:

$$\underline{w} = i \omega \underline{r} \,, \quad \underline{\alpha}_{spin} = \omega^2 \underline{r} \, . \quad -(5)$$

In electrodynamics, eq. (3) translates into:

$$\frac{\underline{A}\,(spin)}{c} = \frac{m}{e} \underline{\nabla} \times \underline{r} \quad -(6)$$

i.e.

$$\boxed{\underline{A}\,(spin) = i \frac{m}{e} \omega \underline{r}} \quad -(7)$$

In the limit of zero spin connection, and using antisymmetry:

$$\underline{E}\,(spin) = -2 \frac{\partial \underline{A}}{\partial t}(spin) \quad -(8)$$

$$= -2 i \frac{m}{e} \omega \frac{\partial \underline{r}}{\partial t}$$

$$= 2 \frac{m}{e} \omega^2 \underline{r} \quad -(9)$$

2)

So:

$$\boxed{\underline{E}(\text{spin}) = 2\frac{m}{e}\omega^2\,\underline{r}} \quad -(10)$$

This has an ~~ω~~ ω^2 dependence which could be detected experimentally.

However, in the limit of zero spin connection

$$\underline{E}(\text{orb}) = -2\frac{\partial \underline{A}(\text{orb})}{\partial t} \quad -(11)$$

and $$\underline{A}(\text{orb}) = 2\frac{m}{e}\frac{\partial \underline{r}}{\partial t} \quad -(12)$$

so $$\underline{E}(\text{orb}) = -4\frac{m}{e}\frac{\partial^2 \underline{r}}{\partial t^2} \quad -(13)$$

For a plane wave such as eq. (1):

$$\boxed{\underline{E}(\text{orb}) = 4\frac{m}{e}\omega^2\,\underline{r}} \quad -(14)$$

and $$|\underline{E}(\text{orb})| = c\left|\frac{\underline{E}(\text{spin})}{c}\right| \quad -(15)$$

The force on ~~an~~ a test electron or charge for eq. (14) is:

$$\boxed{\underline{F} = e\,\underline{E}(\text{orb}) = 4m\omega^2\,\underline{r}} \quad -(16)$$

1) 144(11): Resonant Coulomb Law, Orbital Field

In this case:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - c\underline{\nabla} A_0 - c\omega_{0b}\underline{A}^b + cA^b_0\,\underline{\omega}_b \quad -(1)$$

where:
$$\underline{A} = \frac{m}{e}\left(\frac{\partial \underline{r}}{\partial t} + c\underline{\nabla} r_0 + c\omega_{0b}\underline{r}^b - cr_0^b\,\underline{\omega}_b\right) \quad -(2)$$

and
$$\underline{\nabla}\cdot\underline{E} = \rho/\epsilon_0 \quad -(3)$$

In eq. (1)
$$c\omega_{0b}\underline{A}^b = c\omega_{0b}A\underline{v}^b = c\omega_0\underline{A} \quad -(4)$$

and
$$cA^b_0\,\underline{\omega}_b = \phi\,\cancel{q^b}\,\underline{\omega}_b = \phi\, q^b\,\underline{\omega}_b = \phi\,\underline{\omega} \quad -(5)$$

So
$$\boxed{\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \underline{\nabla}\phi - \omega_0 c\underline{A} + \phi\underline{\omega}} \quad -(6)$$

i.e.
$$\underline{E} = -\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{A} - \left(\underline{\nabla} - \underline{\omega}\right)\phi \quad -(7)$$

In the limit of special relativity:
$$\omega^\mu = (\omega_0, \underline{\omega}) \rightarrow 0 \quad -(8)$$

so
$$\underline{E} \rightarrow -\frac{\partial \underline{A}}{\partial t} - \underline{\nabla}\phi \quad -(9)$$

which is the usual textbook result.

2) By antisymmetry:

$$-\left(\frac{\partial}{\partial t}+\omega_0 c\right)\underline{A} = -\left(\underline{\nabla}-\underline{\omega}\right)\phi \qquad -(10)$$

So:

$$\boxed{\underline{E} = -2\left(\frac{\partial}{\partial t}+\omega_0 c\right)\underline{A} = -2\left(\underline{\nabla}-\underline{\omega}\right)\phi \qquad -(11)}$$

Similarly:

$$\underline{A} = \frac{2m}{e}\left(\left(\frac{\partial}{\partial t}+c\omega_0\right)\underline{r} + c\left(\underline{\nabla}-\underline{\omega}\right)r_0\right) \qquad -(12)$$

By antisymmetry:

$$\boxed{\underline{A} = \frac{4m}{e}\left(\frac{\partial}{\partial t}+c\omega_0\right)\underline{r} - \frac{4mc}{e}\left(\underline{\nabla}-\underline{\omega}\right)r_0 \qquad -(13)}$$

The sign change between eqns. (11) and (13) is a convention.

Therefore:

$$\boxed{\begin{aligned}\underline{E} &= -\frac{8m}{e}\left(\frac{\partial}{\partial t}+\omega_0 c\right)\left(\frac{\partial}{\partial t}+\omega_0 c\right)\underline{r} \\ &= -2\left(\underline{\nabla}-\underline{\omega}\right)\phi\end{aligned}} \qquad -(14)$$

3) and $\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0$ — (15)

In the limit $\omega_0 \rightarrow 0$ — (16)

$$\underline{E} = -2\underline{\nabla}\phi = -\cancel{8}^{\,4}\frac{m}{e}\frac{\partial^2 r}{\partial t^2} = -2\frac{\partial A}{\partial t}$$ — (17)

In this limit $\underline{A} = \cancel{4}^{\,2}\frac{m}{e}\frac{\partial \underline{r}}{\partial t} = \cancel{4}^{\,2}\frac{m}{e}\underline{v}$ — (18)

Eq. (18) is the minimal prescription:

$$p = mv = \frac{e}{2}\underline{A}$$ — (19)

From eqs. (14) and (15)

$$\boxed{\frac{\partial^2}{\partial t^2}(\underline{\nabla}\cdot\underline{r}) + 2\omega_0 c\frac{\partial}{\partial t}(\underline{\nabla}\cdot\underline{r}) + c^2\omega_0^2(\underline{\nabla}\cdot\underline{r}) = -\frac{\rho e}{4m\epsilon_0}}$$ — (20)

Denote $\underline{\nabla}\cdot\underline{r} = f(t)$ — (21)

then: $$\boxed{\frac{\partial^2 f}{\partial t^2} + 2\omega_0 c\frac{\partial f}{\partial t} + c^2\omega_0^2 f = -\frac{\rho e}{4m\epsilon_0}}$$ — (22)

4) This is an Euler resonance equation if:

$$\rho = \bar{\rho}_0 \cos(\omega t) \quad - (23)$$

At resonance the time dependent $f(t)$ becomes very large so $\nabla \cdot \underline{r}(t)$ becomes very large and the electron moves very quickly. From eq. (18), $\underline{A}$ becomes very large and $\underline{E}$ becomes very large.

There is a resonant amplification of the electric field strength $\underline{E}$ (volts per metre) and a sharp maximum in voltage.

144(12) : SCR in the Coulomb Law of the Electric Spin Field.

For the orbital electric field, and if Sy convention we choose consistent signs, then

$$\underline{E} = -\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{A} - (\underline{\nabla} - \underline{\omega})\phi \quad -(1)$$

$$\underline{A} = -\frac{m}{e}\left(\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{r} - c(\underline{\nabla} - \underline{\omega})r_0\right) \quad -(2)$$

giving an SCR equation of the type:

$$\frac{\partial^2 f}{\partial t^2} + 2\omega_0 c\frac{\partial f}{\partial t} + c^2\omega_0^2 f = \frac{\rho e}{4m\epsilon_0} \quad -(3)$$

The Spin Electric Field

This is defined as c times smaller than the orbital electric field, so:

$$\underline{\mathcal{E}} = \underline{E}/c \quad -(4)$$

has the units of magnetic flux density (tesla) but is an electric type of field

$$\underline{\mathcal{E}} := \frac{1}{c}\underline{E} = -\frac{1}{c}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{A} - \frac{1}{c}(\underline{\nabla} - \underline{\omega})\phi \quad -(5)$$

where

$$\frac{1}{c}\underline{A} = -\frac{m}{e}\left(\underline{\nabla} \times \underline{r} - \underline{\omega}_b \times \underline{r}^b\right) \quad -(6)$$

Now eliminate the b indices:

$$\underline{\omega}_b \times \underline{r}^b = r\,\underline{\omega}_b \times \underline{q}^b \quad -(7)$$

$$= \underline{\omega}_b \times (r\underline{q}^b) = \underline{\omega} \times \underline{r}$$

2)

So:
$$\frac{1}{c}\underline{A} = -\frac{m}{e}\left(\underline{\nabla}\times\underline{r} - \underline{\omega}\times\underline{r}\right) \quad -(8)$$

The Coulomb law for $\underline{E}$ is:

$$\underline{\nabla}\cdot\underline{E} = \frac{\rho}{\epsilon_0 c} = c\mu_0\rho \quad -(9)$$

where:
$$\epsilon_0\mu_0 = 1/c^2. \quad -(10)$$

Using antisymmetry:
$$\underline{E} = -\frac{2}{c}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{A} \quad -(11)$$

$$\boxed{\underline{E} = \frac{2m}{e\cancel{c}}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\nabla}\times\underline{r} - \underline{\omega}\times\underline{r}\right)} \quad -(12)$$

From eqs. (9) and (12): $\quad -(13)$

$$-\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{\nabla}\cdot\underline{\omega}\times\underline{r} = \frac{ec\mu_0\rho}{2m}$$

because
$$\underline{\nabla}\cdot\underline{\nabla}\times\underline{r} = 0. \quad -(14)$$

Now use: $\quad -(15)$
$$\underline{\nabla}\cdot\underline{\omega}\times\underline{r} = \underline{\omega}\cdot\underline{\nabla}\times\underline{r} - \underline{r}\cdot\underline{\nabla}\times\underline{\omega}$$

to find:
$$\frac{\partial f}{\partial t} + \omega_0 c f = \frac{ec\mu_0\rho}{2m} \quad -(16)$$

where:

where:

$$f = \underline{\omega} \cdot \underline{\nabla} \times \underline{r} - \underline{r} \cdot \underline{\nabla} \times \underline{\omega} \quad -(17)$$

Differentiate eq (16) to find:

$$\boxed{\frac{\partial^2 f}{\partial t^2} + \omega_0 c \frac{\partial f}{\partial t} + c\left(\frac{\partial \omega_0}{\partial t}\right) f = \frac{e c \mu_0}{2m} \frac{\partial \rho}{\partial t}} \quad -(18)$$

This is an Euler resonance equation in the variable f. If it is assumed that $\underline{\omega}$ is a fixed property of spacetime, then at resonance $\underline{r} \cdot \underline{\nabla} \times \underline{r}$ becomes very large.

Conclusions

The spin electric field $\underline{E}$ is c times smaller than the orbital (usual) electric field $\underline{E}$ and is defined by eq. (12) in terms of the position $\underline{r}$ of the electron or ion. The SCR equation of its Coulomb law is eq. (18).

1)
144(13): Orbital and Spin Magnetic Fields

Orbital Magnetic Field

This is defined by:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(1)$$

$$\underline{A} = -\frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{r} \quad -(2)$$

so:

$$\underline{B}\,(\text{orbital}) = -\frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{\nabla} \times \underline{r} \quad -(3)$$

$$+ \frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{\omega} \times \underline{r}$$

$$\boxed{\underline{B}\,(\text{orbital}) = \frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\omega} \times \underline{r} - \underline{\nabla} \times \underline{r}\right)} \quad -(4)$$

Assuming the existence of a magnetic monopole ρ_m then:

$$\underline{\nabla} \cdot \underline{B}\,(\text{orbital}) = \rho_m \quad -(5)$$

From eqns. (4) and (5):

$$\boxed{\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{\nabla} \cdot (\underline{\omega} \times \underline{r}) = \frac{e}{2m}\rho_m} \quad -(6)$$

because

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{r} = 0 \quad -(7)$$

If there is no magnetic monopole:

$$\left(\frac{\partial}{\partial t} + \omega . c\right) \underline{\nabla} \cdot (\underline{\omega} \times \underline{r}) = 0 \quad -(8)$$

Eqs. (6) and (8) can be solved by computer algebra for various models.

<u>Spin Magnetic Field</u>

$$\frac{\underline{B}}{c}(\text{spin}) = \frac{1}{c}\left(\underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}\right) \quad -(9)$$

$$\frac{\underline{A}}{c} = \frac{m}{e}\left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right) \quad -(10)$$

Here consistent positive signs are used in eqns. (9) and (10). The spin field is c times smaller than the orbital field.

So:

$$\boxed{\frac{\underline{B}}{c}(\text{spin}) = \frac{m}{e}\left(\underline{\nabla} \times \left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right) - \underline{\omega} \times \left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right)\right)} \quad -(11)$$

If it is assumed that there is a magnetic monopole then:

$$\boxed{\underline{\nabla} \cdot \left(\frac{\underline{B}}{c}(\text{spin})\right) = \rho_m / c} \quad -(12)$$

3)

Now use:

$$\underline{\nabla} \cdot \underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = 0 \quad -(13)$$

$$\underline{\nabla} \cdot \underline{\nabla} \times (\underline{\omega} \times \underline{r}) = 0 \quad -(14)$$

so:

$$\boxed{\underline{\nabla} \cdot \left(\underline{\omega} \times (\underline{\omega} \times \underline{r}) - \underline{\omega} \times (\underline{\nabla} \times \underline{r}) \right) = \frac{\rho_m}{c} \frac{e}{m}} \quad -(15)$$

Eq. (15) can be solved with various models.

Some Vector Identities

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = -\nabla^2 \underline{r} + \underline{\nabla}(\underline{\nabla} \cdot \underline{r}) \quad -(16)$$

$$\underline{\nabla} \times (\underline{\omega} \times \underline{r}) = \underline{\omega}\, \underline{\nabla} \cdot \underline{r} - \underline{r}\, \underline{\nabla} \cdot \underline{\omega} + (\underline{r} \cdot \underline{\nabla})\underline{\omega} - (\underline{\omega} \cdot \underline{\nabla})\underline{r} \quad -(17)$$

$$\underline{\omega} \times (\underline{\nabla} \times \underline{r}) = (\underline{\omega} \times \underline{\nabla}) \times \underline{r} + \underline{\omega}\, \underline{\nabla} \cdot \underline{r} - (\underline{\omega} \cdot \underline{\nabla})\underline{r} \quad -(18)$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = (\underline{r} \cdot \underline{\omega})\underline{\omega} - (\underline{\omega} \cdot \underline{\omega})\underline{r} \quad -(19)$$

In the limit of vanishing spin connection,

144(14): Summary of Results to Date

The results of paper 144 to date are as follows:

$$\underline{E}(\text{orbital}) = -\frac{4m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{r} \quad -(1)$$

$$= -2(\underline{\nabla} - \underline{\omega})\,\phi \quad -(2)$$

$$\underline{B}(\text{orbital}) = -\frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right) \quad -(3)$$

$$\frac{\underline{E}}{c}(\text{spin}) = \frac{2m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right) \quad -(3)$$

$$\frac{\underline{B}}{c}(\text{spin}) = \frac{m}{e}\left(\underline{\nabla} - \underline{\omega}\right) \times \left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right) \quad -(4)$$

It is seen that the electric spin field and the magnetic orbital field have the same characteristics. Therefore the magnetic field as conventionally observed, is a spin electric field.

The completely new entity is the spin magnetic field, which is c^2 times smaller than the orbital electric field, i.e. 9×10^{16} times smaller.

The analysis may now be extended using the Faraday law of induction and Ampère Maxwell laws.

144(15): Faraday Law of Induction for Orbital and Spin Fields.

The Faraday law of induction from ECE theory in the assumed absence of a magnetic monopole is:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(1)$$

Orbital Law

$$\underline{E} = 4\frac{m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\frac{\partial}{\partial t} + \omega_0 c\right)\underline{r} \quad -(2)$$

$$\underline{B} = 2\frac{m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\omega} \times \underline{r} - \underline{\nabla} \times \underline{r}\right) \quad -(3)$$

So the law is:

$$\boxed{\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{r} + \underline{\omega} \times \underline{r}\right) + 2\omega_0 c\right) = \underline{0}} \quad -(4)$$

For a model of the spin connection

$$\underline{\omega} = \left(\omega_0 \underline{\omega}\right) \quad -(5)$$

eq. (4) can be solved for the electronic trajectory $\underline{r}$.

Spin Law

$$\frac{\underline{E}}{c} = 2\frac{m}{e}\left(\frac{\partial}{\partial t} + \omega_0 c\right)\left(\underline{\omega} \times \underline{r} - \underline{\nabla} \times \underline{r}\right) \quad -(6)$$

$$\frac{\underline{B}}{c} = \frac{m}{e}\left(\underline{\nabla} - \underline{\omega}\right) \times \left(\underline{\nabla} \times \underline{r} - \underline{\omega} \times \underline{r}\right)$$

$$-(7)$$

2) The law is therefore:

$$\boxed{\frac{\partial}{\partial t}\left(\underline{v}+\underline{\omega}\right)\times\left(\underline{v}\times\underline{r}-\underline{\omega}\times\underline{r}\right) + 2\omega_0 c\left(\underline{v}\times\underline{r}-\underline{\omega}\times\underline{r}\right) = \underline{0} \qquad (8)}$$

It is seen that if the same model of the spin connection is used in eqns. (4) and (8), the trajectory of the electron is different. This means that the orbital law and spin law have a different physical meaning.

In the limit of zero spin connection the orbital law becomes

$$\frac{\partial^2}{\partial t^2}\left(\underline{v}\times\underline{r}\right) = \underline{0} \qquad (9)$$

and the spin law becomes

$$\frac{\partial}{\partial t}\left(\underline{v}\times\left(\underline{v}\times\underline{r}\right)\right) = \underline{0} \qquad (10)$$

This means that in the limit of zero spin connection all fields vanish, because in this limit the torsion vanishes.

144(16): Ampère Maxwell Laws for Orbital and Spin Fields

Orbital Law

$$\left(\frac{\partial}{\partial t}+\omega_{0c}\right)\left(\underline{\nabla}\times\left(\underline{\omega}\times\underline{r}-\underline{\nabla}\times\underline{r}\right)-\frac{2}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\omega_{0c}\right)\underline{r}\right)=\frac{e\mu_0}{2m}\underline{J} \qquad -(1)$$

Spin Law

$$\left(\underline{\nabla}\times\left(\underline{\nabla}-\underline{\omega}\right)+\frac{2}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\omega_{0c}\right)\right)\left(\underline{\nabla}\times\underline{r}-\underline{\omega}\times\underline{r}\right)=\frac{e\mu_0}{2mc}\underline{J} \qquad -(2)$$

In the limit

$$\omega^{\mu}\to 0 \qquad -(3)$$

eq. (1) becomes:

$$\frac{\partial}{\partial t}\left(\underline{\nabla}\times\left(\underline{\nabla}\times\underline{r}\right)+\frac{2}{c^2}\frac{\partial^2\underline{r}}{\partial t^2}\right)=-\frac{e\mu_0}{2m}\underline{J} \qquad -(4)$$

and eq. (2) becomes:

$$\underline{\nabla}\times\left(\underline{\nabla}\times\left(\underline{\nabla}\times\underline{r}\right)\right)+\frac{2}{c^2}\frac{\partial^2}{\partial t^2}\underline{\nabla}\times\underline{r} \qquad -(5)$$
$$=\frac{e\mu_0}{2mc}\underline{J}$$

Using the identity:

2)

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{r}) = -\nabla^2 \underline{r} + \underline{\nabla}(\underline{\nabla} \cdot \underline{r}) \quad -(6)$$

and assuming:

$$\underline{\nabla} \cdot \underline{r} = 0 \quad -(7)$$

then eq. (4) becomes:

$$\frac{\partial}{\partial t}\left(-\nabla^2 \underline{r} + \frac{1}{c^2}\frac{\partial^2 r}{\partial t^2}\right) = -\frac{e\mu_0}{2m}\,\underline{J} \quad -(8)$$

i.e.

$$\frac{\partial}{\partial t}\left(\Box \underline{r} + \frac{1}{c^2}\frac{\partial^2 \underline{r}}{\partial t^2}\right) = -\frac{e\mu_0}{2m}\,\underline{J}$$

Note that the current appears in (7) is $\underline{J}/c$, c times smaller than the current $\underline{J}$ of eq. (1).

4)

$$\underline{B}\,(\text{orbital}) \rightarrow -\frac{2m}{e}\,\underline{\nabla}\times\frac{\partial\underline{r}}{\partial t} \quad -(20)$$

$$\frac{\underline{B}}{c}\,(\text{spin}) \rightarrow \frac{m}{e}\,\underline{\nabla}\times(\underline{\nabla}\times\underline{r}) \quad -(21)$$

$$= \frac{m}{e}\left(-\nabla^2\underline{r}+\underline{\nabla}(\underline{\nabla}\cdot\underline{r})\right) \quad -(22)$$

Computer Modelling

The characteristics of the orbital magnetic field (4) and the spin magnetic field (11) can be worked out by computer algebra and graphed for models of $\underline{r}$, ω_0 and $\underline{\omega}$. The spin magnetic field can be written as:

$$\boxed{\left(\frac{\underline{B}}{c}\right)(\text{spin}) = \frac{m}{e}\left((\underline{\nabla}-\underline{\omega})\times(\underline{\nabla}\times\underline{r}-\underline{\omega}\times\underline{r})\right)} \quad -(23)$$

and the orbital as:

$$\boxed{\underline{B}\,(\text{orbital}) = -\frac{2m}{e}\left(\frac{\partial}{\partial t}+\omega_0 c\right)\left(\underline{\nabla}\times\underline{r}-\underline{\omega}\times\underline{r}\right)} \quad -(24)$$