Note 1: Some Fundamental Mathematical Properties of the Tetrad.

As discussed by Carroll on page 88 of the 1997 notes to his book "Spacetime and Geometry: an Introduction to General Relativity" (Addison-Wesley, NY 2006), the tetrad is a set of vectors comprising an orthonormal basis and can take on any dimension. Denoting the tetrad as q_ν^a, it is an invertible n x n matrix. The inverse of q_ν^a is denoted q_a^ν and:

$$q_a^\mu \; q_\nu^a = \delta_\nu^\mu \tag{1}$$

if a set of basis vectors $\hat{e}_{(a)}$ is orthonormal, the canonical form of the metric is η_{ab}, where:

$$g\,(\hat{e}_{(a)}, \hat{e}_{(b)}) = \eta_{ab} \tag{2}$$

In a Lorentz space-time, η_{ab} is the Minkowski metric. Any vector can be expressed as a linear combination of basis vectors, and two basis are related by the tetrad:

$$\hat{e}_{(\mu)} = q_\mu^a \; \hat{e}_{(a)} \tag{3}$$

The dimension of one basis is usually assumed to be the same as the dimension of the other, so that q_μ^a is a square matrix. However this definition may be extended to m x n dimensional matrices. However, it is best to use a square matrix for the tetrad, because the square matrix has a well defined inverse matrix, that is, invertible. The inverse of the square matrix is its adjunct matrix divided by its determinant.

<u>According to these fundamental properties, therefore, the tetrad may be defined as a 2 x 2 matrix:</u>

$$\begin{bmatrix} V^R \\ V^L \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} \tag{4}$$

i.e

$$V^a = q_\nu^a \; V^\nu \tag{5}$$

where the dimensions of a and ν are two. In general the index ν refers to a manifold with torsion and curvature and the index a to a Lorentzian spacetime with Minkowski metric. The index a refers to a two-dimentional representation space of the Lorentzian spacetime and the index ν to a two dimensional space of the base manifold.

We then have:

$$D\,V = (D_\mu\, V^\nu)\,\mathrm{d}x^\mu \otimes \partial_\nu = (D_\mu\, V^\nu)\,\mathrm{d}x^\mu \otimes\, \hat{e}_{(\nu\,)}$$

$$= (D_\mu\, V^a)\,\mathrm{d}x^\mu \otimes\, \hat{e}_{(a\,)} \tag{6}$$

So: $$D_\mu\, q^a_\nu = 0 \tag{7}$$

with: $$D_\mu\, V^\nu = \partial_\mu\, V^\nu + \Gamma^\nu_{\mu\lambda}\, V^\lambda \tag{8}$$

$$D_\mu\, V^a = \partial_\mu\, V^a + \omega^a_{\mu b}\, V^b \tag{9}$$

The dimensionality of μ may be four :

$$\mu = 0\,,1\,,2\,,3 \tag{10}$$

so: $$D_\mu\, q^a_\nu = \begin{pmatrix} D_\mu\; q^R_1 & D_\mu\, q^R_2 \\ D_\mu\; q^L_1 & D_\mu q^L_2 \end{pmatrix} = 0 \tag{11}$$

where: $$D_\mu\; q^R_1 = (D_0\; q^R_1\,, D_1\; q^R_1\,, D_2\; q^R_1\,, D_3\; q^R_1\,) \tag{12}$$

For example: $$\underline{\sigma}\,.\,\underline{r} = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix} \tag{13}$$

and: $$\partial_\mu\;\; (\underline{\sigma}\,.\,\underline{r}\,) = (\; 0\,, \sigma^1\,, \sigma^2\,, \sigma^3\,)$$
(14)

where: $$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,\quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,,\quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{15}$$

It follows that:

$$D^\mu\,(D_\mu\, q^a_\nu\,)\; := 0 \tag{16}$$

i.e.

$$\Box \, q_\nu^a \; := \; R\, q_\nu^a \tag{17}$$

which is:

$$\Box \begin{pmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{pmatrix} := R \begin{pmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{pmatrix} \tag{18}$$

By hypothesis:

$$R = -\mathrm{k\,T} \tag{19}$$

so:

$$(\Box + \mathrm{k\,T}) \begin{pmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{pmatrix} = 0 \tag{20}$$

In this equation:

$$\boxed{a = R, L \quad ; \quad \nu = 1, 2} \tag{21}$$

Note 2: Symmetry of the Connection in Gravitation and Electromagnetism .

Throughout the 20th century the theory of electromagnetism was based in the standard physics of the 19th century Maxwell Heaviside theory (MH theory). In the last decade of the century a higher symmetry gauge theoretical approach to electromagnetism was developed by several groups independently: Horwitz et al. (1989), Barrett, Lehnert et al., Harmuth et al. and Evans in O(3) electrodynamics (see Omnia Opera in www.aias.us . These are all inter-related theories that produce the fundamental B(3) field of electromagnetism in different nomenclatures and formalisms. The O(3) electrodynamics was a gauge theory whose internal gauge space was ((1), (2) ,(3)) -based on the complex circular representation of spacetime. In 2003 it was realized that the complex circular and Cartesian representations of spacetime are related by a Cartan tetrad. Thereby, it was realized that electromagnetism could be unified with gravitation through use of Cartan´s geometry of the early 1920s. The basis hypothesis is:

$$A^a_\mu = A^{(0)} \; q^a_\mu \tag{1}$$

where A^a_μ Is the electromagnetic potential density and q^a_μ the Cartan tetrad. Therefore , $c\,A^{(0)}$ is a voltage density of general relativity. It is observed in the radiative corrections. The electromagnetic field density is defined by:

$$F^a_{\mu\nu} = A^{(0)} \, T^a_{\mu\nu} \tag{2}$$

where $T^a_{\mu\nu}$ is the Cartan torsion.

Note carefully that this is a development of the original idea of the tetrad by Cartan, who introduced it originally in the context of the covariant derivative of a Cartan spinor (inferred by Cartan in 1913). The connection is thereby defined as the spin connection, and denoted $\omega^a_{\mu b}$. In equation (1) the tetrad is defined by:

$$V^a = q^a_\mu \, V^\mu \tag{3}$$

where: $$V^a = x^a \quad , \qquad V^\mu = x^\mu \tag{4}$$

Here: $$x^a = (\mathrm{ct} \, , x^{(1)}, x^{(2)}, x^{(3)}) \tag{5}$$

$$x^\mu = (\mathrm{ct} \, , X \, , Y, Z \,) \tag{6}$$

The basic vectors of the complex circular representation are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j}) \tag{7}$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j}) \tag{8}$$

$$\underline{e}^{(3)} = \underline{k} \tag{9}$$

where $\underline{i}$, $\underline{j}$ and $\underline{k}$ are the basis of vectors of the Cartesian representation. Both (5) and (6) are component factors in four dimensional spacetime. The tetrad defined by:

$$x^a = q^a_\mu \, x^\mu \tag{10}$$

is therefore a 4 × 4 matrix. It is a Cartan tetrad and is therefore part of Cartan´s geometry in the spacetime with torsion. This is not the Minkowski spacetime of the MH theory, because in the Minkowski spacetime there is no torsion.

The conceptual leap forward is therefore that the electromagnetic field density is the torsion of spacetime in general relativity. In quantum electrodynamics the wave function a is the Cartan tetrad through the ECE Lemma:

$$\Box \, q^a_\mu = R \, q^a_\mu \tag{11}$$

so

$$\Box \, A^a_\mu = R \, A^a_\mu \tag{12}$$

Equation (12) is:

$$(\gamma^\mu \partial_\mu - \frac{mc}{\hbar}) \, A^a_\mu \tag{13}$$

where:

$$2 \, g^{\mu\nu} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \tag{14}$$

In the limit:

$$R \longrightarrow -(\frac{mc}{\hbar})^2 \tag{15}$$

In this limit:

$$(\Box + (\frac{mc}{\hbar})^2) \, A^a_\mu = 0 \tag{16}$$

Equation (16) <u>is the Proca equation</u> for photon mass m , and equation (13) is the Majorana equation for photon mass.

At this point the ECE equation supplants the gauge theory of electromagnetism because the Proca and Majorana equations are not gauge invariant. This is one of the numerous weaknesses of the 20th century standard model. Photon mass is incompatible was gauge invariance . In ECE theory gauge invariance is not used, and is replaced by invariance of the tetrad postulate and ECE Lemma under the general coordinate transformation of geometry.

ECE uses a more general definition of the Cartan tetrad. Writing out equation (10), for example:

$$\begin{pmatrix} x^{(0)} \\ x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{pmatrix} = \begin{pmatrix} q_0^{(0)} & q_1^{(0)} & q_2^{(0)} & q_3^{(0)} \\ q_0^{(1)} & q_1^{(1)} & q_2^{(1)} & q_3^{(1)} \\ q_0^{(2)} & q_1^{(2)} & q_2^{(2)} & q_3^{(2)} \\ q_0^{(3)} & q_1^{(3)} & q_2^{(3)} & q_3^{(3)} \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \tag{17}$$

Thus, for a plane wave:

$$A^{(1)} = A^{(2)*} = \frac{1}{\sqrt{2}} (\underline{\mathrm{i}} - i\underline{\mathrm{j}})\, e^{i\varphi} \tag{18}$$

$$\varphi = \omega t - \kappa z \tag{19}$$

$$\boxed{\begin{array}{ll} A_x^{(1)} = A^{(0)} \frac{1}{\sqrt{2}} e^{i\varphi} \quad , & A_y^{(1)} = - i\, A^{(0)} \frac{1}{\sqrt{2}} e^{i\varphi} \\ A_x^{(2)} = A^{(0)} \frac{1}{\sqrt{2}} e^{-i\varphi} \quad , & A_y^{(2)} = i\, A^{(0)} \frac{1}{\sqrt{2}} e^{-i\varphi} \end{array}} \tag{20}$$

are tetrad elements of Cartan geometry .

Note 3: The Commutator Method in Gravitation and Electromagnetism.

In gravitational theory, fundamental Riemann geometry uses the equation:

$$[D_\mu, D_\nu]\, V^\rho = R^\rho_{\sigma\mu\nu}\, V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \tag{1}$$

to define the curvature tensor $R^\rho_{\sigma\mu\nu}$ and the torsion tensor $T^\lambda_{\mu\nu}$. Here, V^ρ is a vector in any spacetime and any dimension. The commutator of covariant derivatives acts on the vector V^ρ. The covariant derivative is:

$$D_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda} V^\lambda \tag{2}$$

where $\Gamma^\rho_{\mu\lambda}$ Is the general connection. Thus:

$$[D_\mu, D_\nu]\, V^\rho = D_\mu\,(D_\nu V^\rho\,) - D_\nu(D_\mu V^\rho) \tag{3}$$

and is antisymmetric in μ y ν :

$$[D_\mu, D_\nu] = -\ [D_\nu, D_\mu] \tag{4}$$

All quantities and equation (1) generated by the commutator are antisymmetric in μ and ν. The torsion tensor is defined by the action of the antisymmetric commutator as follows:

$$\begin{aligned} T^\lambda_{\mu\nu} &= -\ T^\lambda_{\mu\nu} \\ &= \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \\ &= -(\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}) \end{aligned} \tag{5}$$

It follows that :

$$\boxed{\Gamma^\lambda_{\mu\nu} = -\ \Gamma^\lambda_{\nu\mu}} \tag{6}$$

Also:

$$R^\rho_{\sigma\mu\nu} = -\ R^\rho_{\sigma\mu\nu} \tag{7}$$

It is seen that in equations (4) to (7):

$$\boxed{\mu\nu \longrightarrow -\ \nu\mu} \tag{8}$$

<u>in each occurrence of μν , self consistently .</u>

The two fundamental errors of 20th century gravitational physics are well known to be as follows:

1) The <u>incorrect</u> assertion:

$$T^{\lambda}_{\mu\nu} \ =?\ 0 \tag{9}$$

2) The consequentially <u>incorrect</u> assertion:

$$\Gamma^{\lambda}_{\mu\nu} \ =?\ \Gamma^{\lambda}_{\nu\mu} \tag{10}$$

In consequence of (1) and (2) the Einstein field equation violates the fundamental identity:

$$D_{\mu} \ T^{\kappa\mu\nu} := R_{\mu}^{\kappa\mu\nu} \tag{11}$$

known as <u>the Cartan Evans dual identity</u>.

<u>Torsion is a fundamental part of old physics and the ECE unified field theory.</u>

The electromagnetic field density tensor in ECE is:

$$F^{\lambda}_{\mu\nu} = A^{(0)}\, T^{\lambda}_{\mu\nu} \tag{12}$$

and is the space-time torsion with $A^{(0)}$. It is related to the Cartan torsion form by:

$$T^{a}_{\mu\nu} = q^{a}_{\lambda}\ T^{\lambda}_{\mu\nu} \tag{13}$$

The electromagnetic potential density is:

$$A^{a}_{\mu} = A^{(0)} q^{a}_{\mu} \tag{14}$$

In the notation of differential geometry:

$$\boxed{F^{a} = \mathrm{D} \wedge A^{a}} \tag{15}$$

follows directly from <u>the first Cartan structure equation</u>:

$$T^{a} = \mathrm{D} \wedge q^{a} \tag{16}$$

Also: $$\boxed{D \wedge F^a := A^b \wedge R^a_b} \quad (17)$$

follows directly from the Cartan Bianchi identity:

$$D \wedge T^a := q^b \wedge R^a_b \quad (18)$$

By the rules of wedge product:

$$A^b \wedge R^a_b = R^a_b \wedge A^b \quad (19)$$

So: $$D \wedge F^a := R^a_b \wedge A^b \quad (20)$$

In four dimensions, F^a and R^a_b are two-forms Hodge duals to two forms. It follows immediately that:

$$D \wedge \tilde{F}^a := \tilde{R}^a_b \wedge A^b \quad (21)$$

The identities (20) and (21) are Hodge invariant, and are the ECE field equations of electromagnetism. They are equivalent to:

$$A^{(0)} [D_\mu, D_\nu] V^\rho = A^{(0)} R^\rho_{\sigma\mu\nu} V^\sigma - F^\lambda_{\mu\nu} D_\lambda V^\rho \quad (22)$$

and $$A^{(0)} [D_\mu, D_\nu]_{\mathrm{HD}} V^\rho = \tilde{R}^\rho_{\sigma\mu\nu} V^\sigma - \tilde{F}^\lambda_{\mu\nu} D_\lambda V^\rho \quad (23)$$

It is seen that the electromagnetic field density is generated by the commutator.

$$\boxed{[D_\mu, D_\nu] A^\rho = - F^\lambda_{\mu\nu} D_\lambda V^\rho + R^\rho_{\sigma\mu\nu} A^\sigma} \quad (24)$$

where:

$$A^\rho = A^{(0)} V^\rho \quad (25)$$

Equation (24) is Hodge invariant with:

$$\boxed{[D_\mu, D_\nu]_{\mathrm{HD}} A^\rho = - \tilde{F}^\lambda_{\mu\nu} D_\lambda V^\rho + \tilde{R}^\rho_{\sigma\mu\nu} A^\sigma} \quad (26)$$

The commutator $[D_\mu, D_\nu]$, and its Hodge dual $[D_\mu, D_\nu]_{\mathrm{HD}}$, generates both electromagnetism and gravitation.

Note 4 : Gauge Theory in Electromagnetism.

This is developed extensively in the Omnia Opera of www.aias.us from 1992 onwards. The discovery of the $B^{(3)}$ field necessitated the development of U(1) electromagnetism. In the gauge series of electromagnetism the gauge field is denoted ψ . The covariant derivative at the U(1) level is:

$$D_\mu = \partial_\mu - i g A_\mu \quad (1)$$

and the electromagnetic field is denoted $F_{\mu\nu}$. Thus:

$$[D_\mu, D_\nu]\,\psi = [\partial_\mu - i A_\mu , \partial_\nu - i g A_\nu]\,\psi \quad (2)$$

$$= [\partial_\mu , \partial_\nu]\,\psi - i g [A_\mu , \partial_\nu]\,\psi - i g [\partial_\mu , A_\nu]\,\psi - g^2 [A_\mu , A_\nu]\,\psi$$

Now use:

$$[\partial_\mu , \partial_\nu] = 0 \quad (3)$$

therefore:

$$[D_\mu, D_\nu]\,\psi = i g \left([A_\mu , \partial_\nu]\,\psi + [\partial_\mu , A_\nu]\,\psi \right) - g^2 [A_\mu , A_\nu]\,\psi \quad (4)$$

By definition:

$$[D_\mu, D_\nu] = - [D_\nu, D_\mu] \quad (5)$$

because it is a commutator. In equation (4):

$$[A_\mu , \partial_\nu]\,\psi = A_\mu (\partial_\nu \psi) - \partial_\nu (A_\mu \psi)$$

$$= A_\mu \partial_\nu \psi - (\partial_\nu A_\mu)\psi - A_\mu (\partial_\nu \psi) \quad (6)$$

using the Leibnitz Theorem. Similarly:

$$[\partial_\mu , A_\nu]\,\psi = \partial_\mu (A_\nu \psi) - A_\nu (\partial_\mu \psi)$$

$$= (\partial_\mu A_\nu)\,\psi - A_\nu (\partial_\mu \psi) - A_\nu \partial_\mu \psi \quad (7)$$

Therefore:

$$[D_\mu, D_\nu]\,\psi \;=\; -\,i\,g\,(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\psi - g^2\,[A_\mu, A_\nu]\,\psi \tag{8}$$

The standard procedure is to identify the electromagnetic field as:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{9}$$

and to assert $$[A_\mu, A_\nu] = ?\; 0 \tag{10}$$

<u>However</u>, equation (10) is <u>erroneous</u> because there must be <u>anti-symmetry</u> in μ and ν :

$$\mu\nu \longrightarrow -\nu\mu \tag{11}$$

It is also claimed erroneously in the standard procedure that there is somehow no symmetry in equation (9). The correct <u>anti-symmetry</u> is:

$$\partial_\mu A_\nu = -\,\partial_\nu A_\mu \tag{12}$$

which is a direct consequence of equation (5). By definition:

$$[A_\mu, A_\nu] = A_\mu A_\nu - A_\nu A_\mu \tag{13}$$

and $$\boxed{A_\mu A_\nu = -\,A_\nu A_\mu} \tag{14}$$

in gravitation, the analogue of equation (14) is:

$$\Gamma^\lambda_{\mu\nu} = -\,\Gamma^\lambda_{\nu\mu} \tag{15}$$

<u>Therefore, equation (10) cannot be true and the standard U(1) gauge theory of electromagnetism is erroneous.</u>

The O(3) gauge theory of electrodynamics was introduced in the 90s following the inference of the $B^{(3)}$ field, observable in the inverse Faraday effect . It is defined by:

$$\underline{B}^{(3)} = \underline{B}^{(3)*} = -\,i\,g\,\underline{A}^{(1)} \times \underline{A}^{(2)} \tag{16}$$

where $$g = \frac{\kappa}{A(0)} \tag{17}$$

The vector cross product is antisymmetric:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = - \underline{A}^{(2)} \times \underline{A}^{(1)} \tag{18}$$

where $\underline{A}^{(1)}$ is the complex conjugate of $\underline{A}^{(2)}$. In terms of a commutator, equation (16) is:

$$\boxed{B_z^{(3)} = -i\, g\, [A_x^{(1)}, A_y^{(2)}]} \tag{19}$$

where

$$A_x^{(1)} A_y^{(2)} = - A_y^{(1)} A_x^{(2)} \tag{20}$$

Equation (20) is an example of equation (14), QED.

In O(3) electrodynamics there is an O(3) internal gauge symmetry, so:

$$\boxed{F_{\mu\nu}^{a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - i\, g\, \varepsilon_{abc} A_\mu^b A_\nu^c} \tag{21}$$

in direct analogy to electroweak theory of SU(2) gauge symmetry (Ryder, chapter 3). So:

$$F_{\mu\nu}^{(3)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - i\, g\, (A_\mu^{(1)} A_\nu^{(2)} - A_\nu^{(1)} A_\mu^{(2)}) \tag{22}$$

In ECE theory:

$$\boxed{F_{\mu\nu}^{a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \omega_{\mu b}^{a} A_\nu^b - \omega_{\nu b}^{a} A_\mu^b} \tag{23}$$

but now a and b are no longer abstract. They are defined by geometry. So:

$$F_{\mu\nu}^{(3)} = \partial_\mu A_\nu^{(3)} - \partial_\nu A_\mu^{(3)} + \omega_{\mu b}^{(3)} A_\nu^b - \omega_{\nu b}^{(3)} A_\mu^b \tag{24}$$

Compare equations (22) and (24) for:

$$b = (1) \tag{25}$$

To find:

$$\boxed{\omega_{\mu(2)}^{(3)} = -i\, g\, A_\mu^{(1)}} \tag{26}$$

Therefore, O(3) the electromagnetism is an example of ECE Theory.

Note (5) : General Symmetry Considerations and Consequences of Anti-symmetry.

In general the connection is:

$$\Gamma^{\kappa}_{\mu\nu} = \frac{1}{2} (\Gamma^{\kappa}_{\mu\nu} + \Gamma^{\kappa}_{\nu\mu}) + \frac{1}{2} (\Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu})$$

$$= \Gamma^{\kappa}_{\mu\nu} \text{ (S)} + \Gamma^{\kappa}_{\mu\nu} \text{ (A)} \quad (1)$$

where $\Gamma^{\kappa}_{\mu\nu}$ (S) is a hypothetical symmetric part and where $\Gamma^{\kappa}_{\mu\nu}$ (A) the anti-symmetric part. Similarly:

$$\partial_\mu A_\nu = \frac{1}{2} (\partial_\mu A_\nu + \partial_\nu A_\mu) + \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$= \partial_\mu A_\nu \text{ (S)} + \partial_\mu A_\nu \text{ (A)} \quad (2)$$

The symmetric parts are defined by:

$$\mu = \nu \ , \quad \mu\nu \longrightarrow \nu\mu \quad (3)$$

So:

$$\Gamma^{\kappa}_{\mu\nu} \text{ (S)} = \Gamma^{\kappa}_{\nu\mu} \text{ (S)} \quad (4)$$

$$\partial_\mu A_\nu \text{ (S)} = \partial_\nu A_\mu \text{ (S)} \quad (5)$$

The antisymmetric parts are defined by:

$$\mu\nu = -\nu\mu \quad (6)$$

So:

$$\Gamma^{\kappa}_{\mu\nu} \text{ (A)} = - \Gamma^{\kappa}_{\nu\mu} \text{ (A)} \quad (7)$$

$$\partial_\mu A_\nu \text{ (A)} = - \partial_\nu A_\mu \text{ (A)} \quad (8)$$

However, both equations (1) and (2) must be produced by an anti-symmetric commutator. In ECE theory they are produced by:

$$[D_\mu, D_\nu] V^\rho = R^\rho_{\sigma\mu\nu} V^\sigma - T^\lambda_{\nu\mu} D_\lambda V^\rho \quad (9)$$

The commutator is antisymmetric, and has no symmetric part:

$$[D_\mu, D_\nu] = - [D_\nu, D_\mu] \quad (10)$$

Therefore: $$[D_\mu, D_\nu] = - [D_\mu, D_\nu] \text{ (A)} \tag{11}$$

$$[D_\mu, D_\nu] \text{ (S)} = 0 \tag{12}$$

Thus:

$$[D_\mu, D_\nu]\text{(S)} \, V^\rho = 0 \, V^\sigma - 0 D_\lambda V^\rho \tag{13}$$

and

$$[D_\mu, D_\nu]\text{(A)} \, V^\rho = R^\rho_{\sigma\mu\nu} \, (A) V^\sigma - T^\lambda_{\nu\mu}(A) D_\lambda V^\rho \tag{14}$$

It follows that:

$$T^\lambda_{\mu\nu} \text{ (S)} = \Gamma^\lambda_{\mu\nu} \text{ (S)} - \Gamma^\lambda_{\nu\mu} \text{ (S)} = 0 \tag{15}$$

$$T^\lambda_{\nu\mu} \text{ (S)} = \Gamma^\lambda_{\nu\mu} \text{ (S)} - \Gamma^\lambda_{\mu\nu} \text{ (S)} = 0 \tag{16}$$

In the case: $$T^\lambda_{\mu\nu} \text{ (S)} = \Gamma^\lambda_{\nu\mu} \text{ (S)} \tag{17}$$

<u>there is no curvature</u>: $$R^\rho_{\sigma\mu\nu} \text{ (S)} = R^\rho_{\sigma\nu\mu} \text{(S)} = 0 \tag{18}$$

<u>and no torsion</u>: $$T^\lambda_{\mu\nu} \text{ (S)} = T^\lambda_{\nu\mu} \text{ (S)} = 0 \tag{19}$$

It is concluded that curvature and torsion are both non-zero if and only if we have the case:

$$\boxed{\Gamma^\lambda_{\mu\nu} \text{ (A)} = - \Gamma^\lambda_{\mu\nu} \text{ (A)}} \tag{20}$$

Similarly, in electromagnetism, if:

$$\partial_\mu A_\nu \text{ (S)} = \partial_\nu A_\mu \text{ (S)} \tag{21}$$

then : $$F_{\mu\nu} = 0 \tag{22}$$

In general: $$\partial_\mu A_\nu \neq 0 \tag{23}$$

So: $$\partial_\mu A_\nu \text{ (A)} = - \partial_\nu A_\mu \text{ (A)} \tag{24}$$

The general rule is that whenever any quantity occurs with $\mu = \nu$, and that quantity is generated by the commutator, then it vanishes. It must have the same anti-symmetry as the commutator. If it did not, it would not be generated by the commutator.

Examples in the U(1) level of Electrodynamics

1) Static Electric Field

$$\underline{\mathrm{E}} = \underline{\nabla}\, \varphi - \partial \underline{\mathrm{A}} / \partial \mathrm{t} \tag{25}$$

so:

$$\boxed{\underline{\nabla}\, \varphi = - \partial \underline{\mathrm{A}} / \partial \mathrm{t}} \tag{26}$$

2) Static Magnetic Field

$$\underline{\mathrm{B}} = \underline{\nabla} \times \underline{\mathrm{A}} \tag{27}$$

$$\underline{\mathrm{A}} = \frac{1}{2}\, (\mathrm{Y}\, \underline{\mathrm{i}} - \mathrm{X}\, \underline{\mathrm{j}}) \tag{28}$$

so:

$$\boxed{\frac{\partial \mathrm{A_y}}{\partial x} = - \frac{\partial \mathrm{A_x}}{\partial y}} \tag{29}$$

Equation (26) means that the scalar potential φ and vector potential $\underline{\mathrm{A}}$ are related by symmetry. For example, the static electric field is:

$$\underline{\mathrm{E}} = \underline{\nabla}\, \varphi = \partial \underline{\mathrm{A}} / \partial \mathrm{t} \tag{30}$$

The fact that $\underline{\mathrm{E}}$ is static does not mean that $\underline{\mathrm{A}}$ must be time independent. For example, if:

$$\underline{\mathrm{A}} = \mathrm{A}_z \mathrm{t} \underline{\mathbf{k}} \tag{31}$$

then:

$$\underline{\mathrm{E}} = \mathrm{E}_z \underline{\mathbf{k}} \tag{32}$$

3) Elimination of the Lorenz Condition

In the U(1) standard theory: $\partial_\mu F^{\mu\nu} = \varepsilon_0\, j^\nu$ (33)

so:

$$\partial_\mu\, (\partial^\mu A^\nu - \partial^\nu A^\mu\,) = \varepsilon_0\, j^\nu \tag{34}$$

using:

$$\partial^\mu A^\nu = - \partial^\nu A^\mu \tag{35}$$

we immediately obtain the d´Alembert equation:

$$\Box A^{\nu} = \frac{\varepsilon_0}{2} j^{\nu} \tag{36}$$

The standard procedure uses:

$$\Box A^{\nu} - \partial^{\nu} (\partial_{\mu} A^{\mu}) = \varepsilon_0 j^{\nu} \tag{37}$$

and has to use a gauge condition:

$$\partial_{\mu} A^{\mu} = 0 \tag{38}$$

known as the Lorenz Gauge.

Note 6: Simple Proof of the Anti-symmetry Law.

In basic geometry:

$$[D_\mu, D_\nu] \; V^\rho = - T^\lambda_{\mu\nu} D_\lambda \; V^\rho + R^\rho_{\sigma\mu\nu} \; V^\sigma \tag{1}$$

$$= - (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}) \; D_\lambda \; V^\rho + R^\rho_{\sigma\mu\nu} \; V^\sigma$$

Therefore:

$$\boxed{[D_\mu, D_\nu] \; V^\rho = - \Gamma^\lambda_{\mu\nu} D_\lambda \; V^\rho + \ldots} \tag{2}$$

Let: $\mu \longrightarrow \nu \; , \; \nu \longrightarrow \mu$ (3)

Then $[D_\nu, D_\mu] \; V^\rho = - \Gamma^\lambda_{\nu\mu} D_\lambda \; V^\rho + \ldots$ (4)

However: $[D_\mu, D_\nu] = - [D_\nu, D_\mu]$ (5)

by definition, so:

$$\boxed{\Gamma^\lambda_{\nu\mu} = - \Gamma^\lambda_{\mu\nu}} \tag{6}$$

Q.E.D.

The connection in Riemann geometry is antisymmetric because the commutator is antisymmetric.

The basic error in the standard model was to assume that the first term in equation (1) "does not exist ”. This left :

$$[D_\mu, D_\nu] \; V^\rho = \; ? \; R^\rho_{\sigma\mu\nu} \; V^\sigma \tag{7}$$

and there is no indication that equations (2) and (4) exist. Equation (2) shows that the connection is the direct result of the commutator. If it is assumed that the connection is symmetric:

$$\Gamma^\lambda_{\mu\nu} = \; ? \; \Gamma^\lambda_{\nu\mu} \tag{8}$$

then $[D_\mu, D_\nu] = ? \; [D_\nu, D_\mu]$ (9)

which is incorrect .

In electromagnetism :

$$[D_\mu, D_\nu]\,\psi = -i\,g\,(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\psi - g^2[A_\mu, A_\nu]\,\psi \qquad (10)$$

that is
$$[D_\mu, D_\nu]\,\psi = -i\,g\,\partial_\mu A_\nu\,\psi + \ldots \qquad (11)$$

so
$$\boxed{\partial_\mu A_\nu = -\partial_\nu A_\mu} \qquad (12)$$

and
$$[A_\mu, A_\nu] = -[A_\nu, A_\mu] \qquad (13)$$

$$F_{\mu\nu} = -F_{\nu\mu} \qquad (14)$$

The errors in the U(1) standard model were to assume that:

$$\partial_\mu A_\nu \neq -\partial_\nu A_\mu \qquad (15)$$

and
$$[A_\mu, A_\nu] = 0 \qquad (16)$$

If equation (16) were true, then:

$$[D_\mu, D_\nu] = ?\ 0 \qquad (17)$$

If equation (15) were true <u>then the commutator would have no symmetry</u>. By definition the commutator is always antisymmetric.

Also, if equation (16) were true, as assumed in U(1) electromagnetism, there would be no field $F_{\mu\nu}$ because there would be no commutator.

For some reason, equation (14) is always used in U(1), but equation (12) is not. Equation (12) shows that the scalar and vector potentials cannot be independent, because:

$$\partial_0 A_1 = -\partial_1 A_0 \qquad (18)$$

etc.

so

$$\boxed{\underline{\nabla}\,\varphi = \partial \underline{A}/\partial t} \qquad (19)$$

Therefore, the standard model is erroneous both in its gravitational and electromagnetic sectors.

The ECE Theory is correct in both sectors.

Note 7: Law Relating Scalar and Vector Potentials.

As seen in previous notes the action of the antisymmetric commutator on the gauge field produces a new symmetry law in the U(1) level:

$$[D_\mu, D_\nu]\,\psi = -i\,g\,(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\psi - g^2[A_\mu\,,A_\nu]\,\psi \qquad (1)$$

In which :

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] \qquad (2)$$

$$\partial_\mu A_\nu = -\,\partial_\nu A_\mu \qquad (3)$$

$$[A_\mu\,,A_\nu] = -\,[A_\nu\,,A_\mu] \qquad (4)$$

It is seen immediately that the dogma of the standard dogma is incorrect on two counts.

1) It asserts that $\partial_\mu A_\nu$ Is unrelated to $\partial_\nu A_\mu$, whereas they are related by equation (3).

2) It asserts incorrectly that $[A_\mu\,,A_\nu]$ is zero in which case $[D_\mu, D_\nu]$ is zero, and the electromagnetic field vanishes.

These are typical of the dogmatic errors that have crept into the electrodynamics since Heaviside's time. In standard textbook electrodynamics it is asserted that electrodynamics is a U(1) gauge field theory, but this is clearly incorrect as argued already. The origin of this claim is Heaviside's interpretation of scalar and vector potentials as being "unphysical". This claim is refuted by the use of the minimal prescription and by the Aharonov-Bohm effects.

In tensor notation the U(1) at electrodynamics is described by :

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad (5)$$

and

$$\partial_\mu F^{\mu\nu} = \varepsilon_0\, J^\nu \qquad (6)$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad (7)$$

In differential form notation, U(1) electrodynamics are:

$$\mathrm{d} \wedge F = 0 \qquad (8)$$

$$d \wedge \tilde{F} = 0 \tag{9}$$

There were already several things that have gone wrong, notably it is claimed that equations (5) and (6) are not Hodge invariant, and that equations (5) and (6) are independent. In ECE theory, both claims are refuted. The gauge invariance of equation (7) is claimed to come from:

$$A^\nu \longrightarrow A^\nu + \partial^\nu x \tag{10}$$

under which $F^{\mu\nu}$ is invariant. In form notation,

$$A \longrightarrow A + dx \tag{11}$$

under which:

$$F = d \wedge A \tag{12}$$

is invariant by the Poincaré Lemma:

$$d \wedge d = 0 \tag{13}$$

so:

$$d \wedge dx = 0 \tag{14}$$

for all x . It is then claimed that A can be varied arbitrarily according to equation (11) without affecting F, and on this basis A is claimed to have "no physical meaning ".

This dogma has been rejected in the ECE theory, in which:

$$D \wedge F^a = A^b \wedge R^a_b \tag{15}$$

$$D \wedge \tilde{F}^a = A^b \wedge \tilde{R}^a_b \tag{16}$$

are Hodge invariant and generally covariant. Equations (8) and (9) are Lorentz covariant only, and are not geometrically based. In ECE:

$$F^a = d \wedge A^a + \omega^a_b \wedge A^b \tag{17}$$

and both A^a and ω^a_b are physical, leading to a straightforward explanation of the Aharonov-Bohm effects.

Equation (1) of the standard theory is replaced by:

$$[D_\mu, D_\nu]\, A^\rho = R^\rho_{\sigma\mu\nu} A^\sigma - F^\lambda_{\mu\nu} D_\lambda V^\lambda \qquad (18)$$

where: $$A^\rho = A^{(0)}\, V^\rho \qquad (19)$$

The electromagnetic field tensor is therefore:

$$F^\lambda_{\mu\nu} = A^{(0)}\, (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}) \qquad (20)$$

where: $$\Gamma^\lambda_{\mu\nu} = -\, \Gamma^\lambda_{\nu\mu} \qquad (21)$$

is antisymmetric. The three rank tensor $F^\lambda_{\mu\nu}$ is the electromagnetic field density. The electromagnetic field is:

$$F_{\mu\nu} = \int F^\lambda_{\mu\nu}\, d\sigma_\lambda \qquad (22)$$

and integrates $F^\lambda_{\mu\nu}$ over the infinitesimal hypersurface $d\sigma_\lambda$ in 4 D.

By using the tetrad postulate:

$$D_\mu q^a_\nu = 0 \qquad (23)$$

that is $$D_\mu A^a_\nu = 0 \qquad (24)$$

equation (20) becomes:

$$F^a_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + \omega^a_{\mu b}\, A^b_\nu - \omega^a_{\nu b} A^b_\mu \qquad (25)$$

$$= \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + \omega^a_\mu\, A^b_\nu - \omega^a_\nu A_\mu$$

The differential two-form $F^a_{\mu\nu}$ is the electromagnetic field density, so:

$$F_{\mu\nu} = \int F^a_{\mu\nu} d\sigma_a$$

from which :

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \omega_\mu A_\nu - \omega_\nu A_\mu$$

$$\boxed{\begin{aligned} F_{\mu\nu} &= (\partial_\mu + \omega_\mu)\, A_\nu - (\partial_\nu + \omega_\nu)\, A_\mu \\ &= A^{(0)}\, (\Gamma_{\mu\nu} - \Gamma_{\nu\mu}) \end{aligned}} \quad (26)$$

Defining: $$D_\mu := \partial_\mu + \omega_\mu \quad (27)$$

then $$\Gamma_{\mu\nu} = D_\mu\, q_\nu \quad (28)$$

in short hand notation:

$$\mathrm{F} = \mathrm{D} \wedge \mathrm{A} \quad (29)$$

and is <u>not invariant under</u>:

$$\mathrm{A} \longrightarrow \mathrm{A} + \mathrm{dx} \quad (30)$$

<u>Therefore gauge theory is replaced by your geometrically based theory</u>.

Furthermore: $$(\partial_\mu + \omega_\mu)\, A_\nu = -\, (\partial_\nu + \omega_\nu)\, A_\mu \quad (31)$$

in which:

$$\boxed{\begin{aligned} \partial_\mu A_\nu &= -\, \partial_\nu\, A_\mu \\ \omega_\mu A_\nu &= -\, \omega_\nu A_\mu \end{aligned}} \quad (32)$$

<u>Examples</u>

1) When $\nu = 0$ (33)

then $$\boxed{\underline{\nabla}\, \varphi = \partial \underline{\mathrm{A}} / \partial \mathrm{t}} \quad (34)$$

and $$\boxed{\varphi\, \underline{\omega} = -\, \omega\, \underline{\mathrm{A}}} \quad (35)$$

The electric field is

$$\boxed{\mathrm{E} = -\underline{\nabla}\,\varphi - \frac{\partial \underline{A}}{\partial \mathrm{t}} + \varphi\,\underline{\omega} - \omega\,\underline{\mathrm{A}}} \tag{36}$$

2) When $\nu \neq 0$

then

$$\left.\begin{array}{ll} \partial_1 A_2 = -\partial_2 A_1 & \text{etc.} \\ \omega_1 A_2 = -\omega_2 A_1 & \text{etc.} \end{array}\right\} \tag{38}$$

and the magnetic field is

$$\boxed{\underline{\mathrm{B}} = \underline{\nabla} \times \underline{\mathrm{A}} - \underline{\omega} \times \underline{\mathrm{A}}}$$

<u>The constraints (32) now apply to the ECE engineering model and ECE cosmology and dynamics.</u>

Note 8: Conflict between U(1) Gauge Invariance and the Proca Equation.

As argued in previous notes the idea of U(1) gauge invariance became incorrect dogma in the 20th century. It also obscured the subject of classical and quantum electrodynamics. This can be illustrated with the standard derivation of the d´Alembert wave equation. In tensor notation the field equations of the standard, or U(1), electrodynamics are:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \tag{1}$$

$$\partial_\mu F^{\mu\nu} = \varepsilon_0 J^\nu \tag{2}$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{3}$$

From (3) in (2):

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = \varepsilon_0 J^\nu \tag{4}$$

that its

$$\Box A^\nu - \partial_\mu (\partial^\nu A^\mu) = \varepsilon_0 J^\nu \tag{5}$$

using the Leibnitz Theorem:

$$\partial_\mu (\partial^\nu A^\mu) = \partial_\mu \partial^\nu A^\mu + \partial^\nu \partial_\mu A^\mu \tag{6}$$

$$= \partial^\nu (\partial_\mu A^\mu)$$

Therefore, in the old physics, equation (5) becomes :

$$\Box A^\nu - \partial^\nu (\partial_\mu A^\mu) = \varepsilon_0 J^\nu \tag{7}$$

At this point the idea of gauge transformation is introduced:

$$A^\mu \longrightarrow A^\mu + \partial^\mu \mathrm{x} \tag{8}$$

From equation (8) in equation (3) , gauge transformation produces :

$$F^{\mu\nu} \longrightarrow \partial^\mu (A^\nu + \partial^\nu \mathrm{x}) - \partial^\nu (A^\mu + \partial^\mu \mathrm{x}) \tag{9}$$

However:

$$\partial^\mu \partial^\nu \mathrm{x} = \partial^\nu \partial^\mu \mathrm{x} = 0 \tag{10}$$

by orthogonality of coordinates, so:

$$F^{\mu\nu} \longrightarrow F^{\mu\nu} \qquad (11)$$

The standard dogma was to assert that A^{μ}El
dogmacan be changed arbitrarily, becauseuede cargarse arbitrariamente, porque x can be any function, and that this change in A^{μ} produces no effect on the field tensor $F^{\mu\nu}$. It was then concluded that A^{μ} was "unphysical" and that $F^{\mu\nu}$was "physical" . After the discovery of the Aharonov-Bohm effects the standard dogma has been refuted, but is still used in textbooks.

This situation was further obscured by the use of the 19th century assertion due to Lorenz :

$$\partial_{\mu} A^{\mu} = 0 \qquad (12)$$

called the Lorenz gauge condition. It was asserted that A^{μ} must be unphysical, so that the further assertion (12) is allowed. This was called "gauge freedom". Using equation (12), equation (7) becomes the d´Alembert equation:

$$A^{\mu} = \varepsilon_0 \, J^{\nu} \qquad (13)$$

In the standard dogma, attempts were made to justify equation (12) by assuming that A^{μ} is proportional to four-current density J^{ν}, so:

$$\partial_{\mu} \, J^{\nu} = 0 \qquad (14)$$

which is the continuity equation.

In differential form notation, equations (1) and (2) are :

$$\mathrm{d} \wedge \mathrm{F} = 0 \qquad (15)$$

$$\mathrm{d} \wedge \tilde{\mathrm{F}} = \varepsilon_0 \, \mathrm{J} \qquad (16)$$

and equation (3) is: $$\mathrm{F} = \mathrm{d} \wedge \mathrm{A} \qquad (17)$$

Equation (8) is $$\mathrm{A} \longrightarrow \mathrm{A} + \mathrm{dx} \qquad (18)$$

The Poincaré Lemma is: $$\mathrm{d} \wedge \mathrm{d} := 0 \qquad (19)$$

so it was asserted in the standard dogma that the gauge transformation (18) produces :

$$\mathrm{F} \longrightarrow \mathrm{d} \wedge \mathrm{A} + \mathrm{d} \wedge \mathrm{dx} = \mathrm{F} \qquad (20)$$

because : $$\mathrm{d} \wedge \mathrm{dx} := 0 \qquad (21)$$

The Chambers experiment showed in the early 60s that the potential form A is physical, it has an effect where there is no field form F. Earlier, the minimal prescription:

$$P^{\mu} \longrightarrow P^{\mu} + eA^{\mu} \qquad (22)$$

was routinely used in areas such as ESR and NMR with physical A^{μ}físico. The inverse Faraday effect, shown experimentally in 1965, demonstrates that the conjugate product $\underline{\mathrm{A}} \times \underline{\mathrm{A}^{*}}$ in vector notation is physical.

Despite these clear experimental refutations, field theorists in physics continued to adhere to U(1) gauge invariance, so the idea of a U(1) electrodynamics sector became standard dogma in unified field theory. This dogma is completely incorrect. It leads to insurmountable problems. One of these is illustrated here with the Proca equation for finite photon mass:

$$(\square + \kappa^2)\, A^{\mu} = 0 \qquad (23)$$

where: $$\kappa = \frac{mc}{\hbar} \qquad (24)$$

Here, m is the mass of the photon (Einstein, 1906), c is the speed of light in a vacuum (regarded as a universal constant) and $\hbar$ the reduced Planck constant. So equation (24) means that the photon momentum is :

$$\mathrm{p} = \hbar\, \kappa = mc \qquad (25)$$

The de Broglie wave particle dualism. If the photon mass is zero, there can be no bending of light by gravitation, contrary to experimental data. Yet, the standard dogma asserts:

$$m = ?\, 0 \qquad (26)$$

In this case equation (23) becomes :

$$\square A^{\mu} = 0 \qquad (27)$$

"the free space d´Alembert wave equation”, as it was called in the obsolete dogma.

The Proca equation (23) is equivalent to:

$$\partial_\mu F^{\mu\nu} + \kappa^2 A^\nu = 0 \qquad (28)$$

in the standard literature that mentions it (very few textbooks). The equivalence of equations (23) and (28) is only true however, if the Lorenz gauge is used, because:

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) + \kappa^2 A^\nu = 0 \qquad (29)$$

and this reduces to equation (23) if and only if :

$$\partial_\mu A^\mu = 0 \qquad (30)$$

This result also follows by a applying ∂_ν to equation (28):

$$\partial_\nu \partial_\mu F^{\mu\nu} + \kappa^2 \partial_\nu A^\nu = 0 \qquad (31)$$

that is $$\partial_\nu A^\nu = 0 \qquad (32)$$

by orthogonality of coordinates.

<u>This ought to suggest again that something is wrong with the dogma</u>. The reasons are:

1) <u>The Proca equation is not U(1) gauge invariant</u>, because $F^{\mu\nu}$ndoes not change under the gauge transform (8), but A^ν <u>does</u> .
2) The Lorenz condition (30) is arbitrary mathematics .
3) <u>The standard dogma missed the symmetry condition</u>:

$$\partial_\mu A_\nu = - \partial_\nu A_\mu \qquad (33)$$

<u>Therefore we reject the standard dogma as being full of basic errors .</u>