1) <u>127(1)</u>: <u>Fundamental Origins of Angular Momentum.</u>

In paper 76 the fundamental origin of angular momentum was discussed in terms of the definition of a tetrad. Since then, many advances have been made in ECE theory. Therefore it is now possible to give more than one definition of angular momentum from basic geometry. Angular momentum is of course understood as spacetime angular momentum. In this first note of paper 127 the tetrad definition of angular momentum is reviewed.

The tetrad definition of angular momentum restated as:

$$W^a = V^a_\mu V^\mu \quad -(1)$$

which is the fundamental definition of tetrad. In paper 76 consideration was restricted to three dimensions and the idea of a tangent spacetime labelled a and introduced by Cartan was still used. Since then the ECE theory has been developed in the base manifold in order to give the field equations and equations relating the fields and potentials. In paper 76, eq. (1) was developed as:

$$W^1 = J^1{}_2 V^2 + J^1{}_3 V^3 \quad -(2)$$

$$W^2 = J^2{}_1 V^1 + J^2{}_3 V^3 \quad -(3)$$

$$W^3 = J^3{}_1 V^1 + J^3{}_2 V^2 \quad -(4)$$

because angular momentum must be developed as an anti-symmetric tensor of rank two. Lowering indices in eqs. (2) to (4) gives:

$$W_1 = J_{12} V_2 + J_{13} V_3 \quad -(5)$$

$$W_2 = J_{21} V_1 + J_{23} V_3 \quad -(6)$$

$$W_3 = J_{31} V_1 + J_{32} V_2 \quad -(7)$$

In ~~three~~ three dimensions:

2)

$$J_{ij} = \frac{1}{2}\epsilon_{ijk} J_k \quad -(8)$$

So:

$$W_1 = J_3 V_2 - J_2 V_3 \quad -(9)$$
$$W_2 = J_3 V_1 - J_1 V_3 \quad -(10)$$
$$W_3 = J_1 V_2 - J_2 V_1 \quad -(11)$$

or, in vector notation:

$$\underline{W} = \underline{J} \times \underline{V} \quad -(12)$$

in general under these assumptions.

So far this is just an exercise in geometry, but if

$$\underline{W} := \underline{p} \quad -(13)$$
$$\underline{V} := \underline{r} / r^2 \quad -(14)$$

then:

$$\underline{p} = \underline{J} \times \underline{r} / r^2 \quad -(15)$$

and

$$\boxed{\underline{J} = \underline{r} \times \underline{p}} \quad -(16)$$

This is the starting point of paper 126. Eq. (16) describes a type of spacetime constructed from geometry. Only the definition of the tetrad was used in paper 76, and the tetrad definition was reduced to the definition of a rotation generator, which is angular momentum within $\underline{f}$.

In later papers, angular momentum was identified as the volume integral over torsion.

3) This procedure reduces the rank three torsion tensor to a rank two angular momentum tensor in the base manifold, and eliminates the complication of the Cartan tangent spacetime. For example:

$$\frac{\underline{L}}{V} = \frac{1}{ck}\,\underline{g} \quad -(17)$$

where $\underline{g}$ is the acceleration due to gravity, $\underline{L}/V$ is the orbital angular momentum density, c is the speed of light and k is the Einstein constant. The Cartan structure equation gives:

$$\boxed{\underline{L} = \left(\frac{V}{mck}\right)\left(-\nabla U - \frac{1}{c}\frac{\partial \underline{U}}{\partial t} + U\underline{\omega} - \omega_0 \underline{U}\right)} \quad -(18)$$

where

$$U^\mu = (U, \underline{U}) \quad -(19)$$

is a four-vector of potential energy. The spin angular momentum is then:

$$\boxed{\underline{S} = \left(\frac{V}{mck}\right)\left(\nabla \times \underline{U} - \underline{\omega} \times \underline{U}\right)} \quad -(20)$$

It is possible to define eq. (19) as a potential momentum four vector:

$$p^\mu = \left(\frac{U}{c}, \underline{U}\right) \quad -(21)$$

1)
127(2): Relation Between Angular Momentum and Metric.

This is obtained by using the method of paper 124, in which the angular momentum is defined as:

$$J_{\mu\nu} = \frac{c}{k}\left(\left(\partial_\mu + \omega_\mu\right) q_\nu - \left(\partial_\nu + \omega_\nu\right) q_\mu\right) \quad -(1)$$

Units

$$J_{\mu\nu} = \frac{m^3 m s^{-1}}{m\ kgm^{-1}}\ m^{-1} = kgm\ m^2 s^{-1}$$

The potential energy four-vector is now defined as

$$\boxed{U_\mu = \frac{mc^2}{V} q_\mu} \quad -(2)$$

so:

$$J_{\mu\nu} = \frac{V}{mck}\left(\left(\partial_\mu + \omega_\mu\right) U_\nu - \left(\partial_\nu + \omega_\nu\right) U_\mu\right) \quad -(3)$$

The metric is defined in general by:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab}. \quad -(4)$$

If

$$a = b = 0 \quad -(5)$$

as in paper 124, then:

$$g_{\mu\nu} = q^0_\mu q^0_\nu \eta_{00} = q^0_\mu q^0_\nu \quad -(6)$$

Here:

$$q_\mu = \int q^0_\mu \, dV \quad -(7)$$

$$q_\nu = \int q^0_\nu \, dV \quad -(8)$$

) Therefore:

$$q^{\circ}_{\mu} = q_{\mu} / V \quad -(9)$$

$$q^{\circ}_{\nu} = q_{\nu} / V \quad -(10)$$

and

$$\boxed{g_{\mu\nu} = \frac{U_{\mu} U_{\nu}}{(mc^2)^2}} \quad -(11)$$

There is therefore a direct relation between the metric and the tensor product of the potential four vectors U_{μ} and U_{ν}. In general:

$$\boxed{\begin{aligned} U_{\mu} U_{\nu} &= (mc^2)^2 \, g_{\mu\nu} \\ &= mc^2 \, q^a_{\mu} \, q^b_{\nu} \end{aligned}} \quad -(12)$$

From eqs. (3) and (12) it is seen that spacetime angular momentum originates in tetrads, and therefore in Cartan geometry.

If we define:

$$U_{\mu} = (U, \; -c\underline{\pi}) \quad -(13)$$

then the orbital and spin angular momenta of spacetime are:

$$\underline{L} = \frac{V}{mck}\left(-\underline{\nabla} U - \frac{\partial \underline{\Pi}}{\partial t} + U\underline{\omega} - c\omega_0 \underline{\Pi} \right) \quad -(14)$$

$$\underline{S} = \frac{V}{mk}\left(\underline{\nabla} \times \underline{\Pi} - \underline{\omega} \times \underline{\Pi} \right) \quad -(15)$$

The concept of $\underline{\Pi}$ is that of potential momentum.

It is seen that the spin angular momentum of spacetime is governed by potential momentum. This is analogous to the vector potential $\underline{A}$ in ECE electrodynamics.

In the limit:

$$\underline{\Pi} \rightarrow \underline{0} \quad -(16)$$

then

$$\underline{L} \rightarrow \frac{V}{mck}\left(\left(-\underline{\nabla} + \underline{\omega} \right) U \right) \quad -(17)$$

and in the Newtonian limit:

$$\underline{\omega} \rightarrow \underline{0} \quad -(18)$$

so

$$\underline{L} \rightarrow -\frac{V}{mck} \underline{\nabla} U \quad -(19)$$

$$= \underline{r} \times \underline{p}$$

It is seen that from eq. (12), U originate U originates in a tetrad component, and so U originates in spacetime itself.

1) 127(3): Spacetime Angular Momentum and Quantum Relations

The general expression for spacetime angular momentum derived in note 127(2) is:

$$J_{\mu\nu} = \frac{V}{n k}\left((\partial_\mu + \omega_\mu)\pi_\nu - (\partial_\nu + \omega_\nu)\pi_\mu \right) \quad -(1)$$

where the potential four-momentum of spacetime is:

$$\pi_\mu = \left(\frac{U}{c}, -\underline{\pi} \right) \quad -(2)$$

where U is the potential energy of spacetime and where $\underline{\pi}$ is the potential momentum of spacetime.

Now define the quantum mechanical operator relations:

$$p^\mu = i\hbar \left(\partial^\mu + \omega^\mu \right) \quad -(3)$$

or

$$\boxed{p_\mu = i\hbar \left(\partial_\mu + \omega_\mu \right)} \quad -(4)$$

and

$$\boxed{x_\mu = -\frac{i}{\hbar}\left(\frac{V}{n k} \right) \pi_\mu} \quad -(5)$$

to find

$$\boxed{J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu} \quad -(6)$$

2) Eq. (4) is an extension of the well known $p_\mu = i\hbar \partial_\mu$ of quantum mechanics to a generally covariant theory in which ∂_μ is replaced by $\partial_\mu + \omega_\mu$. This concept also extends gauge theory to a generally covariant theory.

Eq. (5) is a new concept which is an operator relation between x_μ and Π_μ, the potential four-momentum of spacetime. Here:

$$x^\mu = (ct, X, Y, Z) \quad -(7)$$

$$x_\mu = (ct, -X, -Y, -Z) \quad -(8)$$

and Π_μ is defined by eq. (2). In the limit:

$$\Pi \longrightarrow 0 \quad -(9)$$

eq. (5) becomes:

$$ct = -\frac{i}{\hbar}\left(\frac{V}{mk}\right)\frac{U}{c} \quad -(10)$$

i.e.

$$\boxed{t = -\frac{i}{\hbar mc^2}\left(\frac{V}{k}\right)U} \quad -(11)$$

and there is an operator relation between time t and spacetime potential energy U (in joules).

3) Eq (11) shows that time is a form of ~~en~~ spacetime potential energy. The familiar rest energy mc^2 appears in eq (11), together with the reduced Planck constant $\hbar$ and Einstein constant k. The units are:

$$\frac{1}{\hbar m c^2} \frac{V}{k} u = \frac{m^3 \, J}{J^2 \, s \, m \, kgm^{-1}} \, \frac{m^2}{kgm \, m^2 \, s^{-2} \, \cancel{m} \, s \, kgm^{-1}}$$

$$= s$$

Distance is a form of spacetime potential momentum:

$$\boxed{r = \frac{1}{\hbar m c} \left(\frac{V}{k} \right) \pi} \quad - (12)$$

Quantization is described covariantly either by eq (4) or eq (5). The starting point of ECE paper 126 is the space part of eq (6), $\underline{L} = \underline{r} \times \underline{p}$, and conservation of $\underline{L}$ is sufficient to describe all planar orbits.

1) 127(4): Removal of Indeterminacy.

The standard derivation of the Heisenberg principle of indeterminacy is given by Atkins on pp. 93 ff of the 2nd ed. of "Molecular Quantum Mechanics" (OUP 1983).

It starts with:

$$[A, B] = iC. \quad -(1)$$

The expectation values are:

$$\langle A \rangle = \int \psi^* A \psi \, d\tau, \quad \langle B \rangle = \int \psi^* B \psi \, d\tau. \quad -(2)$$

Then Atkins defines:

$$\Delta A = A - \langle A \rangle, \quad \Delta B = B - \langle B \rangle. \quad -(3)$$

and shows that:

$$[\Delta A, \Delta B] = [A, B] = iC. \quad -(4)$$

It is then shown that

$$I = \int |(\alpha \Delta A - i \Delta B)\psi|^2 \, d\tau \geq 0 \quad -(5)$$

This integral is chosen arbitrarily and is not given any justification in physics. It is just chosen to be ≥ 0. The Heisenberg uncertainty principle is just a re-arrangement of this integral as:

$$\delta A \, \delta B \geq \frac{1}{2} |\langle C \rangle| \quad -(6)$$

where:

$$\delta A = \left(\langle A^2 \rangle - \langle A \rangle^2 \right)^{1/2} \quad -(7)$$

and:

$$C = -i[A, B] \quad -(8)$$

2) Remarks

1) The Heisenberg uncertainty principle is simply the integral (5). Atkins gives no derivation of this integral, and it is an arbitrary mathematical construct.

2) If, as in note 127(3), one defines:

$$[A, B] = iC \quad -(9)$$

then:

$$C = -i[A, B] \quad -(10)$$

is imaginary valued if $[A, B]$ is real valued. So C has no real part, its value is imaginary, so in eq. (6):

$$\boxed{\delta A \delta B = 0} \quad -(11)$$

There is no indeterminacy, as found experimentally by Croca and colleagues.

So the idea of indeterminacy is a mirage, by using a commutator where A and B are both imaginary, eq. (11) results. The integral (5) is completely arbitrary, it is chosen to give the desired result. This is an unscientific procedure.

128(3): Position and Momentum Representation.

By reference to the Heisenberg equation:

$$[x, p]\phi = i\hbar\phi \quad -(1)$$

the standard approach is to choose a representation.

Position Representation

The momentum is chosen to be the generator:

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x}, \quad -(2)$$

$$x = x \quad -(3)$$

Momentum Representation

The position is chosen to be the generator:

$$x = -\frac{\hbar}{i}\frac{\partial}{\partial p} \quad -(4)$$

$$p = p \quad -(5)$$

It is accepted in quantum mechanics that some calculations can be carried out without choosing a representation.

In note 127(3) the basic equation is:

$$\boxed{[\hat{x}_\mu, \hat{p}_\nu] = J_{\mu\nu}} \quad -(6)$$

where:

$$\hat{x}_\mu = -\frac{i}{\hbar}\left(\frac{V}{mk}\right)\pi_\mu \quad -(7)$$

and

$$\hat{p}_\mu = i\hbar\left(\partial_\mu + \omega_\mu\right) \quad -(8)$$

It is equally possible to choose another representation so that eq. (6) is true.

The momentum representation of eq. (7) is therefore:

$$\hat{x}_\mu = i\hbar \frac{\partial}{\partial p^\mu} = -\frac{i}{\hbar}\left(\frac{V}{\kappa k}\right)\pi_\mu \quad -(9)$$

So:

$$\frac{\partial}{\partial p^\mu} = \frac{1}{\hbar^2}\left(\frac{V}{\kappa k}\right)\pi_\mu . \quad -(10)$$

The position representation of eq. (8) is:

$$\hat{p}_\mu = i\hbar\left(\partial_\mu + \omega_\mu\right) \quad -(11)$$

$$= i\hbar\left(\frac{\partial}{\partial x^\mu} + \omega_\mu\right) \quad -(12)$$

Here:

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right) \quad -(13)$$

$$\omega_\mu = \left(\omega, -\underline{\omega}\right). \quad -(14)$$

<u>Remarks</u>

The basic quantum operators $\hat{x}_\mu$ and $\hat{p}_\mu$ are defined by the derivation of the classical angular momentum $J_{\mu\nu}$ of spacetime from the Cartan structure equation for spacetime torsion. The proportionality $V/(\kappa k)$ has the units of <u>area.</u> Since $J_{\mu\nu}$ is classical it can go to

3)

zero. The Planck constant $\hbar$ is introduced in eq. (8) for $\hat{p}_\mu$ and in eq. (7) for $\hat{x}_\mu$.

In special relativity there is no spin connection so:

$$\hat{p}_\mu = i\hbar \frac{\partial}{\partial x^\mu} \quad -(15)$$

Eq. (15) gives the Dirac equation from the Einstein equation:

$$p^\mu p_\mu = m c^2 \quad -(16)$$

From eq. (15) in (16):

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \psi = 0 \quad -(17)$$

where ψ is the Dirac four-spinor. In ECE theory eq. (17) is derived from the tetrad postulate. In this procedure the covariant derivative has been simplified to:

$$D_\mu = \partial_\mu + \omega_\mu \quad -(18)$$

by fixing the a index at zero. In a flat spacetime:

$$[\hat{x}_\mu, \hat{p}_\nu] = J_{\mu\nu} = 0 \quad -(19)$$

because spacetime torsion in a flat spacetime is zero.

Eq. (6) in full is:

$$\boxed{[\hat{x}_\mu, \hat{p}_\nu] \psi = J_{\mu\nu} \psi} \quad -(20)$$

1)

P27(6): The Internal Structure of Spacetime Angular Momentum

The starting point of paper 126 was the concept of spacetime angular momentum J_Z. It was shown that the conservation of J_Z gives all planar orbits. The geometrical structure of J_Z is revealed by the definition of torsion:

$$[D_\mu, D_\nu] V^\sigma = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\sigma \quad -(1)$$

where:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = 2\Gamma^\lambda_{\mu\nu} \quad -(2)$$

using the antisymmetry of the connection:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}. \quad -(3)$$

As in previous work we introduce the hypothesis:

$$\boxed{J^\lambda_{\mu\nu} = \frac{c}{k} T^\lambda_{\mu\nu}} \quad -(4)$$

where $J^\lambda_{\mu\nu}$ is the canonical angular energy / momentum density in S.I. units of $kgm\, m^{-1} s^{-1}$. In eq. (4) c is the speed of light and k is the Einstein constant. The classical angular momentum of spacetime is then the volume integral:

$$J_{\mu\nu} = \int J^0_{\mu\nu}\, dV = \frac{c}{k} \int T^0_{\mu\nu}\, dV \quad -(5)$$

using eq (2):

$$\boxed{J_{\mu\nu} = 2\frac{c}{k} \int \Gamma^0_{\mu\nu}\, dV} \quad -(6)$$

showing that the classical angular momentum of spacetime

2) is the volume integral over $\Gamma^0_{\mu\nu}$, a component of the antisymmetric connection. Thus:

$$J_Z = J_{12} = \frac{2c}{k}\int \Gamma^0_{12}\, dV \quad -(7)$$

and the origin of J_Z is:

$$\Gamma^0_{12} = -\Gamma^0_{21}. \quad -(8)$$

In Einsteinian and Newtonian theory these concepts do not exist, yet they provide a straightforward route to the understanding of all planar orbits, where:

$$\frac{dJ_Z}{dt} = 0. \quad -(9)$$

Now introduce the Cartan torsion to develop the internal structure of J_Z:

$$T^a_{\mu\nu} = q^a_\lambda T^\lambda_{\mu\nu} \quad -(10)$$

$$T^\lambda_{\mu\nu} = q^\lambda_a T^a_{\mu\nu} \quad -(11)$$

$$q^a_\lambda q^\lambda_a = 1. \quad -(12)$$

Thus:

$$T^a_{\mu\nu} = \left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu\right) - \left(\partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu\right) \quad -(13)$$

$$= q^a_\lambda \left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right)$$

and the tetrad postulate gives:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu = q^a_\lambda \Gamma^\lambda_{\mu\nu} \quad -(14)$$

$$\partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu = q^a_\lambda \Gamma^\lambda_{\nu\mu} \quad -(15)$$

3) Therefore:

$$\boxed{\begin{aligned} J_2 = J_{12} &= \frac{2c}{k}\int q^0_a\Big(\left(\partial_1 q^a_2 + \omega^a_{1b}\, q^b_2\right) \\ &\qquad - \left(\partial_2 q^a_1 + \omega^a_{2b}\, q^b_1\right)\Big)\, dV \\ &= \frac{2c}{k}\int \Gamma^0_{12}\, dV \end{aligned}} \qquad -(16)$$

This is the most general expression for the structure of J_2 in Cartan geometry. This expression may be simplified in various ways. Eq. (16) is an expression of the tetrad postulate for Γ^0_{12}, and using the antisymmetry of Γ^0_{12} proven in paper 122:

$$J_2 = J_{12} = \frac{2c}{k}\int q^0_a\left(\partial_1 q^a_2 + \omega^a_{1b}\, q^b_2\right) dV \qquad -(17)$$

If there is no intrinsic volume dependence of the function inside the integral of eq. (17):

$$J_2 = J_{12} = \left(\frac{2cV}{k}\right) q^0_a\left(\partial_1 q^a_2 + \omega^a_{1b}\, q^b_2\right) \qquad -(18)$$

i.e. the angular momentum is essentially the covariant Cartan derivative of the tetrad.

1.127(7): The Spin Connection in Flat Spacetime.

1 The torsion tensor in the general spacetime is:

$$T^{\lambda}_{\mu\nu} = 2\Gamma^{\lambda}_{\mu\nu} \quad -(1)$$

The tetrad postulate states that:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu = q^a_\lambda \Gamma^{\lambda}_{\mu\nu} \quad -(2)$$

so the connection is:

$$\Gamma^{\lambda}_{\mu\nu} = q^{\lambda}_{a}\left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu\right). \quad -(3)$$

Thus:

$$J^{\lambda}_{\mu\nu} = \frac{c}{k} T^{\lambda}_{\mu\nu} = 2\frac{c}{k} q^{\lambda}_{a}\left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu\right) \quad -(4)$$

is the canonical angular energy / momentum density. The angular momentum tensor is:

$$J_{\mu\nu} = -J_{\nu\mu} = \int J^{0}_{\mu\nu}\, dV. \quad -(5)$$

Therefore in general relativity the angular momentum is non-zero if and only if $\Gamma^{\lambda}_{\mu\nu}$ is non-zero. Angular momentum requires a non-flat spacetime with an anti-symmetric connection:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu} \quad -(6)$$

that is non-zero.

2) If the connection is zero there is no angular momentum, but there may be a non-zero spin connection, defined by:

$$\boxed{\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu = 0} \quad -(7)$$

The Cartan torsion for zero $\Gamma^\lambda_{\mu\nu}$ is zero:

$$T^a_{\mu\nu} = q^a_\lambda \Gamma^\lambda_{\mu\nu} = 0 \quad -(8)$$

i.e.:

$$T^a_{\mu\nu} = \left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu\right) - \left(\partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu\right) \quad -(9)$$

$$= 0.$$

Therefore the spin connection in a flat spacetime is, for zero Γ:

$$\omega^a_{\mu b} = -q^\nu_b \partial_\mu q^a_\nu \quad -(10)$$

with

$$q^a_\nu q^\nu_a = 1 \quad -(11)$$

by definition. In previous work the spin connection for rotation was defined as:

$$\omega^a_{\mu b} = -\frac{1}{2} \kappa \epsilon^a{}_{bc} q^c_\mu \quad -(12)$$

where κ is a wavenumber. Therefore rotation i

3) a flat spacetime is the o(3) cyclic equation:

$$\partial_\mu q^a_\nu = \frac{1}{2}\kappa\,\epsilon^a{}_{bc}\, q^b_\mu q^c_\nu \quad -(13)$$

which is a coordinate definition in flat spacetime.

In flat spacetime there may exist plane wave tetrads such as:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \exp\left(i(\omega t - \kappa Z)\right) \quad -(14)$$

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \exp\left(-i(\omega t - \kappa Z)\right) \quad -(15)$$

which are conjugate plane wave tetrads. In this case:

$$a = (1), (2), (3) \quad -(16)$$

and

$$\underline{q}^{(1)} \times \underline{q}^{(2)} = i\,\underline{q}^{(3)*} \quad -(17)$$

et cyclicum

an o(3) symmetry coordinate relation of flat spacetime. So the flat spacetime spin connection in this case is worked out from eqs. (10), (14) and (15) as in the next note.

1. III(8): Two Basic Tests of Geometry.

1) Computer Algebra Test

Evaluate the dual identity:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \mu\nu}_{\ \mu} \quad -(1)$$

by computer algebra. Show that no solution of the Einstein field equation obey the basic geometry (1).

2) The Dual Identity

a) Show that the basic commutator equation:

$$[D_\mu, D_\nu] V^\rho = R^\rho_{\ \sigma\mu\nu} V^\sigma - T^\lambda_{\ \mu\nu} D_\lambda V^\rho \quad -(2)$$

is equivalent to:

$$D \wedge T := R \wedge q. \quad -(3)$$

Use the proofs in paper 99 ff.

b) Derive the Hodge dual of eq. (2) and show that it is an example of eq. (2). I give details of this proof as follows. The Hodge dual of the commutator is:

$$[D^\alpha, D^\beta]_{HD} = \frac{1}{2}||g||^{1/2} \epsilon^{\alpha\beta\mu\nu}[D_\mu, D_\nu] \quad -(4)$$

For example:

2.

$$[D^0, D^1]_{HD} = \|g\|^{1/2} [D_2, D_3] \quad -(5)$$

So:

$$[D^0, D^1]_{HD} V^\sigma = \tilde{R}^{\rho}{}_{\sigma}{}^{01} V^\sigma - \tilde{T}^{\lambda 01} D_\lambda V^\rho \quad -(6)$$

where:

$$\tilde{R}^{\rho}{}_{\sigma}{}^{01} = \|g\|^{1/2} R^{\rho}{}_{\sigma 23} \quad -(7)$$

$$\tilde{T}^{\lambda 01} = \|g\|^{1/2} T^{\lambda}{}_{23} \quad -(8)$$

It is seen that eq. (6) is generated from:

$$[D_2, D_3] V^\sigma = R^{\rho}{}_{\sigma 23} V^\sigma - T^{\lambda}{}_{23} D_\lambda V^\rho . \quad -(9)$$

by multiplying both sides by $\|g\|^{1/2}$, which is a number, the positive value of the square root of the determinant of the metric.

So the dual equation (6) is an example of the original equation (2), QED.

We may lower indices in eq. (6) by:

$$[D_0, D_1]_{HD} = g_{00} g_{11} [D^0, D^1]_{HD} \quad -(10)$$

etc., so:

$$[D_0, D_1]_{HD} V^\sigma = \tilde{R}^{\rho}{}_{\sigma 01} V^\sigma - \tilde{T}^{\lambda}{}_{01} D_\lambda V^\rho \quad -(11)$$

and:

$$\boxed{D \wedge \tilde{T} := \tilde{R} \wedge q .} \quad -(12)$$

1) 127(9): Symmetries of the Cartan Identity.

1) Cartan Bianchi Identity

The basic reason for the existence of this fundamental identity of geometry is:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(1)$$

an equation which defines the structures of the curvature and torsion tensors. These structures are related by:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu}$$

which is the Cartan Bianchi identity. This is a well known result, going back to 1925. It is proven in great detail in pages 99 ff. Eq. (2) may be reduced without loss of generality to: $-(2)$

$$D_\mu T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\rho\mu} := R^\kappa{}_{\mu\nu\rho} + R^\kappa{}_{\rho\mu\nu} + R^\kappa{}_{\nu\rho\mu}. \quad -(3)$$

which is the same as:

$$\boxed{D_\mu \tilde{T}^{\kappa\mu\nu} := \tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}} \quad -(4)$$

These equations are true in general, they do not rely on any assumption about the symmetry of the connection or metric, and do not even need metric compatibility.

Einstein in 1915 used the assumptions made by Ricci and Levi-Civita in 1900:

$$\Gamma^\kappa{}_{\mu\nu} = ? \, \Gamma^\kappa{}_{\nu\mu} \quad -(5)$$

$$g_{\mu\nu} = g_{\nu\mu} \quad -(6)$$

2) These assumptions produce:

$$\tilde{T}^{\kappa\mu\nu} =? 0, \quad -(7)$$

and

$$R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\nu\rho\mu} =? 0, \quad -(8)$$

so eq. (4) is:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} =? 0 \quad -(9)$$

Eq. (8) is known in the standard model as the first Bianchi identity. It is, however, true only if the assumptions (5) and (6) are made. So it is not a true identity. The true identity is eq. (4), in which both sides may be non-zero in general.

2) Cartan Evans Dual Identity.

This is an example of eq. (1):

$$[D_\mu, D_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma\mu\nu} V^\sigma - \tilde{T}^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(10)$$

generated by the Hodge dual of the commutator. The Hodge dual curvature and torsion tensors are related by

$$D_\mu \tilde{T}^a{}_{\nu\rho} + D_\rho \tilde{T}^a{}_{\mu\nu} + D_\nu \tilde{T}^a{}_{\rho\mu} := \tilde{R}^a{}_{\mu\nu\rho} + \tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} \quad -(11)$$

which is

$$\boxed{D_\mu T^{\kappa\mu\nu} := R^{\kappa}{}_{\mu}{}^{\mu\nu}} \quad -(12)$$

which is duality invariant with eq. (4). Eqs. (4) and (12) make statements which should be

3) obvious, that torsion and curvature are both non-zero in general. The Einsteinian era incorrectly used eq. (5), so produced:

$$D_\mu T^{\kappa\mu\nu} \stackrel{?}{=} 0, \quad T^{\kappa\mu\nu} \stackrel{?}{=} 0 \qquad — (13)$$

but computer algebra shows that:

$$R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0 \qquad — (14)$$

for ALL solutions of the Einstein field equation in the presence of matter.

Furthermore, the correct symmetry of the connection is given by eq. (1):

$$\boxed{\Gamma^{\kappa}_{\mu\nu} = -\Gamma^{\kappa}_{\nu\mu}} \qquad — (15)$$

and not eq. (5). The latter was chosen arbitrarily in 1900, apparently by Ricci and Levi-Civita to simplify the calculation. The Einstein field equation is geometrically incorrect, and Einsteinian gravitational physics is meaningless.

1) **127(10): Calculation of the Spin Connection in Flat Spacetime for Plane Wave Tetrads.**

The plane wave tetrads were introduced in the early stages of ECE theory and are:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})\exp(i(\omega t - \kappa Z)) \quad -(1)$$

and its complex conjugate

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j})\exp(-i(\omega t - \kappa Z)) \quad -(2)$$

These tetrads exist in a spacetime which is still propagating and spinning. This is a key difference between the ECE theory and the older Maxwell Heaviside theory, in which fields and potentials existed in a flat spacetime. In earlier notes 127 it was shown that the spin connection in a flat spacetime is:

$$\omega^a_{\mu b} = -q^\nu_b \partial_\mu q^a_\nu \quad -(3)$$

In this note, spin connections are evaluated from eq. (3) using eqs. (1) and (2). In so doing the normalization conditions in Carroll chapter three are used:

$$q^\mu_a q^a_\nu = \delta^\mu_\nu \quad -(4)$$

$$q^a_\mu q^\mu_b = \delta^a_b \quad -(5)$$

where δ^μ_ν or δ^a_b are 1 if $\mu = \nu$ or $a = b$, and 0 otherwise. The relations between the gamma and spin connections are as follows:

$$\Gamma^\nu_{\mu\lambda} = q^\nu_a(\partial_\mu q^a_\lambda + q^b_\lambda \omega^a_{\mu b}) \quad -(6)$$

2) and $$\omega^a_{\mu b} = q^\lambda_b \left(q^a_\nu \Gamma^\nu_{\mu\lambda} - \partial_\mu q^a_\lambda \right). \quad -(7)$$

The covariant derivative is defined as:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \quad -(8)$$

and by $$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} X^b . \quad -(9)$$

We have:

$$DX = D_\mu X^\nu \, dx^\mu \otimes \partial_\nu \quad -(10)$$

$$= D_\mu X^a \, dx^\mu \otimes \hat{q}_{(a)} \quad -(11)$$

Eqs. (10) and (11) lead to the tetrad postulate as described in Carroll chapter 3. The tetrad postulate is obtained by expanding eq. (11) using:

$$\hat{q}_{(a)} = q^\sigma_a \partial_\sigma \quad -(12)$$

and $$X^a = q^a_\nu X^\nu \quad -(13)$$

So:

$$DX = \left(\partial_\mu \left(q^a_\nu X^\nu \right) + \omega^a_{\mu b} q^b_\lambda X^\lambda \right) dx^\mu \otimes \left(q^\sigma_a \partial_\sigma \right)$$

$$= q^\sigma_a \left(q^a_\nu \partial_\mu X^\nu + X^\nu \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\lambda X^\lambda \right) dx^\mu \otimes \partial_\sigma \quad -(14)$$

The dummy σ indices in eq. (14) are now relabelled as ν indices:

$$DX = q^\nu_a \left(q^a_\nu \partial_\mu X^\nu + X^\nu \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\lambda X^\lambda \right) dx^\mu \otimes \partial_\nu \quad -(15)$$

3) Eq. (15) is now compared with eq. (10):

$$DX = \left(\partial_\mu X^\nu + \Gamma^\nu_{\mu\lambda} X^\lambda\right) dx^\mu \otimes \partial_\nu \quad -(16)$$

In so doing, Cartan used

$$\boxed{q^\nu_a \, q^a_\nu = 1} \quad -(17)$$

for the first term on the left hand side of eq. (15), which becomes:

$$DX = \left(\partial_\mu X^\nu + \left(q^\nu_a \partial_\mu q^a_\nu X^\nu + q^\nu_a \omega^a_{\mu b} q^b_\lambda X^\lambda\right)\right) dx^\mu \otimes \partial_\nu \quad -(18)$$

The dummy ν indices in the second term on the right hand side of eq. (18) are relabelled λ, so:

$$DX = \left(\partial_\mu X^\nu + q^\nu_a\left(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda\right) X^\lambda\right) dx^\mu \otimes \partial_\nu \quad -(19)$$

From eqs. (16) and (19):

$$\Gamma^\nu_{\mu\lambda} = q^\nu_a\left(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda\right) \quad -(20)$$

QED. By using:

i.e. the tetrad postulate,

$$q^\nu_a \, q^a_\nu = 1 \quad -(21)$$

$$-(22)$$

The postulate is written as:

$$\boxed{D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0}$$

4) In a flat spacetime the covariant derivative is replaced by the ordinary derivative, so eqns. (8) and (9) mean that in a flat spacetime:

$$\Gamma^{\nu}_{\mu\lambda} = 0, \quad -(23)$$

$$\omega^{a}_{\mu b} = 0. \quad -(24)$$

From eq. (3):

$$\boxed{q^{\nu}_{b}\, \partial_{\mu} q^{a}_{\nu} = 0} \quad -(25)$$

in a flat spacetime. It is now possible to use eq. (25) to investigate the correct interpretation of the plane wave tetrad (1) and (2). It is known that in ECE theory they must obey the tetrad postulate (23), so both $\Gamma^{\nu}_{\mu\lambda}$ and $\omega^{a}_{\mu b}$ must be non-zero. In Maxwell Heaviside (MH) theory, eqns. (1) and (2) are interpreted as flat spacetime equations. In a generally covariant field theory, the ECE interpretation must be preferred. In the next note the ECE interpretation is proven by showing that eq. (25) is not obeyed by eqs. (1) and (2). Therefore eqs (1) and (2) are those of a spinning spacetime propagating forward.

1) 127(11): Evaluation of Plane Wave Tetrads.

The plane wave tetrads are:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) \exp\left(i(\omega t - \kappa Z) \right) \quad -(1)$$

and

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}} \left(\underline{i} + i \underline{j} \right) \exp\left(-i(\omega t - \kappa Z) \right). \quad -(2)$$

In this note it is shown that these tetrads are not those of a flat spacetime. The proof directly evaluates:

$$\omega^a_{\mu b} = - q^{\nu}_{b} \partial_\mu q^a_{\nu}, \quad -(3)$$

using eqns. (1) and (2). In the Maxwell Heaviside theory of the standard model eqns. (1) and (2) are regarded as being written in a flat spacetime, because in MH theory the electromagnetic potentials are:

$$\underline{A}^{(1)} = A^{(0)} \underline{q}^{(1)}, \quad \underline{A}^{(2)} = A^{(0)} \underline{q}^{(2)}. \quad -(4)$$

It will be shown that this procedure of the MH theory is self-inconsistent, because eqns (1) and (2) produce:

$$\omega^a_{\mu b} \neq ? \ 0 \quad -(5)$$

when

$$\Gamma^\lambda_{\mu\nu} = 0. \quad -(6)$$

The correct result is that eqns. (1) and (2) must produce:

$$\Gamma^\lambda_{\mu\nu} \neq 0, \quad \omega^a_{\mu b} \neq 0, \quad -(7)$$

because the plane wave tetrads represent a rigorously non-flat spacetime. This result is an extension of the original interpretation of a in Cartan geometry.

2) Proof

Eqs. (1) and (2) produce:

$$q^{(1)}_{x} = \frac{1}{\sqrt{2}} \exp\left(i\left(\omega t - \kappa Z\right)\right) \quad -(8)$$

$$q^{(1)}_{y} = -\frac{i}{\sqrt{2}} \exp\left(i\left(\omega t - \kappa Z\right)\right). \quad -(9)$$

So:

$$q^{(2)}_{x} = \frac{1}{\sqrt{2}} \exp\left(-i\left(\omega t - \kappa Z\right)\right) \quad -(10)$$

$$q^{(2)}_{y} = \frac{i}{\sqrt{2}} \exp\left(-i\left(\omega t - \kappa Z\right)\right). \quad -(11)$$

The inverse tetrads are found from:

$$q^{(1)}_{x}\, q^{x}_{(1)} = 1 \quad -(12)$$

and so on. Therefore:

$$q^{x}_{(1)} = \sqrt{2} \exp\left(-i\left(\omega t - \kappa Z\right)\right) \quad -(13)$$

$$q^{y}_{(1)} = \sqrt{2}\, i \exp\left(-i\left(\omega t - \kappa Z\right)\right) \quad -(14)$$

so

$$q^{x}_{(2)} = \sqrt{2} \exp\left(i\left(\omega t - \kappa Z\right)\right) \quad -(15)$$

and

$$q^{y}_{(2)} = -\sqrt{2}\, i \exp\left(i\left(\omega t - \kappa Z\right)\right). \quad -(16)$$

Eq. (3) may now be evaluated directly. For example:

$$\partial_0 q^{(1)}_{x} = \frac{1}{c}\frac{\partial}{\partial t} q^{(1)}_{x} = i\frac{\omega}{c} q^{(1)}_{x} \quad -(17)$$

and

$$\partial_3 q^{(1)}_{x} = \frac{\partial q^{(1)}_{x}}{\partial Z} = -i\kappa q^{(1)}_{x}. \quad -(18)$$

Therefore:

3) $\omega^{(1)}{}_{0(1)} = -q^{\sim}_{(1)} \partial_0 q^{(1)}{}_{\sim}$

$= -\left(q^{X}_{(1)} \partial_0 q^{(1)}{}_{X} + q^{Y}_{(1)} \partial_0 q^{(1)}{}_{Y} \right)$

$= -i\frac{\omega}{c}\left(q^{X}_{(1)} q^{(1)}{}_{X} + q^{(1)}{}_{Y} q^{Y}_{(1)} \right)$

$$\boxed{\omega^{(1)}{}_{0(1)} = -2i\frac{\omega}{c}} \quad -(19)$$

Similarly: $\boxed{\omega^{(1)}{}_{Z(1)} = 2i\kappa.}$ $-(20)$

and these two elements are non-zero, and so eqns. (1) and (2) cannot be equations of a flat spacetime in which:

$$\Gamma^{\kappa}{}_{\mu\nu} = 0, \quad \omega^{a}{}_{\mu b} = 0. \quad -(21)$$

The correct procedure is to use the tetrad postulate:

$$D_\mu q^{a}{}_{\nu} = 0 \quad -(22)$$

and to evaluate $\Gamma^{\kappa}{}_{\mu\nu}$ and $\omega^{a}{}_{\mu b}$ from eqns. (1), (2) and (22).

Conclusion

The indices $a = (1), (2), (3)$, when superimposed on X, Y, Z, produce a rigorously non-flat spacetime. In the Maxwell Heaviside theory the potential is an entity superimposed on a flat spacetime. This procedure is not generally covariant as required.

127(12): General Integration of Torsion

In general, the rank three torsion form and torsion tensor may be integrated to give a rank two tensor as follows:

$$T_{\mu\nu} = \int T^a{}_{\mu\nu}\, d\sigma_a = \int T^\kappa{}_{\mu\nu}\, d\sigma_\kappa \quad -(1)$$

where the hypersurfaces are related by:

$$\sigma_\kappa = q^a{}_\kappa\, \sigma_a \quad -(2)$$

$$\sigma_a = V^\kappa{}_a\, \sigma_\kappa \quad -(3)$$

The rule in Cartan geometry is:

$$q^a{}_\kappa\, V^\kappa{}_a = 1. \quad -(4)$$

The angular momentum tensor in $kg\, m^2\, s^{-1}$ is:

$$\boxed{J_{\mu\nu} = \frac{c}{k}\, T_{\mu\nu}} \quad -(5)$$

by hypothesis. The units of $T_{\mu\nu}$ are $m^{-1} m^3 = m^2$, the units of the hypersurface are those of volume, m^3.

The angular momentum in general relativity is therefore a property of a spacetime with torsion. In earlier work the spacetime $((0), (1), (2), (3))$ was superimposed on (ct, X, Y, Z), i.e. the complex circular basis $((1), (2), (3))$ was superimposed on the Cartesian basis (X, Y, Z). These are both $O(3)$ space representations. The time coordinate is the same:

$$(0) = ct. \quad -(6)$$

It may be shown as follows that this

2) superimposition creates a rigorously non-flat spacetime. In a flat spacetime, the tetrad postulate is:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu = 0 \quad -(7)$$

The plane wave tetrads are:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \exp\left(i(\omega t - \kappa Z)\right) \quad -(8)$$

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \exp\left(-i(\omega t - \kappa Z)\right) \quad -(9)$$

and there are also (0) and (3) polarizations. Therefore for example:

$$q_X^{(1)} = \frac{1}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right), \quad -(10)$$

$$q_Y^{(1)} = -\frac{i}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right). \quad -(11)$$

It is seen that X and Y (1) is superimposed on $\exp\left(i(\omega t - \kappa Z)\right)$. The phase with the phase has the effect of driving the coordinate system itself forward in a helix around the Z axis. So there are results such as:

$$\partial_0 q_X^{(1)} = \frac{i\omega}{c} q_X^{(1)} \neq 0 \quad -(12)$$

and so on. Q.E.D. So the spacetime is not a flat spacetime. This is an important distinction between special and general relativity. The Minkowski spacetime of special relativity is a flat spacetime, and this is not a self-consistent description of electromagnetism.

3) Therefore there exist torsion tensors such as:

$$T^{(1)}_{\mu\nu} = \partial_\mu q^{(1)}_\nu - \partial_\nu q^{(1)}_\mu + \omega_{\mu b}\, q^{b}_{\nu} - \omega_{\nu b}\, q^{(1)b}_{\mu} \quad (13)$$

— (14)

where: $a = (1)$.

This is interpreted as a state of circular polarization: The basic tetrad is defined by

$$V^{(1)} = q^{(1)}_{\mu} V^{\mu} \quad (15)$$

i.e. by the superposition of ((1), (2), (3)) on (X, Y, Z). This is a development of Cartan's original theory, where the tetrad was defined by the index of a tangent spacetime at point P on a base manifold. In general, the tetrad is defined by

$$V^{a} = q^{a}_{\mu} V^{\mu} \quad (16)$$

In general:

$$T_{\mu\nu} = \int T^{a}_{\mu\nu}\, d\sigma_a = \int T^{(0)}_{\mu\nu}\, d\sigma_{(0)} + \int T^{(1)}_{\mu\nu}\, d\sigma_{(1)} + \int T^{(2)}_{\mu\nu}\, d\sigma_{(2)} + \int T^{(3)}_{\mu\nu}\, d\sigma_{(3)} \quad (17)$$

and $J_{\mu\nu} = \frac{c}{k} T_{\mu\nu}$. — (18)

1. 127(13): Some Development of the Time-Like Tetrad.

The torsion form and torsion tensor are related by:

$$T^{\kappa}_{\mu\nu} = q^{\kappa}_{a} T^{a}_{\mu\nu} \quad -(1)$$

and

$$T^{a}_{\mu\nu} = q^{a}_{\kappa} T^{\kappa}_{\mu\nu}. \quad -(2)$$

The normalization condition is:

$$q^{a}_{\kappa} q^{\kappa}_{a} = 1 \quad -(3)$$

and the tetrad is defined by:

$$V^{a} = q^{a}_{\mu} V^{\mu}. \quad -(4)$$

Therefore there are results such as:

$$V^{(0)} = q^{(0)}_{\mu} V^{\mu} = q^{(0)}_{0} V^{0} + q^{(0)}_{1} V^{1} + q^{(0)}_{2} V^{2} + q^{(0)}_{3} V^{3}. \quad -(5)$$

Plane Wave Tetrads

In this case:

$$q^{(0)}_{0} = 1, \; q^{(0)}_{1} = q^{(0)}_{2} = q^{(0)}_{3} = 0. \quad -(6)$$

The tetrad postulate becomes:

$$D_{\mu} q^{a}_{\nu} = \partial_{\mu} q^{a}_{\nu} + \omega^{a}_{\mu b} q^{b}_{\nu} - \Gamma^{\lambda}_{\mu\nu} q^{a}_{\lambda} = 0 \quad -(7)$$

with:

$$\partial_{\mu} q^{a}_{\nu} = 0. \quad -(8)$$

Therefore:

$$\omega^{a}_{\mu b} q^{b}_{\nu} = \Gamma^{\lambda}_{\mu\nu} q^{a}_{\lambda}, \quad -(9)$$

with:

$$a = (0). \quad -(10)$$

2.) From eq. (6) it is seen that:

$$b = (0), \; \nu = 0, \; \lambda = 0, \quad - (11)$$

so:

$$\boxed{\omega^{(0)}_{\mu(0)} = \Gamma^{(0)}_{\mu 0}} \quad - (12)$$

For the plane wave tetrad the spin and gamma connections are the same, as taking the index (0) the other taking the index 0. This means that one frame is spinning and translating with respect to the other.

Plane Wave Fields and Potentials

The fields are described by torsion tensors or torsion forms. If we focus attention on the transverse polarizations (1) and (2), then the torsion forms are of the type:

$$T^{(1)}_{\mu\nu} = \partial_\mu q^{(1)}_\nu - \partial_\nu q^{(1)}_\mu + \omega^{(1)}_{\mu b} q^b_\nu - \omega^{(1)}_{\nu b} q^b_\mu \quad - (13)$$

For the (0) polarization:

$$T^{(0)}_{\mu\nu} = \omega^{(0)}_{\mu(0)} - \omega^{(0)}_{\nu(0)} = 0 \quad - (14)$$

because $b = (0), \mu = \nu = 0.$ $\quad - (15)$

The angular momentum of the field resides in the (3) transverse components (1) and (2), and in the transverse component:

$$\boxed{T^{(3)}_{12} = \omega^{(3)}_{1b} q^b_2 - \omega^{(3)}_{2b} q^b_1} \quad - (16)$$

1.

127(14): Fundamental Origin of Angular Momentum.

An expression for the angular momentum tensor can be derived from the anti-symmetric connection as follows. This procedure inter-relates Cartan geometry and angular momentum theory. The tetrad postulate is:

$$D_\mu q^a_{\ \nu} = \partial_\mu q^a_{\ \nu} + \omega^a_{\ \mu b} q^b_{\ \nu} - \Gamma^\lambda_{\mu\nu} q^a_{\ \lambda} = 0. \quad -(1)$$

Now use:

$$q^a_{\ \lambda} q^\lambda_{\ a} = 1 \quad -(2)$$

to obtain:

$$\boxed{\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} = q^\lambda_{\ a}\left(\partial_\mu q^a_{\ \nu} + \omega^a_{\ \mu b} q^b_{\ \nu}\right)} \quad -(3)$$

Cartan geometry may therefore be seen as a method of developing the antisymmetric connection. The torsion tensor is

$$T^\lambda_{\ \mu\nu} = 2\Gamma^\lambda_{\mu\nu} = q^\lambda_{\ a} T^a_{\ \mu\nu} \quad -(4)$$

so the first Cartan structure equation simplifies to:

$$\boxed{T^a_{\ \mu\nu} = 2\left(\partial_\mu q^a_{\ \nu} + \omega^a_{\ \mu b} q^b_{\ \nu}\right)} \quad -(5)$$

so in a flat spacetime:

$$\partial_\mu q^a_{\ \nu} = 0. \quad -(6)$$

Using the methods developed in previous notes for page 127:

2)

$$T_{\mu\nu} = \int T^{a}_{\mu\nu}\, d\sigma_a = \int T^{\kappa}_{\mu\nu}\, d\sigma_\kappa \quad -(7)$$

By hypothesis:

$$J_{\mu\nu} = \frac{c}{k} T_{\mu\nu} \quad -(8)$$

which may be thought of as a correction to the Einstein hypothesis.

Therefore:

$$\boxed{\begin{aligned} J_{\mu\nu} &= 2\frac{c}{k}\int \Gamma^{\kappa}_{\mu\nu}\, d\sigma_\kappa \\ &= 2\frac{c}{k}\int \left(\partial_\mu q^{a}_{\nu} + \omega^{a}_{\mu b} q^{b}_{\nu}\right) d\sigma_a \end{aligned}} \quad -(9)$$

where:

$$\sigma^{a} = q^{a}_{\kappa}\, \sigma^{\kappa} \quad -(10)$$

The hypersurface is

$$\sigma^{\kappa} = (V, \underline{V}) \quad -(11)$$

$$\sigma_{\kappa} = (V, -\underline{V}) \quad -(12)$$

If in eq. (9):

$$\kappa = 0 \quad -(13)$$

we obtain:

$$J_{\mu\nu} = 2\frac{c}{k}\int \Gamma^{0}_{\mu\nu}\, dV \quad -(14)$$

3)

Eq. (14) shows that the angular momentum of spacetime originates in a volume integral over an antisymmetric connection:

$$\Gamma^{0}_{\mu\nu} = -\Gamma^{0}_{\nu\mu} \quad -(15)$$

More generally, eq. (9) must be used. However, eq. (14) shows that angular momentum is a property of non-Minkowski spacetime, specifically a property of spinning spacetime.

Conservation of angular momentum as in page 126 leads to a description of all known orbits without the use of any concept of the standard model. The latter omits torsion and spacetime angular momentum. The latter is defined by:

$$\boxed{J_{\mu\nu} = \frac{c}{k} \int T^{\kappa\tau}_{\ \ \mu\nu}\, d\sigma_{\kappa\tau}} \quad -(16)$$

1) 127(16) : The Electromagnetic ~~and~~ Potential and Field in Cartesian and Circular Polar Vector Notation.

The fundamental ECE hypothesis is that:

$$A^a_\mu = A^{(0)} q^a_\mu, \quad -(1)$$

and

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu}. \quad -(2)$$

Both the potential A^a_μ and the field $F^a_{\mu\nu}$ are vector valued. Therefore they can be written as ~~the~~ vector $\underline{A}_\mu$ and the vector $\underline{F}_{\mu\nu}$. In Cartesian coordinates for example:

$$\underline{A}_\mu = A^0_\mu \underline{e}_0 + A^1_\mu \underline{i} + A^2_\mu \underline{j} + A^3_\mu \underline{k} \quad -(3)$$

and

$$\underline{F}_{\mu\nu} = F^0_{\mu\nu} \underline{e}_0 + F^1_{\mu\nu} \underline{i} + F^2_{\mu\nu} \underline{j} + F^3_{\mu\nu} \underline{k}. \quad -(4)$$

In complex circular coordinates:

$$\underline{A}_\mu = A^{(0)}_\mu \underline{e}^{(0)} + A^{(1)}_\mu \underline{e}^{(1)} + A^{(2)}_\mu \underline{e}^{(2)} + A^{(3)}_\mu \underline{e}^{(3)} \quad -(5)$$

and

$$\underline{F}_{\mu\nu} = F^{(0)}_{\mu\nu} \underline{e}^{(0)} + F^{(1)}_{\mu\nu} \underline{e}^{(1)} + F^{(2)}_{\mu\nu} \underline{e}^{(2)} + F^{(3)}_{\mu\nu} \underline{e}^{(3)} \quad -(6)$$

In a generally covariant unified field theory the components of $\underline{A}_\mu$ and $\underline{F}_{\mu\nu}$ are defined as densities. Similarly, the angular energy / momentum density is also vector valued:

d) and in the complex circular basis is:

$$\underline{J}_{\mu\nu} = J_{\mu\nu}^{(0)} \underline{e}^{(0)} + J_{\mu\nu}^{(1)} \underline{e}^{(1)} + J_{\mu\nu}^{(2)} \underline{e}^{(2)} + J_{\mu\nu}^{(3)} \underline{e}^{(3)} \quad -(7)$$

For each component, the angular energy / momentum tensor is:

$$J_{\mu\nu} = \int \underline{J}_{\mu\nu} \cdot d\underline{V} \quad -(8)$$

$$= \int J_{\mu\nu}^{a} \, d\sigma_a$$

where:

$$\sigma_a = (V, -\underline{V}). \quad -(9)$$

<u>Electric and Magnetic Fields</u>

In general, the electric field density is: $-(10)$

$$\underline{E}_{\mu\nu} = E_{\mu\nu}^{0} \underline{e}_{0} + E_{\mu\nu}^{1} \underline{i} + E_{\mu\nu}^{2} \underline{j} + E_{\mu\nu}^{3} \underline{k}$$

and the magnetic field density is: $-(11)$

$$\underline{B}_{\mu\nu} = B_{\mu\nu}^{0} \underline{e}_{0} + B_{\mu\nu}^{1} \underline{i} + B_{\mu\nu}^{2} \underline{j} + B_{\mu\nu}^{3} \underline{k}.$$

What are usually known as electric and magnetic fields are integrals over these field densities. For the radiated electromagnetic field it is known by experiment that the two transverse components are components of the complex circular basis, whose unit vectors are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \quad -(12)$$

and

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \quad -(13)$$

$$\underline{e}^{(3)} = \underline{k}, \quad -(14)$$

$$\underline{e}^{(0)} = \underline{e}_0. \quad -(15)$$

The radiated electric and magnetic fields are therefore:

$$\underline{E}_{\mu\nu} = E_{\mu\nu}^{(0)}\underline{e}^{(0)} + E_{\mu\nu}^{(1)}\underline{i} + E_{\mu\nu}^{(2)}\underline{j} + E_{\mu\nu}^{(3)}\underline{k} \quad -(16)$$

and

$$\underline{B}_{\mu\nu} = B_{\mu\nu}^{(0)}\underline{e}^{(0)} + B_{\mu\nu}^{(1)}\underline{i} + B_{\mu\nu}^{(2)}\underline{j} + B_{\mu\nu}^{(3)}\underline{k} \quad -(17)$$

The components are the timelike $E_{\mu\nu}^{(0)}$ and $B_{\mu\nu}^{(0)}$, the longitudinal $E_{\mu\nu}^{(3)}$ and $B_{\mu\nu}^{(3)}$, and the transverse $E_{\mu\nu}^{(1)} = E_{\mu\nu}^{(2)*}$ and $B_{\mu\nu}^{(1)} = B_{\mu\nu}^{(2)*}$. Similarly, the electromagnetic potential density has four components: $A_\mu^{(0)}$, $A_\mu^{(1)} = A_\mu^{(2)*}$ and $A_\mu^{(3)}$. For example:

$$\boxed{\underline{B}_{12} = B_{12}^{(3)}\underline{k} = \underline{B}^{(3)}} \quad -(17)$$