1. 125(1): Properties of the Logarithmic Spiral.
(Ref. www.2dcurves.com/spiral/spirallo.html)

The logarithmic spiral is:

$$r = r_0 \exp(a\theta). \quad —(1)$$

It was first studied by Rene Descartes in 1638, and developed by Jakob Bernoulli (1654 – 1705). The curve is identical with its own caustic, evolute, inverse, involute, orthoptic, pedal and radial. The radius of curvature is equal to the reciprocal of the curvature is equal to the arc length. When a log spiral rolls over a line, the path of end point a the spiral is a line. The multiplication of the log spiral is equivalent to rotation. The length from the origin to a point $P(r_1, \theta_1)$ is $r_1 \sec b$ where $a = \cot b$.

The spiral occurs in nature in such things as the Nautilus shell, a hurricane and whirlpool galaxy. The force that makes a point move in a log spiral is proportional to $1/r^3$! A charged particle moving perpendicular to a uniform magnetic field forms a logarithmic spiral. Therefore a mass moving perpendicular to a uniform gravito magnetic field moves in a log spiral.

1) Constant spacetime torsion produces a uniform gravitomagnetic field.

2) It is also of interest that rotation is equivalent to multiplication of the logarithmic spiral. This is a fundamental mathematical property.

2)

The Super-Spiral is:

$$r = e^{f\theta}\left(|\cos(d\theta)|^{a} + |\sin(d\theta)|^{b}\right)^{c} \quad -(2)$$

and produces many spiral-like pattern which could account for irregularities in a whirlpool galaxy. It is necessary to find the force law due to a super-spiral. The log spiral is described by the equation:

$$\frac{dy}{dx} = \frac{\tan b + y/x}{1 - \frac{y}{x}\tan b} \quad -(3)$$

All these equations could be used for animations.

The angular momentum of a star moving on a log spiral is:

$$J = \frac{mvr}{(1+\alpha^2)^{1/2}} \quad -(4)$$

If:

$$\alpha \rightarrow 0 \quad -(5)$$

this becomes the angular momentum of a circle:

$$J \rightarrow mvr \quad -(6)$$

and the spiral looks like a slowly increasing circle. This could model the core of a whirlpool galaxy.

In the opposite limit:

$$\alpha \to \infty \qquad -(7)$$

then:

$$J \to 0 \qquad -(8)$$

and the spiral begins to look like a straight line:

$\alpha \to 0$ — Central core

$\alpha \to \infty$ — outer arms

The spacetime torsion is integrated over volume to give the angular momentum.

For a given α, J is conserved. This is a conservative, or non-dissipative system in which angular momentum and total energy is conserved. However, if α gradually changes, the angular momentum may dissipate. In this case:

$$J(r) = \frac{mvr}{\left(1+\alpha(r)^2\right)^{1/2}} \qquad -(9)$$

Experimentally:

$$\boxed{mv = \frac{1}{r} J(r)\left(1+\alpha(r)^2\right)^{1/2} = \text{constant}} \qquad -(10)$$

1) 125(2): Resonance Structures in the Equation of the Spiral.

Start with the Euler Lagrange equation of a particle moving in a plane subject to the force law $F(r)$:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{J^2} F(r) \quad -(1)$$

where J is a constant. Let:

$$-\frac{mr^2}{J^2} F(r) = \cos(\beta\theta) \quad -(2)$$

i.e.

$$F(r) = -\frac{J^2}{mr^2}\cos(\beta\theta), \quad -(3)$$

then eq. (1) becomes an Euler Bernoulli resonance equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \cos(\beta\theta). \quad -(4)$$

The solution of eq. (4) is:

$$\frac{1}{r} = \frac{\cos(\beta\theta)}{1-\beta^2} \quad -(5)$$

and when:

$$\boxed{\beta = 1, \quad r \to 0} \quad -(6)$$

Check

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\beta\frac{\sin(\beta\theta)}{1-\beta^2} \quad ✓$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\beta^2\frac{\cos(\beta\theta)}{1-\beta^2}$$

2)

Eq. (6) means that when:

$$F(r) = -\frac{J^2}{mr^3}\cos\beta\theta \quad -(7)$$

then:

$$\frac{1}{r} = \frac{\cos(\beta\theta)}{\beta^2} \to \infty \quad -(8)$$

when:

$$\beta = 1 \quad -(9)$$

The potential energy corresponding to eq. (7) is:

$$U(r) = -\frac{J^2}{mr}\cos(\beta\theta) \quad -(10)$$

The Lagrangian is:

$$\mathcal{L} = T - U \quad -(11)$$

where:

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad -(12)$$

$$= \frac{1}{2}mv^2$$

and the total energy is:

$$E = T + U \quad -(13)$$

The Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(14)$$

$$J = mr\dot{\theta} = \text{constant} \quad -(15)$$

3) Eq. (1) is found from eq. (14) as follows, viz:

$$u = 1/r \quad -(16)$$

Firstly:

$$\frac{du}{d\theta} = -\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\theta} = -\frac{1}{r^2}\frac{\dot r}{\dot\theta} \quad -(17)$$

From eq. (15):

$$\dot\theta = \frac{J}{mr^2}, \quad \frac{du}{d\theta} = -\frac{m}{J}\dot r \quad -(18)$$

Therefore:

$$\frac{d^2u}{d\theta^2} = \frac{d}{d\theta}\left(-\frac{m}{J}\dot r\right) = \frac{dt}{d\theta}\frac{d}{dt}\left(-\frac{m}{J}\dot r\right) = -\frac{m}{J\dot\theta}\ddot r = -\frac{m^2 r^2}{J^2}\ddot r \quad -(19)$$

So:

$$\ddot r = -\frac{J^2}{m^2}u^2\frac{d^2u}{d\theta^2}, \quad r\dot\theta^2 = \frac{J^2}{m^2}u^3 \quad -(20)$$

Finally:

$$F(r) = -\frac{\partial U}{\partial r} = m\left(\ddot r - r\dot\theta^2\right) \quad -(21)$$

So from eqs. (10) to (21):

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{J^2 u^2}F(u) \quad -(22)$$

which is eq. (1), QED.

Here:

$$\underline{v} = \dot r\,\underline{e}_r + r\dot\theta\,\underline{e}_\theta \quad -(23)$$

$$v^2 = \dot r^2 + r^2\dot\theta^2 \quad -(24)$$

$$\underline{v} = v_x\,\underline{i} + v_y\,\underline{j} \quad -(25)$$

$$v^2 = v_x^2 + v_y^2. \quad -(26)$$

4.)

For this system the Euler Lagrange equation:

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = -U_0(r)\,\beta \sin(\beta\theta) \quad (27)$$

$$= \frac{dJ}{dt}$$

if $U(\ ) = U_0 \cos \beta\theta$ — (28)

Here $J = \dfrac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$ (29)

So there exists the torque

$$\boxed{T_Q = \frac{dJ}{dt} = -\beta U_0(r) \sin(\beta\theta)} \quad (30)$$

If θ increases from 0 to $360°$ the average torque is zero:

$$\langle T_Q \rangle = \left\langle \frac{dJ}{dt} \right\rangle = 0 \quad (31)$$

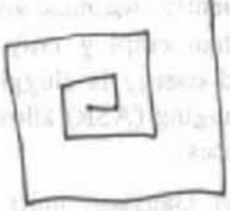
Thus:

1. 125(3): Force Law for a Super-spiral Orbit.

The super-spiral is defined by:

$$r = e^{f\theta}\left(|\cos(d\theta)|^{a} + |\sin(d\theta)|^{b}\right)^{c} \quad -(1)$$

Fig (1)



$a = b = 5,\ c = -1/5,$
$d = 5/2,\ f = 1/5$

$a = b = 100,$
$c = -0.01,$
$d = 1,\ f = 1/5$

This can be used to model the irregularities and pattern observed in a whirlpool galaxy. To find the force law that generates an orbit of type (1), we use the Euler Lagrange equation for planar motion in the form:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2}F(r) \quad -(2)$$

From eq. (1):

$$\frac{1}{r} = \exp(-f\theta)\, f_1(\theta) \quad -(3)$$

where:

$$f_1(\theta) = \left(|\cos(d\theta)|^{a} + |\sin(d\theta)|^{b}\right)^{-c} \quad -(4)$$

Therefore:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -f e^{-f\theta} f_1(\theta) + e^{-f\theta} f_1'(\theta) \quad -(5)$$

2.)

Here:

$$f_1'(\theta) = \frac{d}{d\theta}\left(|\cos(d\theta)|^a + |\sin(d\theta)|^b\right)^{-c} \quad -(6)$$

$$= \left(\frac{df_2(\theta)}{d\theta}\right) f_1(\theta)^{-(c+1)} \quad -(7)$$

where:

$$f_2(\theta) = |\cos(d\theta)|^a + |\sin(d\theta)|^b \quad -(8)$$

We now use Maxima to evaluate $df_2(\theta) / d\theta$.

So:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) e^{-\int f\, d\theta} f_1(\theta)\left(-f + \frac{df_2}{d\theta} f_1^{-1}(\theta)\right)$$

$$= \frac{1}{r}\left(\frac{df_2(\theta)}{d\theta} f_1^{-1}(\theta) - f\right) \quad -(9)$$

$$\boxed{\frac{d}{d\theta}\left(\frac{1}{r}\right) = \frac{1}{r} f_3(\theta)} \quad -(10)$$

where:

$$f_3(\theta) = \frac{df_2(\theta)}{d\theta} f_1^{-1}(\theta) - f \quad -(11)$$

Therefore

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = f_3(\theta)\frac{d}{d\theta}\left(\frac{1}{r}\right) + \frac{1}{r}\frac{df_3(\theta)}{d\theta}$$

$$= f_3^2(\theta)\frac{1}{r} + \frac{1}{r}\frac{df_3(\theta)}{d\theta} \quad -(12)$$

3) Therefore:

$$\boxed{\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = f_4(\theta)\,\frac{1}{r}} \quad -(13)$$

where:

$$f_4(\theta) = f_3^2(\theta) + \frac{d f_3(\theta)}{d\theta} \quad -(14)$$

Therefore:

$$\boxed{F(r) = -\frac{J^2}{m r^3}\left(1 + f_4(\theta)\right)} \quad -(15)$$

This is an inverse cubed force law modulated by the function $f_4(\theta)$ which can be calculated from Maxima. In eq. (15) the mean square angular momentum can be regarded as constant and directed perpendicular to the plane of the paper in fig. (1). So:

$$F(r) = -\frac{\langle J^2 \rangle}{m r^3}\left(1 + f_4(\theta)\right), \quad -(16)$$

$$\langle J^2 \rangle = \text{constant}$$

1) 125(4) : Central Motion is Motion in a Time Independent Spacetime Torsion.

Central motion is traditionally described as the motion due to an attractive force between two particles. The potential energy of attraction depends only on the distance of a particle from the force centre, defined by the reduced mass:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad -(1)$$

Rotation of the system about any fixed axis through the centre of force cannot affect the equation of motion. Under these circumstances the angular momentum is conserved:

$$\boxed{\frac{d\underline{J}}{dt} = 0}, \quad -(2)$$

$$\underline{J} = \underline{r} \times \underline{p}. \quad -(3)$$

However:

$$\boxed{\underline{\nabla} \cdot \underline{J} \neq 0.} \quad -(4)$$

This view is entirely equivalent to considering a spacetime torsion that does not change with time. Such a torsion can be integrated over volume to produce equation (2). It follows that a time-independent, i.e. conserved, torsion produces an attraction potential that is a function only of r.

This is an important new theorem because

2) Orbital theory of any kind is due to a conserved torsion. The conserved torsion of spacetime is responsible for the orbit and also for any force of attraction that depends only on r. This is a physical insight to the workings of nature produced by considering torsion. Thus universal gravitation is universal torsion. We have already deduced this from ECE field theory. The mathematics of the classical theory remain the same but became part of a unified field theoretical approach, ECE theory.

Therefore the lagrangian is:

$$\mathcal{L} = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r), \quad (5)$$

and the Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = 0 \quad (6)$$

and

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \neq 0 \quad (7)$$

Eq. (6) defines the angular momentum:

$$J = \mu r^2 \dot{\theta} \quad (8)$$

and eq (7) can be rewritten as:

3)

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{J^2} F(r) \quad \text{— (9)}$$

where:

$$F = -\frac{\partial U}{\partial r} \quad \text{— (10)}$$

In general it is possible to develop $U(r)$ as:

$$U(r) = \left(\frac{k_1}{r} + \frac{k_2}{r^2} + \cdots + \frac{k_n}{r^n}\right) \quad \text{— (11)}$$

so

$$F(r) = \left(\frac{k_1}{r^2} + \frac{2k_2}{r^3} + \cdots + \frac{n k_n}{r^{n+1}}\right) \quad \text{— (12)}$$

All these force laws are due to conserved torsion, non-relativistic and relativistic. Some examples are given as follows.

1) Newton / Kepler

Here

$$F = \frac{k_1}{r^2}, \quad U = -\frac{k_1}{r} \quad \text{— (13)}$$

and

$$\frac{1}{r} = \frac{1}{d}\left(1 + \epsilon \cos\theta\right). \quad \text{— (14)}$$

In this case conserved torsion produces universal gravitation, a major new insight.

2) Whirlpool Galaxy

Here:

$$F = -\frac{J^2}{\mu r^3}\left(1 + \alpha\right) \quad \text{— (15)}$$

$$U = -\frac{J^2}{2 m r^2}\left(1 + \alpha\right). \quad \text{— (16)}$$

and: $r = r_0 \exp(\alpha\theta)$ — (17)

In this case conserved torsion produces the whirlpool galaxy's arms.

3) The Precessing Elliptical Orbit (Relativistic Kepler Problem)

Here: $F(r) = -\frac{k}{r^2} - \frac{\lambda}{r^3}$ — (18)

All the features of relativistic orbits are produced by conserved torsion.

4) Other Orbits

a) The spiral orbit: $r = r_0 \theta^2$ — (19)

is produced by: $F(r) = -\frac{J^2}{m}\left(\frac{6 r_0}{r^4} + \frac{1}{r^3}\right)$ — (20)

b) If $F(r) = -\frac{k}{r^5}$ — (21)

the orbit is circular and passes through the force centre.

All features of all planar orbits are produced by conserved torsion.

125(5): ECE Equation in Terms of Angular Momentum

It may be proven as follows that the ECE equations of motion of dynamics and electrodynamics are angular momentum equations. The basic ECE hypothesis is:

$$F^{\kappa\mu\nu} = A^{(0)} T^{\kappa\mu\nu} \quad -(1)$$

where $F^{\kappa\mu\nu}$ is the electromagnetic field, $T^{\kappa\mu\nu}$ the torsion, and $A^{(0)}$ is the potential magnitude. Here $cA^{(0)}$ has the units of volts. If we consider the Einstein field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad -(2)$$

it is seen that k has the units of $m\,kgm^{-1}$, and the left hand side has the units of m^{-2}. So $T_{\mu\nu}$ has the units of $kgm\,m^{-3}$. This means that in field theory $T^{\mu\nu}$ has the units of linear momentum divided by cV, where V is a volume, and $J^{\kappa\mu\nu}$ has the units of angular momentum ($kgm\,m^2\,s^{-1}$) divided by cV. These are the units in field theory. In these non-SI units:

$$\boxed{T^{\kappa\mu\nu} = k J^{\kappa\mu\nu}} \quad -(3)$$

Now define:

$$T^{\mu\nu} = \int T^{0\mu\nu}\, dV = V T^{0\mu\nu} \quad -(4)$$

$$J^{\mu\nu} = \int J^{0\mu\nu}\, dV = V J^{0\mu\nu} \quad -(5)$$

The volume V is taken to be a fixed volume which does not fluctuate with time. This is the basis in field theory of the derivation of conservation of total angular momentum of a dynamically conservative system, contained within the volume V. Since V is a constant:

$$D_\mu \left(V T^{0\mu\nu} \right) = D_\mu T^{\mu\nu} = V D_\mu T^{0\mu\nu} \quad -(6)$$

2) Similarly:

$$R_{\mu}{}^{\mu\nu} = \int_0^V R^{\circ}_{\mu}{}^{\mu\nu}\, dV = V R^{\circ}_{\mu}{}^{\mu\nu} \quad (7)$$

Therefore:

$$\partial_\mu T^{\mu\nu} = R_{\mu}{}^{\mu\nu} \quad (8)$$

or

$$\boxed{\partial_\mu T^{\mu\nu} = J^{\nu}} \quad (9)$$

The dual is:

$$\boxed{\partial_\mu \tilde{T}^{\mu\nu} = \tilde{J}^{\nu}} \quad (10)$$

Here:

$$\boxed{T^{\mu\nu} = \frac{k}{c} J^{\mu\nu}} \quad (11)$$

so eqns. (9) and (10) are equations of angular momentum,

QED.

It follows that:

$$F^{\mu\nu} = \frac{A^{(0)}}{V} T^{\mu\nu} \quad (12)$$

Therefore:

$$F^{\mu\nu} = A^{(0)} \frac{k}{cV} J^{\mu\nu}. \quad (13)$$

The magnetic flux density is:

$$\boxed{B^{\mu\nu} = A^{(0)} \frac{k}{cV} J^{\mu\nu}} \quad (14)$$

and the electric field strength is:

$$\boxed{E^{\mu\nu} = A^{(0)} \frac{k}{V} J^{\mu\nu}} \quad (15)$$

3) In eq. (15) the units of $E^{\mu\nu}$ are volts m^{-1}. The units of $cA^{(0)}$ are volts. So the RHS is:

$$A^{(0)} \frac{k}{V} J^{\mu\nu} = \text{volt}\, s\, m^{-1}\, m\, kg\, m^{-1}\, \frac{kg\, m^2\, s^{-1}}{m^3}$$

$$= \text{volt}\, m^{-1}$$

In S.I. units $B^{\mu\nu}$ is c times less than $E^{\mu\nu}$, so the units of $B^{\mu\nu}$ are volt m^{-1} s m^{-1} = volt s m^{-2} = tesla

Electrodynamics

If it is assumed that there is no magnetic monopole, (that $\tilde{j}^{\mu}$ is zero) then eqs. (8) and (10) become:

$$\nabla \cdot \mathbf{B} = 0 ; \quad \nabla \times \mathbf{E} + \partial \mathbf{B} / \partial t = \mathbf{0} \quad -(16)$$

and:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 ; \quad \nabla \times \mathbf{B} - (1/c^2)\, \partial \mathbf{E} / \partial t = \mu_0 \mathbf{J} \quad -(17)$$

It is seen from eqs. (14) and (15) that these are angular momentum equations. Eqs. (16) are:

$$\boxed{\partial_\mu \tilde{J}^{\mu\nu} = 0} \quad -(18)$$

and eqs. (17) are:

$$\boxed{\partial_\mu J^{\mu\nu} = \frac{kJ}{c}} \quad -(19)$$

In ECE theory both eqs. (18) and (19) are used in the field theory of dynamics & electrodynamics.

1) 125(6): ECE Dynamical Equations

These are:

$$\underline{\nabla} \cdot \underline{L} = \frac{1}{2} c V \rho_m \quad — (1)$$

$$\underline{\nabla} \times \underline{S} - \frac{1}{c} \frac{\partial \underline{L}}{\partial t} = \frac{1}{2} V \underline{J}_m \quad — (2)$$

$$\underline{\nabla} \cdot \underline{S} = \frac{1}{2} c V \rho_m \quad — (3)$$

$$\underline{\nabla} \times \underline{L} + \frac{1}{c} \frac{\partial \underline{S}}{\partial t} = \frac{1}{2} V \underline{J}_m \quad — (4)$$

$$j^\mu = \left(\rho_m, \underline{J}_m / c \right) \quad — (5)$$

These will be proven in the next note. Here:

$\underline{L}$ = orbital angular momentum of spacetime

$\underline{S}$ = spin angular momentum of spacetime

ρ_m = mass density

$\underline{J}_m$ = mass-current density

V = volume of the conservative system

Eq. (1) is equivalent to the Coulomb law

Eq. (2) " " " AM law

Eq. (3) " " " Gauss law

Eq. (4) " " " Faraday law

1) 125(7): ECE Field Equations in Terms of Angular Momentum

It has been shown in previous notes that the angular momentum structure of the field equation is:

$$\partial_\mu \tilde{J}^{\mu\nu} = \tilde{j}^{\nu} \quad -(1)$$

$$\partial_\mu J^{\mu\nu} = j^{\nu} \quad -(2)$$

in direct analogy with the equations of electrodynamics

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(3)$$

$$\partial_\mu F^{\mu\nu} = J^{\nu}/\epsilon_0 \quad -(4)$$

where in eq. (3) it has been assumed that the magnetic monopole is zero. In general, ECE allows a non-zero monopole. The electromagnetic field tensor is:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix} \quad -(5)$$

Therefore:

$$J^{\mu\nu} = \begin{bmatrix} 0 & -L_x & -L_y & -L_z \\ L_x & 0 & -S_z & S_y \\ L_y & S_z & 0 & -S_x \\ L_z & -S_y & S_x & 0 \end{bmatrix} \quad -(6)$$

and consists of orbital (L) and spin (S) angular momentum.

It follows that eq. (2) is:

$$\boxed{\underline{\nabla} \cdot \underline{L} = j^0} \quad -(7)$$

2) and

$$\boxed{\underline{\nabla} \times \underline{S} - \frac{1}{c}\frac{\partial \underline{L}}{\partial t} = \underline{j}} \quad -(8)$$

Here:

$$L_x = J^{10} = -J^{01}, \quad S_z = J^{21} = -J^{12}$$
$$L_y = J^{20} = -J^{02}, \quad S_y = J^{13} = -J^{31}$$
$$L_z = J^{30} = -J^{03}, \quad S_x = J^{32} = -J^{23} \quad -(9)$$

and:

$$\underline{L} = L_x \underline{i} + L_y \underline{j} + L_z \underline{k},$$
$$\underline{S} = S_x \underline{i} + S_y \underline{j} + S_z \underline{k}. \quad -(10)$$

Acceleration due to Gravity ($m s^{-2}$)

This is defined as:

$$\boxed{\underline{g} = \frac{ck}{V}\underline{L}} \quad -(11)$$

Electric Field Strength ($volt\ m^{-1}$)

This is defined as:

$$\boxed{\underline{E} = A^{(0)}\frac{k}{V}\underline{L}} \quad -(12)$$

In previous work eq. (7) has been deduced in the form:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m = c^2(R - \omega T) \quad -(13)$$

where:

$$k = \frac{8\pi G}{c^2} \quad -(14)$$

3) Therefore:

$$\underline{\nabla} \cdot \underline{L} = \frac{1}{2} c V \rho_m . \quad -(15)$$

If it is assumed that:

$$\rho_m = m / V \quad -(16)$$

then:

$$\boxed{\underline{\nabla} \cdot \underline{L} = \frac{1}{2} mc} \quad -(17)$$

Similarly:

$$\boxed{\underline{\nabla} \times \underline{S} - \frac{1}{c} \frac{\partial \underline{L}}{\partial t} = \frac{1}{2} V \underline{J}_m} \quad -(18)$$

where:

$$j^\mu = \left(\rho_m , \underline{J}_m / c \right) . \quad -(19)$$

The dual equation (1) gives:

$$\boxed{\underline{\nabla} \cdot \underline{S} = \frac{1}{2} c V \tilde{\rho}_m} \quad -(20)$$

and

$$\boxed{\underline{\nabla} \times \underline{L} + \frac{1}{c} \frac{\partial \underline{S}}{\partial t} = \frac{1}{2} V \tilde{\underline{J}}_m} \quad -(21)$$

) 125(9): Force Law for Variable $\alpha(r)$

Consider a log spiral orbit where α is r dependent:

$$r = r_0 \exp(\alpha(r)\theta) \quad -(1)$$

so.

$$\frac{1}{r} = \frac{1}{r_0}\exp(-\alpha(r)\theta) \quad -(2)$$

and

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{\alpha'(r)}{r_0}\exp(-\alpha(r)\theta) \quad -(3)$$

with

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{\alpha'(r)^2}{r} \quad -(4)$$

The Euler Lagrange equation is

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{L^2} F(r) \quad -(5).$$

so:

$$F(r) = -\frac{L^2}{\mu r^3}\left(1 + \alpha'(r)^2\right) \quad -(6)$$

The orbital velocity is

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 \quad -(7)$$

$$\boxed{v^2 = \left(\frac{L}{\mu r}\right)^2\left(1 + \alpha'(r)^2\right)} \quad -(8)$$

This is eq. (29) of paper 123, with the $\alpha'(r)$ made r dependent. Thus:

$$1 + \alpha'(r)^2 = r^2\left(\frac{\mu^2 v^2}{L^2}\right) \quad -(9)$$

2) If $v \to v_0$ — (10)

then:

$$\alpha'(r)^2 \to \left(\frac{\mu^2 v_0^2}{L^2}\right) r^2 - 1 \qquad (11)$$

$$\boxed{\alpha'(r) \to \left(\left(\frac{\mu v_0}{L}\right)^2 r^2 - 1\right)^{1/2}} \qquad (12)$$

Therefore:

$$\alpha(r) \to \int \left(\left(\frac{\mu v_0}{L}\right)^2 r^2 - 1\right)^{1/2} dr \qquad (13)$$

If $\left(\frac{\mu v_0}{L}\right)^2 r^2 >> 1$ — (14)

then

$$\boxed{\alpha(r) \to \left(\frac{\mu v_0}{L}\right) \int r \, dr = \left(\frac{\mu v_0}{L}\right) \frac{r^2}{2}} \qquad (15)$$

1) 125(10): Field Potential Relations in Terms of Angular Momentum.

It is possible to write:

$$\underline{g} = \frac{1}{m}\left(-\nabla U - \frac{1}{c}\frac{\partial \underline{U}}{\partial t} + U\underline{\omega} - \omega_0 \underline{U}\right) \quad -(1)$$

where:

$$U^{\mu} = \left(U, \underline{U}\right) \quad -(2)$$

is the four-vector of potential energy. Then, as in previous notes:

$$\underline{L} = \frac{V}{ck}\underline{g} \quad -(3)$$

and

$$\underline{\nabla} \cdot \underline{L} = j^{0} \quad -(4)$$

$$= \frac{1}{2} c V \rho_m . \quad -(5)$$

Similarly:

$$\underline{h} = \frac{1}{m}\left(\underline{\nabla} \times \underline{U} - \underline{\omega} \times \underline{U}\right) \quad (6)$$

where $\underline{h}$ is defined in eq. (6) has having the same units of $\underline{g}$, and so $\underline{h}$ is the gravitational equivalent of $\underline{B}$.

Spin Connection Resonance

Eq. (4) is:

$$\underline{\nabla} \cdot \underline{L} = \frac{V}{ckm}\underline{\nabla} \cdot \left(-\underline{\nabla} U - \frac{1}{c}\frac{\partial \underline{U}}{\partial t} + U\underline{\omega} - \omega_0 \underline{U}\right) \quad -(7)$$

$$= \frac{1}{2} c V \rho_m$$

2)

So:

$$\boxed{\nabla^2 u + \frac{1}{c}\,\underline{\nabla}\cdot\frac{\partial \underline{u}}{\partial t} - u\,\underline{\nabla}\cdot\underline{\omega} + \omega_0\,\underline{\nabla}\cdot\underline{u} = -\frac{1}{2} m c^2 k \rho_m} \quad -(8)$$

As shown in previous work this has a Bernoulli: Euler resonant structure under well defined conditions. This means there is resonance in u and also resonance in $\underline{L}$, because:

$$\underline{L} = \frac{V}{ckm}\left(-\underline{\nabla} u - \frac{1}{c}\frac{\partial \underline{u}}{\partial t} + u\,\underline{\omega} - \omega_0\,\underline{u}\right). \quad -(9)$$

Resonant Initial Condition

This assumes that the initial condition of eq. (8) is the resonant condition. This means that a very small amount of mass density ρ_m (of any kind, e.g. plasma) creates a large amount of U^μ and angular momentum, $\underline{L}$. As evolution proceeds (e.g. in a galaxy), the $\underline{L}$ evolves to a constant, giving the galaxy as we see it "today" (in our terms).

1) A Note 125(11): Improvement of Paper 125.

The Lagrangian is defined by:

$$\mathcal{L} = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r) \quad -(1)$$

with

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(2)$$

so:

$$\mu\left(\ddot{r} - r\dot{\theta}^2\right) = -\frac{\partial U}{\partial r} = F(r), \quad -(3)$$

i.e.

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{J^2}F(r). \quad -(4)$$

Now define the logarithmic spiral orbit

$$r = r_0 \exp\left(\alpha(r)\theta\right) \quad -(5)$$

so:

$$F(r) = -\frac{\partial U}{\partial r} = -\frac{J^2}{\mu r^3}\left(1 + \alpha^2(r)\right) \quad -(6)$$

and

$$U(r) = -\frac{J^2}{2\mu r^2} - \frac{J^2}{\mu}\int \frac{\alpha^2(r)}{r^3}\, dr \quad -(7)$$

The orbital velocity is:

$$v^2 = \frac{J^2}{\mu^2 r^2}\left(1 + \alpha(r)\right) \quad -(8)$$

It is observed that $v \to v_0$ as $r \to \infty$, so:

$$\left(1 + \alpha(r)\right)/r^2 \xrightarrow[r\to\infty]{} \left(1 + x r^2\right)/r^2 = x \quad -(9)$$

i.e.

$$\boxed{\alpha(r) \to x r^2 - 1} \quad -(10)$$

so $\cancel{U(r)}$

$$\alpha^2(r) \xrightarrow[r\to\infty]{} \left(\left(\frac{v_0\mu}{J}\right)^2 r^2 - 1\right)^2$$

$$\to \left(\frac{v_0 \mu r}{J}\right)^4. \quad -(11)$$

2) Therefore in eq. (6):

$$F(r) = -\frac{\partial U}{\partial r} \rightarrow -\frac{J^2}{\mu r^3}\left(1 + \left(\frac{v_0 \mu r}{J}\right)^2\right)$$

$$\boxed{F(r) \xrightarrow[r \to \infty]{} -\frac{J^2}{\mu r^3} - \frac{v_0^2 \mu}{r}} \quad -(12)$$

This is the force needed to give logarithmic spiral orbits and to give a constant v_0 as $r \to \infty$. No dark matter is used and the result (12) is a consideration of a $1/r^3$ and $1/r$ force law.

ie 123(12) : Improvement of Paper 123

We have:
$$F(r) = -\frac{J^2}{\mu r^3}\left(1+\alpha^2(r)\right) \quad -(1)$$

where:
$$1+\alpha(r) = \left(\frac{\mu v r}{J}\right)^2. \quad -(2)$$

So:
$$\alpha(r) \xrightarrow[r \to \infty]{} \left(\frac{\mu v_0 r}{J}\right)^2 \quad -(3)$$

So:
$$\boxed{\begin{aligned} F(r) \xrightarrow[r \to \infty]{} & -\frac{J^2}{\mu r^3}\left(1+\left(\frac{\mu v_0 r}{J}\right)^4\right) \quad -(4) \\ & = -\frac{\mu^3 v_0^4}{J^2}\, r \quad -(5) \end{aligned}}$$

The orbit in general is:
$$r = r_0 \exp\left(\alpha(r)\theta\right) \quad -(6)$$

where
$$\alpha(r) = \left(\frac{\mu v r}{J}\right)^2 - 1 \quad -(7)$$

with
$$v^2 = \frac{J^2}{\mu^2 r^2}\left(1+\alpha(r)\right) \quad -(8)$$

Using the limit (3), then:
$$\boxed{r \to r_0 \exp\left(\left(\frac{\mu v_0 r}{J}\right)^2 \theta\right)} \quad -(9)$$