122(7): Exact Cartan Evans Dual Identity without Assuming Metric Compatibility.

The identity in the base manifold is:

$$D_\mu \tilde{T}^\kappa{}_{\nu\rho} + D_\rho \tilde{T}^\kappa{}_{\mu\nu} + D_\nu \tilde{T}^\kappa{}_{\rho\mu} := \tilde{R}^\kappa{}_{\mu\nu\rho} + \tilde{R}^\kappa{}_{\rho\mu\nu} + \tilde{R}^\kappa{}_{\nu\rho\mu} . \quad -(1)$$

This is an exact identity that makes no assumption other than the tetrad postulate. It does not assume metric compatibility. In the obsolete Einsteinian theory the left hand side is zero through use of a symmetric connection:

$$\Gamma^\kappa_{\mu\nu} = \Gamma^\kappa_{\nu\mu} . \quad -(2)$$

Therefore computer algebra can be used to test the right hand side of the identity.

By definition of the Hodge dual operation:

$$\tilde{R}^\kappa{}_{\mu\nu\rho} = \frac{1}{2} \|g\|^{1/2} \epsilon_{\nu\rho\alpha\beta} R^\kappa{}_\mu{}^{\alpha\beta} \quad -(3)$$

$$\tilde{R}^\kappa{}_{\rho\mu\nu} = \frac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu\alpha\beta} R^\kappa{}_\rho{}^{\alpha\beta} \quad -(4)$$

$$\tilde{R}^\kappa{}_{\nu\rho\mu} = \frac{1}{2} \|g\|^{1/2} \epsilon_{\rho\mu\alpha\beta} R^\kappa{}_\nu{}^{\alpha\beta} \quad -(5)$$

So the test is:

$$\boxed{\epsilon_{\nu\rho\alpha\beta} R^\kappa{}_\mu{}^{\alpha\beta} + \epsilon_{\mu\nu\alpha\beta} R^\kappa{}_\rho{}^{\alpha\beta} + \epsilon_{\rho\mu\alpha\beta} R^\kappa{}_\nu{}^{\alpha\beta} \stackrel{?}{=} 0} \quad -(6)$$

2) In eq. (6):

$$R^{\kappa}{}_{\mu}{}^{\alpha\beta} = g^{\alpha\gamma} g^{\beta\delta} R^{\kappa}{}_{\mu\gamma\delta} \quad -(7)$$

$$R^{\kappa}{}_{\rho}{}^{\alpha\beta} = g^{\alpha\gamma} g^{\beta\delta} R^{\kappa}{}_{\rho\gamma\delta} \quad -(8)$$

$$R^{\kappa}{}_{\nu}{}^{\alpha\beta} = g^{\alpha\gamma} g^{\beta\delta} R^{\kappa}{}_{\nu\gamma\delta} \quad -(9)$$

where:

$$R^{\kappa}{}_{\mu\gamma\delta} = \partial_\gamma \Gamma^{\kappa}_{\delta\mu} - \partial_\delta \Gamma^{\kappa}_{\gamma\mu} + \Gamma^{\kappa}_{\gamma\lambda}\Gamma^{\lambda}_{\delta\mu} - \Gamma^{\kappa}_{\delta\lambda}\Gamma^{\lambda}_{\gamma\mu} \quad -(10)$$

and so on.

Using computer algebra, the Christoffel symbols are evaluated from a metric that is an exact solution of the Einstein field equation. They are used to evaluate the Riemann tensor of eq (10). Indices are raised with metric elements as in eqs. (7) to (9), and the totally anti-symmetric unit tensor in 4-D is used to evaluate the sum (6). The code tests whether eq. (6) is zero or non-zero. The factor $\frac{1}{2}\|g\|^{1/2}$ is a constant scalar premultiplier, and can be left out. Here $\|g\|^{1/2}$ is the square root of the modulus of the determinant of the metric.

3) Example Test

Choose: $\nu = 1, \rho = 2, \mu = 3$ — (11)

and the test reduces to:

$$\frac{1}{2}\left(\epsilon_{12\alpha\beta} R^{\kappa}{}_{\mu}{}^{\alpha\beta} + \epsilon_{31\alpha\beta} R^{\kappa}{}_{\rho}{}^{\alpha\beta} + \epsilon_{23\alpha\beta} R^{\kappa}{}_{\nu}{}^{\alpha\beta}\right)$$

$$=? \ 0 \quad — (12)$$

Using the properties of the totally anti-symmetric unit tensor in four dimensions, eq (12) becomes:

$$\epsilon_{1230} R^{\kappa}{}_{3}{}^{30} + \epsilon_{3120} R^{\kappa}{}_{2}{}^{20} + \epsilon_{2310} R^{\kappa}{}_{1}{}^{10}$$

$$=? \ 0 \quad — (13).$$

From review paper 100, Eq. (D23):

$$\epsilon^{1230} = \epsilon^{3120} = \epsilon^{2310} = -1. \quad — (14)$$

By convention: $\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$ — (15)

so eq. (13) becomes:

$$R^{\kappa}{}_{1}{}^{10} + R^{\kappa}{}_{2}{}^{20} + R^{\kappa}{}_{3}{}^{30} =? \ 0 \quad — (16)$$

i.e. $\boxed{R^{\kappa}{}_{\mu}{}^{\mu\nu} =? \ 0}$ — (17)

where $\nu = 0, \mu = 1, 2, 3$ — (18)

In paper 93 this was found to be <u>non-zero</u> in general for various κ.

1) 122(8): Cartan Evans Dual Identity with the Assumption of Metric Compatibility.

Taking an example of the rigorous identity:

$$D_1 \tilde{T}^{\kappa}{}_{23} + D_3 \tilde{T}^{\kappa}{}_{12} + D_2 \tilde{T}^{\kappa}{}_{31} := \tilde{R}^{\kappa}{}_{123} + \tilde{R}^{\kappa}{}_{312} + \tilde{R}^{\kappa}{}_{231} \quad -(1)$$

it is found that:

$$\tilde{T}^{\kappa}{}_{23} = \|g\|^{1/2} T^{\kappa 10} \quad -(2)$$

$$\tilde{T}^{\kappa}{}_{12} = \|g\|^{1/2} T^{\kappa 30} \quad -(3)$$

$$\tilde{T}^{\kappa}{}_{31} = \|g\|^{1/2} T^{\kappa 20} \quad -(4)$$

using:

$$\tilde{T}^{\kappa}{}_{\mu\nu} = \frac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu\alpha\beta} T^{\kappa\alpha\beta} \quad -(5)$$

Therefore eq. (1) is:

$$D_\mu \left(\|g\|^{1/2} T^{\kappa\mu\nu} \right) := \|g\|^{1/2} R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(6)$$

This is an exact identity which has assumed only the tetrad postulate. Using the Leibnitz Theorem:

$$D_\mu \left(\|g\|^{1/2} T^{\kappa\mu\nu} \right) = \|g\|^{1/2} D_\mu T^{\kappa\mu\nu} + T^{\kappa\mu\nu} D_\mu \left(\|g\|^{1/2} \right) \quad -(7)$$

Assuming metric compatibility:

$$D_\mu \left(\|g\|^{1/2} \right) = 0 \quad -(8)$$

2) Proof of Eq. (8)

If for example we assume for simplicity a diagonal metric:

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ 0 & 0 & 0 & g_{33} \end{bmatrix} \quad -(9)$$

its determinant is:

$$g = |g_{\mu\nu}| = g_{00} \begin{vmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{vmatrix} \quad -(10)$$

$$\boxed{g = g_{00}\, g_{11}\, g_{22}\, g_{33}}$$

Metric compatibility means:

$$D_\mu g_{\nu\rho} = 0 \quad -(11)$$

so:

$$D_\mu (g_{00}\, g_{11}\, g_{22}\, g_{33})$$
$$= g_{11}\, g_{22}\, g_{33}\, D_\mu g_{00} + g_{00}\, D_\mu (g_{11}\, g_{22}\, g_{33}) \quad -(12)$$
$$= 0$$

because:

$$D_\mu g_{00} = D_\mu g_{11} = D_\mu g_{22} = D_\mu g_{33} \quad -(13)$$
$$= 0,$$

Q.E.D.

With this assumption we obtain

$$\boxed{D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu}} \quad — (14)$$

Similarly, the Cartan identity becomes:

$$\boxed{D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu}} \quad — (15)$$

Metric compatibility is very fundamental, and it is difficult to conceive of any situation in physics where it does not hold. Nevertheless I will give some notes next on the origin and meaning of metric compatibility for the sake of completeness.

122(9): Proof of Metric Compatibility

The symmetric metric is defined by:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad -(1)$$

where q^a_μ is the Cartan tetrad and where η_{ab} is the Minkowski metric, diag $(-1, 1, 1, 1)$. Now use the tetrad postulate:

$$D_\nu q^a_\mu = 0 \quad -(2)$$

in the equation:

$$D_\rho g_{\mu\nu} = D_\rho \left(q^a_\mu q^b_\nu \eta_{ab} \right). \quad -(3)$$

Using the Leibniz Theorem:

$$D_\rho \left(q^a_\mu q^b_\nu \eta_{ab} \right) = q^b_\nu D_\rho \left(q^a_\mu \eta_{ab} \right) + q^a_\mu \eta_{ab} D_\rho q^b_\nu$$

$$= q^b_\nu D_\rho \left(q^a_\mu \eta_{ab} \right)$$

$$= q^b_\nu q^a_\mu D_\rho \eta_{ab} + q^b_\nu \eta_{ab} D_\rho q^a_\mu$$

$$= 0. \quad -(4)$$

Therefore

$$\boxed{D_\rho g_{\mu\nu} = 0} \quad -(5)$$

which is the metric compatibility condition, QED.

It follows that:

$$D_\rho \|g\|^{1/2} = 0 \quad -(6)$$

as used in note 122(8).

2)

Therefore we have rigorously proven the Cartan Evans dual identity:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(7)$$

and the Cartan Bianchi identity:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}. \quad -(8)$$

The proofs depend only on the tetrad postulate and on the commutator of covariant derivatives and its Hodge dual commutator. Eqs. (7) and (8) are therefore very fundamental and have two momentous consequences.

1) They signal the end of the Einsteinian era in gravitational physics and cosmology.
2) They are the basis of the ECE equations of dynamics and electrodynamics.

1)
122(10): Simple Demonstration of the Fact that Torsion must be Finite.

The torsion and curvature tensors are defined by:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(1)$$

where:

$$\boxed{[D_\mu, D_\nu] = -[D_\nu, D_\mu]} \quad -(2)$$

so that:

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu} \quad -(3)$$

$$T^\lambda{}_{\mu\nu} = -T^\lambda{}_{\nu\mu}. \quad -(4)$$

Here

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}. \quad -(5)$$

and:

$$\boxed{\Gamma^\lambda{}_{\mu\nu} = -\Gamma^\lambda{}_{\nu\mu}}. \quad -(6)$$

Therefore the gamma connections cannot be symmetric unless they are zero, in which case both $T^\lambda{}_{\mu\nu}$ and $R^\rho{}_{\sigma\mu\nu}$ are zero. Therefore:

$$\boxed{\Gamma^\lambda{}_{\mu\nu} \neq \Gamma^\lambda{}_{\nu\mu}}. \quad -(7)$$

For 107 years it was assumed that:

$$\Gamma^\lambda{}_{\mu\nu} =? \Gamma^\lambda{}_{\nu\mu} \quad -(8)$$

122(11): Developing the Cartan Maurer Structure Equation.

The first Cartan Maurer structure equation defines the torsion form in terms of the Cartan tetrad and the spin connection:

$$T^a = D \wedge q^a := d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(1)$$

in standard notation. In full notation:

$$T^a{}_{\mu\nu} = (D \wedge q^a)_{\mu\nu} \quad -(2)$$

The torsion tensor is defined as:

$$T^\kappa{}_{\mu\nu} = q^\kappa{}_a T^a{}_{\mu\nu} \quad -(3)$$
$$= \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu},$$

with:

$$T^\kappa{}_{\mu\nu} = - T^\kappa{}_{\nu\mu}. \quad -(4)$$

Therefore:

$$\boxed{\Gamma^\kappa{}_{\mu\nu} = - \Gamma^\kappa{}_{\nu\mu}}. \quad -(5)$$

The gamma connection cannot be symmetric. The only possibility is:

$$\boxed{\Gamma^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\nu\mu} = 0} \quad -(6)$$

and therefore the physical sciences must be developed on the basis of torsion.

From the tetrad postulate:

$$\Gamma^\kappa{}_{\mu\nu} = q^\kappa{}_a \left(\partial_\mu q^a{}_\nu + q^b{}_\mu \omega^a{}_{\nu b} \right) -(7)$$

i.e.

$$\Gamma^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu + q^b{}_\mu \omega^a{}_{\nu b} \quad -(8)$$
$$= q^a{}_\kappa \Gamma^\kappa{}_{\mu\nu}$$

2) which may be written as:

$$\boxed{\partial_\mu q^a_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}} \quad -(9)$$

where
$$\omega^a_{\mu\nu} = q^b_\mu \, \omega^a_{\nu b}. \quad -(10)$$

In electrodynamics:

$$\partial_\mu A^a_\nu = A^{(0)} \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(11)$$

and
$$\Box A^a_\nu = A^{(0)} \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(12)$$

$$:= \partial^\mu \partial_\mu A^a_\nu \quad -(13)$$

Eq. (12) is an ECE generalization of the d'Alembert equation, and its structure is:

$$\boxed{\Box A^a_\nu = A^{(0)} \partial^\mu \left(\Gamma^a - \omega^a \right)_{\mu\nu}} \quad -(14)$$

The tetrad postulate itself is:

$$\Box q^a_\nu = \partial^\mu \left(\Gamma^a - \omega^a \right)_{\mu\nu}, \quad -(15)$$

i.e.
$$\boxed{\Box q^a_\nu = R \, q^a_\nu} \quad -(16)$$

where:
$$R = q^\nu_a \, \partial^\mu \left(\Gamma^a - \omega^a \right)_{\mu\nu}$$

<u>Eq. (16)</u> is <u>the ECE Lemma.</u>

3)

The electromagnetic potential A^a_μ therefore obeys the wave equation:

$$\boxed{\Box A^a_\mu = R A^a_\mu} \quad -(17)$$

which naturally quantizes the electromagnetic field. The vacuum in ECE theory is filled with the primordial voltage $cA^{(0)}$, which is observed in the radiative corrections such as the Lamb shift.

The d'Alembertian operator is:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(18)$$

where:

$$\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \quad -(19)$$

so eq. (17) is:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} A^a_\mu - \nabla^2 A^a_\mu = R A^a_\mu \quad -(20)$$

i.e.

$$\boxed{\nabla^2 A^a_\mu + R A^a_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} A^a_\mu} \quad -(21)$$

This is an Euler Bernoulli equation which gives resonance. Its structure is

$$\nabla^2 A^a_\mu + R A^a_\mu = A^{(0)}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} q^a_\mu\right) \quad -(22)$$

At resonance, the potential becomes very large.

4)

The typical Euler Bernoulli equation is:

$$\ddot{x} + \omega_0^2 x = A \cos\omega t \quad -(23)$$

whose particular integral is:

$$x_p(t) = \frac{A}{\omega_0^2 - \omega^2} \cos\omega t \quad -(24)$$

and in which resonance occurs at:

$$\omega_0 = \omega \quad -(25)$$

Eq. (22) has the structure of eq. (23) if:

$$\frac{1}{c^2} \frac{\partial^2 q^a_\mu}{\partial t^2} := q^a_\mu \cos(\underline{\kappa} \cdot \underline{r}) \quad -(26)$$

where:

$$\underset{\sim}{r} = X \underline{i} + Y \underline{j} + Z \underline{k} . \quad -(27)$$

Thus:

$$\boxed{\nabla^2 A^a_\mu + R A^a_\mu = A^{(0)} q^a_\mu \cos(\underline{\kappa} \cdot \underline{r})} \quad -(28)$$

For given indices a and μ, and in one dimension:

$$\frac{\partial^2 A}{\partial Z^2} + RA = A^{(0)} \cos(\kappa Z) \quad -(29)$$

and there is resonance at

$$\boxed{\kappa = R^{1/2}} \quad -(30)$$

) The particular solution of eq. (29) is:

$$\boxed{A = \frac{A^{(0)}}{R - \kappa^2} \cdot \cos \kappa Z} \quad — (31)$$

These resonances can be thought of as transition between quantum states of eq. (17).

Therefore the basic tetrad postulate itself can give rise to resonances in A^a_μ. These give rise to resonances in the electromagnetic field via:

$$F^a_{\mu\nu} = \left(D \wedge A^a \right)_{\mu\nu} \quad — (32)$$

2(12): Galaxy Evolution for Animation

Start with the dual identity:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(1)$$

Expanding the covariant derivative:

$$\partial_\mu T^{\kappa\mu\nu} + \omega^{\kappa}{}_{\mu\lambda} T^{\lambda\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(2)$$

Therefore:

$$\partial_1 T^{010} + \partial_2 T^{020} + \partial_3 T^{030}$$
$$+ \omega^{0}{}_{1\lambda} T^{\lambda 10} + \omega^{0}{}_{2\lambda} T^{\lambda 20} + \omega^{0}{}_{3\lambda} T^{\lambda 30}$$
$$= R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}. \quad -(3)$$

Define:

$$\underline{g} = c^2 \underset{\sim}{T} = c^2 \left(T^{010} \underline{i} + T^{020} \underline{j} + T^{030} \underline{k} \right) \quad -(4)$$

to obtain:

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 (R - \omega T)} \quad -(5)$$

where:

$$R - \omega T := R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}$$
$$- \left(\omega^{0}{}_{1\lambda} T^{\lambda 10} + \omega^{0}{}_{2\lambda} T^{\lambda 20} + \omega^{0}{}_{3\lambda} T^{\lambda 30} \right) \quad -(6)$$

In general, λ in eq. (6) sums from 0 to 3.

2)

The evolution of the galaxy depends on the model taken for the spin connection. Its velocity curve is described in paper 76.

Newtonian Dynamics

In this case:

$$\omega \to 0 \qquad -(7)$$

in eq. (5) and:

$$\underline{\nabla} \cdot \underline{g} = c^2 R = 4\pi G\rho \qquad -(8)$$

This gives Kepler's Laws.

Constant v Dynamics

In this case:

$$R = \omega T \qquad -(9)$$

so

$$\underline{\nabla} \cdot \underline{g} = 0 \qquad -(10)$$

i.e.

$$\boxed{\frac{\partial T^{010}}{\partial X} + \frac{\partial T^{020}}{\partial Y} + \frac{\partial T^{030}}{\partial Z} = 0} \qquad -(11)$$

The angular momentum density tensor is defined by (k = Einstein constant):

$$J^{\kappa\mu\nu} = \frac{1}{k} T^{\kappa\mu\nu} \quad - (12)$$

so eq. (11) means that there is no dependence on distance of the angular momentum density. The angular momentum is the volume integral:

$$J^{\mu\nu} = \int J^{0\mu\nu} d^3x. \quad - (13)$$

Consider the galaxy to be in the X – Y plane, and consider the angular momentum about Z:

$$J_Z = \int J^{030} dV \quad - (14)$$

Then:

$$\underline{J} = J_Z \underline{k}. \quad - (15)$$

= constant

This equation is:

$$J = mrv = \text{constant} \quad - (16)$$

$$= mr^2 \frac{d\theta}{dt}$$

So

$$\frac{d\theta}{dt} = \frac{\text{constant}}{mr^2},$$

$$\theta = \left(\frac{\text{constant}}{m} \int_0^{\tau} dt \right) \cdot \frac{1}{r^2} \quad - (17)$$

$$\boxed{\theta = \left(\frac{\text{constant}\ \tau}{m} \right) \cdot \frac{1}{r^2}} \quad - (18)$$

4) This is the equation of a spiral (cf paper 76).

Intermediate Case

In general the galactic evolution is governed by eq. (5), with:

$$\underline{g} = c^2 k \left(J^{010} \underline{i} + J^{020} \underline{j} + J^{030} \underline{k} \right) \quad -(19)$$

and:

$$\underline{\nabla} \cdot \underline{g} = c^2 (R - \omega T) \quad -(20)$$

For given R and ω, this is a differential equation in J. For simplicity and illustration:

$$\boxed{\frac{\partial J^{030}}{\partial Z} = \left(\frac{R^{0}{}_{3}{}^{30}}{k} - \omega^{0}{}_{30} J^{030} \right)} \quad -(21)$$

Integrate this for a given initial and boundary condition to give J^{030}, the density of the galactic angular momentum about the Z axis. The plane of the galaxy is X-Y. This gives:

$$J = mrv \quad -(22)$$

with varying v.

1) 122(13): Proof of the Antisymmetry of the Connection in General

The commutator of covariant derivatives is anti-symmetric:

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] \quad — (1)$$

It may operate on any tensor in any spacetime in any dimensions. For example:

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho \quad — (2)$$

where R is the curvature tensor and T is the torsion tensor. Eq. is antisymmetric in μ and ν by construction. Therefore:

$$T^\lambda_{\mu\nu} = -T^\lambda_{\nu\mu} \quad — (3)$$

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu} \quad — (4)$$

By construction:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \quad — (5)$$

so:

$$\boxed{\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}} \quad — (6)$$

Q.E.D

It is seen that if:

$$\mu = \nu \quad — (7)$$

then

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\mu\nu} = 0 \quad — (8)$$

$$T^\lambda_{\mu\nu} = -T^\lambda_{\mu\nu} = 0 \quad — (9)$$

$$[D_\mu, D_\nu] = -[D_\mu, D_\nu] = 0 \quad — (10)$$

) Therefore the use of a symmetric connection:

$$\Gamma^{\lambda}_{\mu\nu} = ?\ \Gamma^{\lambda}_{\nu\mu} \quad -(11)$$

is erroneous, because it leads to zero R and T.

<u>Examples</u>

Let:

$$\mu = 0,\ \nu = 1,\ \rho = 2 \quad -(12)$$

in eq. (2). Then:

$$[D_0, D_1] V^2 = R^2{}_{\sigma 01} V^\sigma - T^\lambda{}_{01} D_\lambda V^2 \quad -(13)$$

It follows that:

$$T^\lambda{}_{01} = \Gamma^\lambda{}_{01} - \Gamma^\lambda{}_{10} = -T^\lambda{}_{10} \quad -(14)$$

i.e

$$\Gamma^\lambda{}_{01} = -\Gamma^\lambda{}_{10} \quad -(15)$$

We may now consider the curvature tensor:

$$R^2{}_{\sigma 01} = -R^2{}_{\sigma 10}. \quad -(16)$$

For example:

$$R^2{}_{001} = \partial_0 \Gamma^2{}_{10} - \partial_1 \Gamma^2{}_{00} + \Gamma^2{}_{0\lambda}\Gamma^\lambda{}_{10} - \Gamma^2{}_{1\lambda}\Gamma^\lambda{}_{00} \quad -(17)$$

in which:

$$\partial_0 \Gamma^2{}_{10} = -\partial_1 \Gamma^2{}_{00} = -\partial_0 \Gamma^2{}_{01} = \partial_1 \Gamma^2{}_{00} \quad -(18)$$

i.e

$$\Gamma^2{}_{00} = -\Gamma^2{}_{00} = 0 \quad -(19)$$

$$\Gamma^2{}_{01} = -\Gamma^2{}_{10} \neq 0 \quad -(20)$$

Q.E.D.

3)

Similarly we may consider all other μ and ν indices:

$$\left.\begin{array}{l}\mu = 1,\ \nu = 2;\ \mu = 1,\ \nu = 3, \\ \mu = 0,\ \nu = 3;\ \mu = 0,\ \nu = 2\end{array}\right\} \quad -(21)$$

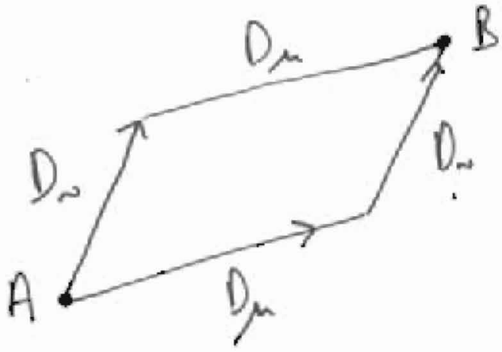
to find that:

$$\boxed{\Gamma^{\lambda}_{00} = \Gamma^{\lambda}_{11} = \Gamma^{\lambda}_{22} = \Gamma^{\lambda}_{33} = 0} \quad -(22)$$

for all λ.

It is seen immediately that any metric of gravitational theory that does not obey eq. (22) is incorrect. Therefore the Einsteinian era is obsolete and gravitational theory must be based on a non-zero torsion as in the ECE equations. In particular, it is <u>incorrect</u> to assert that R is non-zero and T is zero in eq. (2) by use of the <u>incorrect</u> symmetric connection.

The commutator of covariant derivatives arises from the fact that the covariant derivative of a tensor in a given direction measures how much the tensor changes relative to what it would have been if it had been parallel transported. The covariant derivative of a tensor (e.g. the metric $g_{\mu\nu}$) in the direction along which it is parallel transported is zero. The commutator measures the difference between parallel transporting the tensor first one way and then the other versus the opposite ordering.



i.e we go from A to B clockwise, then anti-clockwise. The result is not the same in a general spacetime. It is the same in Minkowski spacetime because:

$$[\partial_\mu, \partial_\nu] = \overbrace{-[\partial_\nu, \partial_\mu]}^{=0} \quad -(23)$$

and all Γ are zero by construction.

For the arbitrary tensor in any spacetime:

$$\begin{aligned}
[D_\rho, D_\sigma] X^{\mu_1 \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} &= -T^\lambda{}_{\rho\sigma} D_\lambda X^{\mu_1 \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} \\
&+ R^{\mu_1}{}_{\lambda\rho\sigma} X^{\lambda \mu_2 \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} + R^{\mu_2}{}_{\lambda\rho\sigma} X^{\mu_1 \lambda \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} + \ldots \\
&- R^\lambda{}_{\nu_1\rho\sigma} X^{\mu_1 \ldots \mu_k}{}_{\lambda \nu_2 \ldots \nu_l} - R^\lambda{}_{\nu_2\rho\sigma} X^{\mu_1 \ldots \mu_k}{}_{\nu_1 \lambda \ldots \nu_l} - \ldots
\end{aligned} \quad -(24)$$

and there is antisymmetry in ρ and σ by construction.

<u>The torsion</u> is <u>always</u> <u>non-zero</u> <u>and</u> <u>is always</u>:

$$T^\lambda{}_{\rho\sigma} = -T^\lambda{}_{\sigma\rho} \quad -(25)$$

Q.E.D.

122(14): Development of the Cartan Maurer Structure Equation.

In its most concise format this is:

$$T = D \wedge q \quad -(1)$$

where T denotes the Cartan torsion and q the Cartan tetrad. Here $D\wedge$ denotes the covariant exterior derivative. When written out more fully:

$$T = d \wedge q + \omega \wedge q \quad -(2)$$

where $d\wedge$ denotes the exterior derivative and ω the spin connection form. In this notation indices have been left out to emphasize the simple basic structure. In ECE theory the electromagnetic potential is defined as:

$$A = A^{(0)} q \quad -(3)$$

and the electromagnetic field as:

$$F = A^{(0)} T \quad -(4)$$

so:

$$F = D \wedge A = d \wedge A + \omega \wedge A. \quad -(5)$$

In the standard model:

$$F = d \wedge A \quad -(6)$$

and the spin connection ω is missing. The reason is that in the standard model the spacetime is Minkowski spacetime with zero connection.

Restoring the indices in eq. (5):

$$F^a_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + (\omega^a{}_b \wedge A^b)_{\mu\nu} \quad -(7)$$

i.e for each μ and ν:

2)

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(8)$$

in the standard notation of differential geometry.

Therefore in ECE theory the electromagnetic field is a mixed index rank three tensor, and the e/m potential is a mixed index rank two tensor. The index a is that of the tangential spacetime at point P in the base manifold. The tangential spacetime is a Minkowski spacetime by construction. In the standard model the tangential spacetime is not considered, and the base manifold is a Minkowski spacetime. During the course of development of ECE theory it has been shown that a may denote a frame ((1), (2), (3)) superimposed on a frame (X, Y, Z). In this case a = (1), (2), (3) denotes polarization. This interpretation is useful for circularly polarized electromagnetic plane waves, which are characterized by tetrads such as:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) \exp \left(i(\omega t - \kappa Z) \right) \quad -(8)$$

in vector format. These tetrads have symmetry:

$$\underline{q}^{(1)} \times \underline{q}^{(2)} = i \underline{q}^{(3)*} \quad -(9)$$

et cyclicum

where:

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}} \left(\underline{i} + i \underline{j} \right) \exp \left(-i(\omega t - \kappa Z) \right) \quad -(10)$$

Elliptical polarization is described by:

$$\underline{v}^{(1)} = \frac{1}{\sqrt{2}} \left(a \underline{i} - i b \underline{j} \right) \exp \left(i(\omega t - \kappa Z) \right) \quad -(11)$$

and in general any electromagnetic wave is a wave of spacetime.

Extending this description to spacetime there is a frame ((0), (1), (2), (3)) superimposed on the frame (ct, X, Y, Z). The components (1), (2) and (3) are space-like and therefore (0) is time-like. The four components may appear both in F^a and in A^a. Therefore there can be time-like waves (scalar potential waves) and space-like waves (vector potential waves). The basis vectors of the complex circular representation are:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) \quad -(12)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} \left(\underline{i} + i \underline{j} \right) \quad -(13)$$

$$\underline{e}^{(3)} = \underline{k} \quad -(14)$$

and there is also a time-like unit vector.

In the next note the tensor and vector formats are developed of the structure equation.

) 122(13): Tensor and Vector Notation for the First Cartan Maurer Structure Equation.

In note 122(4) it was shown that:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b. \quad -(1)$$

In tensor notation this becomes:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu. \quad -(2)$$

Raising indices:

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}{}_b A^{b\nu} - \omega^{a\nu}{}_b A^{b\mu}. \quad -(3)$$

This expression can be simplified using:

$$\omega^{a\mu}{}_b A^{b\nu} = A^{(0)} \omega^{a\mu}{}_b q^{b\nu} = A^{(0)} \omega^{a\mu\nu}, \quad -(4)$$

so:

$$\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right)} \quad -(5)$$

In this format, it is seen clearly that the e/m field contains a term due to the difference of spin connections, and a term due to the partial four derivative of potential.

In gravitational theory, eq. (5) becomes:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu\nu} - \omega^a_{\nu\mu}, \quad -(6)$$

$$T^\kappa_{\mu\nu} = \left(\Gamma^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) q^\kappa_a. \quad -(7)$$

In the polarization interpretation of a, there are fields such as:

$$F^{(1)}_{\mu\nu} = \partial_\mu A^{(1)}_\nu - \partial_\nu A^{(1)}_\mu + A^{(0)}\left(\omega^{(1)}_{\mu\nu} - \omega^{(1)}_{\nu\mu}\right) \quad -(8)$$

2) and so on. The magnetic flux density known as $\underline{B}^{(3)}$ in vector notation is:

$$B^{(3)}_{12} = A^{(0)} \left(\omega^{(3)}_{12} - \omega^{(3)}_{21} \right) \quad -(9)$$

i.e.

$$\boxed{B^{(3)}_{12} = 2A^{(0)} \omega^{(3)}_{12}} \quad -(10)$$

It is seen that the $B^{(3)}_{12}$ field is generated directly by the $\omega^{(3)}_{12}$ spin connection multiplied by the fundamental potential magnitude $A^{(0)}$ of ECE theory. In vector notation:

$$\boxed{\underline{B}^{(3)} = 2A^{(0)} \underline{\omega}^{(3)}} \quad -(11)$$

and is observed experimentally in the inverse Faraday effect. The latter therefore observes the spin connection directly, the spinning of spacetime is observed directly in the inverse Faraday effect.

The electric field in tensor notation is defined by:

$$E^{a\mu\nu} = c \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}_{\ \ b} A^{b\nu} - \omega^{a\nu}_{\ \ b} A^{b\mu} \right) \quad -(12)$$

Specifically:

$$E^{a10} = c \left(\partial^1 A^{a0} - \partial^0 A^{a1} + \omega^{a1}_{\ \ b} A^{b0} - \omega^{a0}_{\ \ b} A^{b1} \right)$$
$$E^{a20} = c \left(\partial^2 A^{a0} - \partial^0 A^{a2} + \omega^{a2}_{\ \ b} A^{b0} - \omega^{a0}_{\ \ b} A^{b2} \right)$$
$$E^{a30} = c \left(\partial^3 A^{a0} - \partial^0 A^{a3} + \omega^{a3}_{\ \ b} A^{b0} - \omega^{a0}_{\ \ b} A^{b3} \right) \quad -(13)$$

In vector notation this is:

$$\underline{E}^a = -\underline{\nabla} A^{a0} - \frac{\partial \underline{A}^a}{\partial t} + A^{b0} \underline{\omega}^a{}_b - \omega^{a0}{}_b \underline{A}^b \qquad -(14)$$

For each polarization index a, and in the absence of the vector potential:

$$\boxed{\underline{E}^a = -\underline{\nabla} \phi^a + \underline{\omega}^a{}_b \phi^b} \qquad -(15)$$

The electric field is defined as:

$$E^a{}_{\mu\nu} = q^a{}_\kappa E^\kappa{}_{\mu\nu} \qquad -(16)$$

or

$$E^{a\mu\nu} = q^a{}_\kappa E^{\kappa\mu\nu} \qquad -(17)$$

For the Coulomb law:

$$\kappa = 0, \mu = 1, \nu = 0 \text{ etc.} \qquad -(18)$$

so

$$E^{a10} = q^a{}_0 E^{010} \text{ etc.} \qquad -(19)$$

The space polarization indices $a = (1), (2), (3)$ have no time-like meaning, so the only possibility in eq. (19) is:

$$E^{010} = q^0{}_0 E^{010}, \qquad -(20)$$

$$q^0{}_0 = \underline{1}$$

22(16): Electric Field of the Coulomb Law in the Cartesian and Complex Circular Basis.

The electric field in the Cartesian basis is:

$$\underline{E} = E_X \underline{i} + E_Y \underline{j} + E_Z \underline{k} \quad -(1)$$
$$= E^{010} \underline{i} + E^{020} \underline{j} + E^{030} \underline{k} \quad -(2)$$

In general, a change of basis is represented by:

$$E^a_{\mu\nu} = q^a_{\ k} E^k_{\mu\nu}. \quad -(3)$$

The complex circular basis is defined by the unit vectors:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i \underline{j}) \quad -(4)$$
$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} (\underline{i} + i \underline{j}) \quad -(5)$$
$$\underline{e}^{(3)} = \underline{k}. \quad -(6)$$

The complete vector field $\underline{E}$ is the same in both representations, and in the complex circular one is:

$$\underline{E} = E^{(1)} \underline{e}^{(1)} + E^{(2)} \underline{e}^{(2)} + E^{(3)} \underline{e}^{(3)} \quad -(7)$$
$$\underline{E}^* = E^{(2)} \underline{e}^{(2)} + E^{(1)} \underline{e}^{(1)} + E^{(3)} \underline{e}^{(3)} \quad -(8)$$

so:
$$2 E^{(1)} E^{(2)} = E_X^2 + E_Y^2 \quad -(9)$$

and
$$E^{(1)} = \frac{1}{\sqrt{2}} (E_X - i E_Y), \quad -(10)$$
$$E^{(2)} = \frac{1}{\sqrt{2}} (E_X + i E_Y). \quad -(11)$$

Using eq. (2), eqns. (10) and (11) may be written so as to define the transformation tetrads as follows:

2)

$$E^{0(1)0} = q^{(1)}_1 E^{010} + q^{(1)}_2 E^{020} \quad -(12)$$

$$E^{0(2)0} = q^{(2)}_1 E^{010} + q^{(2)}_2 E^{020} \quad -(13)$$

i.e.
$$\begin{bmatrix} E^{0(1)0} \\ E^{0(2)0} \end{bmatrix} = \begin{bmatrix} q^{(1)}_1 & q^{(1)}_2 \\ q^{(2)}_1 & q^{(2)}_2 \end{bmatrix} \begin{bmatrix} E^{010} \\ E^{020} \end{bmatrix} \quad -(14)$$

Here:
$$\left. \begin{aligned} q^{(1)}_1 &= \frac{1}{\sqrt{2}}, \quad q^{(1)}_2 = -\frac{i}{\sqrt{2}}, \\ q^{(2)}_1 &= \frac{1}{\sqrt{2}}, \quad q^{(2)}_2 = \frac{i}{\sqrt{2}}. \end{aligned} \right\} \quad -(15)$$

In the base manifold the electric field is the rank three tensor $E^{\kappa\mu\nu}$. In the Coulomb law the index κ is fixed at zero, a is the index ν. The index μ is 1, 2, 3. This choice of indices indicates that there are three components present of orbital torsion, E^{010}, E^{020} and E^{030}. These are proportional to three components J^{010}, J^{020} and J^{030} of orbital angular momentum/angular energy density. If

$$F^{a\mu\nu} = q^a_\kappa F^{\kappa\mu\nu} \quad -(16)$$

and κ is fixed at zero, then:

$$F^{a\mu\nu} = q^a_0 F^{0\mu\nu}. \quad -(17)$$

The indices μ and ν represent the space-like properties of both $F^{a\mu\nu}$ and $F^{\kappa\mu\nu}$ and therefore the only possibility in eq. (17) is the time-like:

$$a = 0 \quad -(18)$$

and

$$q^0_0 = 1. \quad -(19)$$

In general:

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}{}_b A^{b\nu} - \omega^{a\nu}{}_b A^{b\mu} \quad -(20)$$

so:

$$E^{010} = c\left(\partial^1 A^{00} - \partial^0 A^{01} + \omega^{01}{}_b A^{b0} - \omega^{00}{}_b A^{b1}\right)$$
$$E^{020} = c\left(\partial^2 A^{00} - \partial^0 A^{2} + \omega^{02}{}_b A^{b0} - \omega^{00}{}_b A^{b2}\right)$$
$$E^{030} = c\left(\partial^3 A^{00} - \partial^0 A^{3} + \omega^{03}{}_b A^{b0} - \omega^{00}{}_b A^{b3}\right) \quad -(21)$$

Now use:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) \quad -(22)$$

$$\underline{A} = A^{01}\underline{i} + A^{02}\underline{j} + A^{03}\underline{k} \quad -(23)$$

$$\phi = A^{00} \quad -(24)$$

The simplest model of the spin connection is obtained if b is assumed to be 0. In this case the spin connection scalar and vector are:

$$\omega = \omega^{00}{}_0 \quad -(25)$$

$$\underline{\omega} = \omega^{01}{}_0\,\underline{i} + \omega^{02}{}_0\,\underline{j} + \omega^{03}{}_0\,\underline{k} \quad -(26)$$

Thus we obtain the engineering model equations:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} + \phi\,\underline{\omega} - \omega\underline{A} \quad -(27)$$

$$\underline{\nabla}\cdot\underline{E} = \rho/\epsilon_0 \quad -(28)$$

122(17): Simplification and Development of the ECE Engineering Model.

The inhomogeneous field equations of the model are based on:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(1)$$

and the homogeneous equation on:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(2)$$

These equations may be simplified by using:

$$T^{\mu\nu} = \int T^{\kappa\mu\nu} d\sigma_\kappa \quad -(3)$$

i.e. by integrating over a hyper-surface $d\sigma_\kappa$. When:

$$\kappa = 0 \quad -(4)$$

$$T^{\mu\nu} = \int T^{0\mu\nu} d^3x = \int T^{0\mu\nu} dV. \quad -(4)$$

In the same way, the angular momentum tensor is obtained by integrating over the component $J^{0\mu\nu}$ of the canonical angular energy / angular momentum density tensor $J^{\kappa\mu\nu}$, which in ECE theory is proportional to $T^{\kappa\mu\nu}$, the torsion tensor.

Integrating eqns. (1) and (2) uses:

$$T^{\mu\nu} = \int T^{0\mu\nu} dV, \quad R_\mu{}^{\mu\nu} = \int R^0{}_\mu{}^{\mu\nu} dV \quad -(5)$$

$$\tilde{T}^{\mu\nu} = \int \tilde{T}^{0\mu\nu} dV, \quad \tilde{R}_\mu{}^{\mu\nu} = \int \tilde{R}^0{}_\mu{}^{\mu\nu} dV \quad -(6)$$

) to give:

$$D_\mu T^{\mu\nu} = R_\mu{}^{\mu\nu} := J^\nu \quad -(7)$$

$$D_\mu \tilde{T}^{\mu\nu} = \tilde{R}_\mu{}^{\mu\nu} := \tilde{J}^\nu \quad -(8)$$

The covariant derivative in eq. (1) is:

$$D_\mu T^{\kappa\mu\nu} = \partial_\mu T^{\kappa\mu\nu} + \omega^\kappa{}_{\mu\lambda} T^{\lambda\mu\nu} \quad -(9)$$

Denote:

$$\omega_\mu T^{\mu\nu} = \int \omega^0{}_{\mu 0} T^{0\mu\nu} \, dV \quad -(10)$$

then:

$$(\partial_\mu + \omega_\mu) T^{\mu\nu} = J^\nu \quad -(11)$$

$$(\partial_\mu + \omega_\mu) \tilde{T}^{\mu\nu} = \tilde{J}^\nu \quad -(12)$$

i.e.

$$\boxed{\begin{aligned} \partial_\mu T^{\mu\nu} &= J^\nu - \omega_\mu T^{\mu\nu} \\ \partial_\mu \tilde{T}^{\mu\nu} &= \tilde{J}^\nu - \omega_\mu \tilde{T}^{\mu\nu} \end{aligned}} \quad \begin{aligned} -(13) \\ -(14) \end{aligned}$$

Electrodynamics

$$\partial_\mu F^{\mu\nu} = A^{(0)} \left(J^\nu - \omega_\mu T^{\mu\nu} \right) \quad -(15)$$

and

$$\partial_\mu \tilde{F}^{\mu\nu} = A^{(0)} \left(\tilde{J}^\nu - \omega_\mu \tilde{T}^{\mu\nu} \right), \quad -(16)$$

$$D_\mu = \partial_\mu + \omega_\mu \quad -(16a)$$

3) These equations are the generally covariant form of the Maxwell Heaviside equations. They may be written as:

$$\partial_\mu F^{\mu\nu} = A^{(0)} j^\nu \quad -(17)$$

$$\partial_\mu \tilde{F}^{\mu\nu} = A^{(0)} \tilde{j}^\nu \quad -(18)$$

where:

$$j^\nu = R_\mu{}^{\mu\nu} - \omega_\mu T^{\mu\nu} \quad -(19)$$

$$\tilde{j}^\nu = \tilde{R}_\mu{}^{\mu\nu} - \omega_\mu \tilde{T}^{\mu\nu} \quad -(20)$$

If there is no magnetic monopole:

$$\tilde{R}_\mu{}^{\mu\nu} = \omega_\mu \tilde{T}^{\mu\nu} \quad -(21)$$

and:

$$\boxed{\begin{aligned} \partial_\mu F^{\mu\nu} &= A^{(0)} j^\nu \quad -(22) \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0 \quad -(23) \end{aligned}}$$

In vector notation:

$$\boxed{\begin{aligned} &\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \\ &\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \\ &\underline{\nabla} \cdot \underline{B} = 0 \\ &\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \end{aligned}} \quad -(24)$$

which have the same structure as the Maxwell Heaviside equations but which are generally covariant.

122(18): Simplified Derivation of the Field Potential Relation of the ECE Engineering Model.

In general:

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}{}_b A^{b\nu} - \omega^{a\nu}{}_b A^{b\mu} \quad -(1)$$

Now let: $a = 0$ $\quad -(2)$

so: $F^{\mu\nu} = \int F^{0\mu\nu} \, dV$ $\quad -(3)$

and: $\boxed{F^{\mu\nu} = (\partial^\mu + \omega^\mu) A^\nu - (\partial^\nu + \omega^\nu) A^\mu} \quad -(4)$

Electric Field

$$F^{10} = (\partial^1 + \omega^1) A^0 - (\partial^0 + \omega^0) A^1 \quad -(5)$$

etc.

i.e. $\boxed{\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} + \phi \underline{\omega} - \omega \underline{A}} \quad -(6)$

where

$$A^\mu = (c\phi, \underline{A}) \quad -(7)$$

$$\omega^\mu = (\omega, \underline{\omega}) \quad -(8)$$

Magnetic Field

$$F^{12} = (\partial^1 + \omega^1) A^2 - (\partial^2 + \omega^2) A^1 \quad -(9)$$

etc.

$\boxed{\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}} \quad -(10)$

$$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \quad -(11)$$

$$\omega = \omega^0 \quad -(12)$$