1)

Note 119(3): The Frame Invariance Relations under the Lorentz Boost.

Under the Lorentz boost along an axis such as the z axis there are basis vector relations such as:

$$\underline{e}_0' = \underline{e}_0 \cosh\phi - \underline{e}_1 \sinh\phi \quad -(1)$$
$$\underline{e}_1' = -\underline{e}_0 \sinh\phi + \underline{e}_1 \cosh\phi \quad -(2)$$
$$\underline{e}_2' = \underline{e}_2 \quad\quad\quad\quad\quad\quad -(3)$$
$$\underline{e}_3' = \underline{e}_3 \quad\quad\quad\quad\quad\quad -(4)$$

There are therefore $O(3)$ symmetry relations in 4-D spacetime such as:

$$\underline{e}_0' \times \underline{e}_1' = (\underline{e}_0 \cosh\phi - \underline{e}_1 \sinh\phi) \times$$
$$(-\underline{e}_0 \sinh\phi + \underline{e}_1 \cosh\phi)$$
$$= (\underline{e}_0 \times \underline{e}_1)(\cosh^2\phi - \sinh^2\phi)$$
$$\quad\quad\quad\quad\quad\quad\quad\quad -(5)$$
$$= \underline{e}_0 \times \underline{e}_1$$

QED

$$= \underline{e}_2 .$$

In 4-D spacetime there are four unit vectors instead of the usual three in 3-D space. The Lorentz boost involves a change in the time, the Lorentz rotation does not. In the Lorentz group there are three rotation generators

2) and three boost generators. The relation between
generators must be covariant. This is the very
basis of relativity. If one considers a Lorentz
boost along $\underline{k}$, then:

$$\underline{k}' = - \underline{e}_o \sinh\phi + \underline{k}\cosh\phi \quad - (6)$$

$$\underline{i}' = \underline{i} \quad - (7)$$

$$\underline{j}' = \underline{j} \quad - (8)$$

In frame K:

$$\underline{i} \times \underline{j} = \underline{k} \quad - (9)$$

but in frame K':

$$\underline{i}' \times \underline{j}' = \underline{i} \times \underline{j} \neq? \ \underline{k}' \quad - (10)$$

unless:
$$v = 0. \quad - (11)$$

The reason for this is that a Lorentz boost is
independent of a Lorentz rotation. Eq. (9) is
a relation between rotation generators, and as
such is covariant (indeed invariant) only
under Lorentz rotation. The relation (5) on
the other hand is one between boost generators,
and is covariant under the Lorentz boost.
Indeed the boost relations are invariant under the
Lorentz boost.