1) Note 119(2): Invariance of the Frame of Reference under the Rotational Lorentz Transform.

This is an example of Carroll's page 9:

$$V = V^{\mu} e_{\mu} = V^{\mu'} e_{\mu'} \quad -(1)$$

$$= \Lambda^{\mu'}{}_{\mu} V^{\mu} e_{\mu'}$$

$$= V^{\mu} \Lambda^{\mu'}{}_{\mu} e_{\mu'}$$

Therefore:

$$e_{\mu} = \Lambda^{\mu'}{}_{\mu} e_{\mu'} \quad -(2)$$

and

$$\boxed{e_{\mu'} = \left(\Lambda^{\mu'}{}_{\mu}\right)^{-1} e_{\mu}} \quad -(3)$$

For a rotational Lorentz transform about the Z axis:

$$\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad -(4)$$

so

$$\left(\Lambda^{\mu'}{}_{\mu}\right)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad -(5)$$

and

$$V^{\mu'} e_{\mu'} = \Lambda^{\mu'}{}_{\mu} \left(\Lambda^{\mu'}{}_{\mu}\right)^{-1} V^{\mu} e_{\mu}$$

$$= V^{\mu} e_{\mu} \quad -(6)$$

Therefore

$$\begin{bmatrix} \underline{i}' \\ \underline{j}' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(7)$$

2)

i.e.
$$\boxed{\begin{aligned} \underline{i}' &= \underline{i}\cos\theta - \underline{j}\sin\theta \\ \underline{j}' &= \underline{i}\sin\theta + \underline{j}\cos\theta \end{aligned}} \quad -(8)$$

The rotation of a vector with respect to a fixed frame is the same as rotation of the frame with respect to a fixed vector. The $\underline{k}$ vector is fixed:

$$\underline{k}' = \underline{k} \quad -(9)$$

From eq. (8):

$$\underline{i}' \times \underline{j}' = \left(\underline{i}\cos\theta - \underline{j}\sin\theta\right) \times \left(\underline{i}\sin\theta + \underline{j}\cos\theta\right)$$
$$= \left(\sin^2\theta + \cos^2\theta\right) \underline{i} \times \underline{j}$$
$$= \underline{i} \times \underline{j} \quad -(10)$$

Therefore: $\underline{i}' \times \underline{j}' = \underline{k}' \quad -(11)$

follows from the rotational Lorentz transform of

$$\underline{i} \times \underline{j} = \underline{k} \quad -(12)$$

QED. The B Cyclic Theorem is Lorentz invariant.