1) 115(6): Free Field and Interacting Field

In this note the free and interacting fields are defined from the Bianchi identity:

$$D \wedge T := R \wedge q \quad -(1)$$

and its Hodge dual:

$$D \wedge \tilde{T} := \tilde{R} \wedge q. \quad -(2)$$

These equations are written as:

$$d \wedge T = j = R \wedge q - \omega \wedge T \quad -(3)$$

$$d \wedge \tilde{T} = \tilde{j} = \tilde{R} \wedge q - \omega \wedge \tilde{T} \quad -(4)$$

Free Fields

These are defined by the geometry:

$$j = \tilde{j} = 0 \quad -(5)$$

i.e.

$$R \wedge q = \omega \wedge T \quad -(6)$$

$$\tilde{R} \wedge q = \omega \wedge \tilde{T}. \quad -(7)$$

As in previous work, possible solutions of (6) and (7) are $R$ dual to $T$ and $\omega$ dual to $q$. The concept of free field is pure mathematics, because a field cannot exist without a source. In physics, a field far from a source is defined by:

$$j \rightarrow 0, \quad -(8)$$

$$\tilde{j} \rightarrow 0, \quad -(9)$$

but not identically zero. Therefore the free field geometry is:

2)

$$\boxed{\begin{array}{l} d \wedge T \longrightarrow 0 \\ d \wedge \tilde{T} \longrightarrow 0 \end{array}} \quad \begin{array}{l} -(10) \\ -(11) \end{array}$$

a) Free Electromagnetic Field

Use the ECE hypothesis:

$$F = A^{(0)} T \quad -(12)$$

to find the ECE equations of the free electromagnetic field:

$$\boxed{\begin{array}{l} d \wedge F = 0 \\ d \wedge \tilde{F} = 0 \end{array}} \quad \begin{array}{l} -(12) \\ -(13) \end{array}$$

for all practical purposes (FAPP). These equations imply:

$$\boxed{\begin{array}{l} R \wedge A = \omega \wedge F \\ \tilde{R} \wedge A = \omega \wedge \tilde{F}, \end{array}} \quad \begin{array}{l} -(14) \\ -(15) \end{array}$$

so for the free field, propagating at c in the vacuum, the spin connection is of the same order as the torsion and curvature. Therefore this is not a Minkowski spacetime at all in ECE theory because the free field is due to spacetime torsion. In a Minkowski spacetime the torsion, curvature and spin connection all vanish.

Translating into vector notation, eqs. (12) and (13) become the familiar:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(16)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(17)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(18)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(19)$$

For example, the Coulomb law for the free field is eq. (18), with the solution:

$$\boxed{\underline{E} \rightarrow \underline{0}} \quad -(20)$$

This means that in electrostatics, the static electric field tends to zero if the distance between two charges approaches infinity. This result can be seen from the Coulomb law:

$$\underline{F} = \frac{e_1 e_2}{4\pi\epsilon_0 r^2} \underline{r} \quad -(21)$$

$$= e_1 \underline{E} \quad -(22)$$

where:

$$\boxed{\underline{E} := \frac{e_2}{4\pi\epsilon_0 r^2} \underline{r}} \quad -(23)$$

In electrodynamics it is well known that there are mathematical plane wave solutions of eqs. (16) to (19):

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left( \underline{i} - i\underline{j} \right) \exp\left( i\left( \omega t - \underline{\kappa} \cdot \underline{r} \right) \right) \quad -(24)$$

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}} \left( i\underline{i} + \underline{j} \right) \exp\left( i\left( \omega t - \underline{\kappa} \cdot \underline{r} \right) \right). \quad -(25)$$

There are also many other solutions such as spherical waves.

b) Free Gravitational Field

In the Eckardt engineering model, the free gravitational field is given by:

$$\underline{\nabla} \cdot \underline{h} = 0 \quad -(26)$$

$$\underline{\nabla} \times \underline{g} + \frac{1}{c} \frac{\partial \underline{h}}{\partial t} = \underline{0} \quad -(27)$$

$$\underline{\nabla} \cdot \underline{g} = 0 \quad -(28)$$

$$\underline{\nabla} \times \underline{h} - \frac{1}{c} \frac{\partial \underline{g}}{\partial t} = \underline{0} \quad -(29)$$

The result (28) is interpreted in the same way as the static electric field, using the Newton inverse square law:

$$\underline{F} = - \frac{m M G}{r^2} \underline{r} \quad -(30)$$

$$\boxed{\underline{g} = - \frac{M G}{r^2} \underset{\sim}{r}} \quad -(31)$$

The acceleration due to gravity approaches zero for an infinitely distant mass.

5)

If the classical vacuum is defined by the absence of all mass and charge, then:

$$\underline{E} \rightarrow \underline{0}, \quad \underline{g} \rightarrow \underline{0}. \quad -(32)$$

in the vacuum.

There are also radiated gravitational plane waves:

$$\boxed{\begin{aligned} \underline{g} &= \frac{g^{(0)}}{\sqrt{2}} \left( i\underline{i} - \underline{j} \right) \exp\left( i \left( \omega t - \underline{\kappa} \cdot \underline{r} \right) \right) \quad -(33) \\ \underline{h} &= \frac{h^{(0)}}{\sqrt{2}} \left( i\underline{i} + \underline{j} \right) \exp\left( i \left( \omega t - \underline{\kappa} \cdot \underline{r} \right) \right) \quad -(34) \end{aligned}}$$

But the gravitational waves are many orders of magnitude weaker than the electromagnetic waves.

Interaction of Fields with Matter

The controlling geometry in this case is:

$$d \wedge T \rightarrow 0 \quad -(35)$$

$$d \wedge \tilde{T} = \underline{\tilde{j}} = \left( \tilde{R \wedge q - \omega \wedge T} \right)_{\text{interaction}} \quad -(36)$$

a) In electrodynamics these become:

$$d \wedge F \rightarrow 0 \quad -(37)$$

$$d \wedge \tilde{F} = \underline{\tilde{j}} / \epsilon_0 \quad -(38)$$

Eqs. (37) and (38) translate into:

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad -(39)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \qquad -(40)$$

$$\underline{\nabla} \cdot \underline{D} = \rho \qquad -(41)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \qquad -(42)$$

where: $\underline{D} = \epsilon_0 \underline{E} + \underline{P}, \quad \underline{B} = \mu_0 (\underline{H} + \underline{M})$ $-(43)$

is the usual S.I. notation. Therefore the interaction of the electromagnetic field with matter is described by eq. (38). Eq. (37) is still that of the free field.

In general, an asymmetric connection must be used to find $\underline{J}$ in eq. (38). Also, $(\tilde{R} \wedge q - \omega \wedge \tilde{T})_{\text{interaction}}$ is not the Hodge dual of $(R \wedge q - \omega \wedge T)_{\text{free}}$. The latter's Hodge dual is $(\tilde{R} \wedge q - \omega \wedge \tilde{T})_{\text{free}}$. It is also now known that the Einstein field equation cannot be used to calculate $\tilde{R} \wedge q$.

7)

b) The interaction of the gravitational field with matter is given by

$$d \wedge \tilde{T} = \tilde{j}_{int} = \left( \tilde{R} \wedge q - \omega \wedge \tilde{T} \right)_{interaction} \quad -(39)$$

For example:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(40)$$

is the gravitational analogue of eq. (41). There is also a gravitational analogue of eq. (42), yet to be observed.

Homogeneous Current $j$

This must be interpreted as:

$$\boxed{j = \left( R \wedge q - \omega \wedge T \right)_{free} \neq 0} \quad -(41)$$

and is due to the interaction of the free gravitational and electromagnetic fields. In electrodynamics it means the existence of a magnetic monopole type of charge density. There is also a similar gravitational concept yet to be observed.