1) 115(5): Standard Representation of the Poisson Equation, Torsion and Curvature.

The gravitational Poisson equation is given erroneously by Carroll as

$$\nabla^2 \Phi = 4\pi G \rho_m \quad -(1)$$

whereas in Marion and Thornton:

$$\underline{g} = -\underline{\nabla} \Phi \quad -(2)$$

and

$$\Phi = -\frac{GM}{r} \quad -(3)$$

so

$$\underline{F} = -\underline{\nabla} U, \quad -(4)$$

$$U = m\Phi. \quad -(5)$$

In Jackson (3rd ed.) the Poisson equation is given as:

$$\nabla^2 \Phi = -\rho / \epsilon_0 \quad -(6)$$

so there are differences in convention and sign. The Einstein and Newton constants are related by:

$$k = 8\pi G / c^2 = 1.86595 \times 10^{-26} \; m \; kgm^{-1} \quad -(7)$$

In the ECE theory and its engineering model eq. (2) is extended to include the spin connection, and also the vector potential of gravitation. The origin of the factor 4π is the surface area of a sphere of unit radius:

$$\int_0^{2\pi} \int_0^{\pi} \sin\phi \, d\phi = 4\pi \quad -(8)$$

and:

$$S = \int_0^{2\pi} d\theta \int_0^{\pi} r^2 \sin\phi \, d\phi = 4\pi r^2 \quad -(9)$$

so:

$$V = \int_0^r 4\pi r^2 dr = \frac{4}{3}\pi r^3. \quad -(10)$$

2)

is the volume of a sphere. The factor 4π therefore enters into the definition of the mass density ρ_m in eq. (1). If Φ is defined as in Marion and Thornton, eq. (3), then:

$$\underline{\nabla}\Phi = \frac{\partial \Phi}{\partial r} = \frac{GM}{r^2} \quad -(11)$$

and the Newton force equation is:

$$\underline{F} = -m\underline{\nabla}\Phi, \quad -(12)$$

the potential energy being defined by m:

$$U = m\Phi. \quad -(13)$$

Therefore the origin of m is the ratio of potential energy to gravitational potential:

$$m = \frac{U}{\Phi} \quad -(14)$$

From eq. (11):

$$\nabla^2\Phi = -2\frac{GM}{r^3} \quad -(15)$$

Therefore the error in Carroll is that there should be a minus sign in the Poisson equation. If in eq. (15):

$$\rho_m := \frac{2M}{r^3} \quad -(16)$$

then:

$$\nabla^2\Phi = -G\rho_m \quad -(17)$$

By analogy with the derivation of the Coulomb law the following relation is obtained:

3)

$$\oint_S \underline{g} \cdot \underline{n} \, da = 4\pi G \int_V \rho_m (\underline{x}) \, d^3x \quad - (18)$$

the divergence theorem means that:

$$\oint_S \underline{g} \cdot \underline{n} \, da = 4\pi \int_V \underline{\nabla} \cdot \underline{g} \, d^3x \quad - (19)$$

which implies that:

$$\boxed{\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m} \quad - (20)$$

This result is obtained by considering a point mass M inside a closed surface S. If r is the distance from the mass to a point on the surface, $\underline{n}$ is an outward unit normal, da is an element of surface area, then:

$$\underline{g} \cdot \underline{n} = MG \frac{\cos\theta}{r^2} da, \quad - (21)$$

which comes from:

$$\underline{F} = -mMG \frac{\underline{r}}{r^2} \quad - (22)$$

$$= m\underline{g} \quad - (23)$$

so:

$$\underline{g} = -\frac{MG}{r^2} \underline{r} \quad - (24)$$

the situation is sketched as follows:

4)

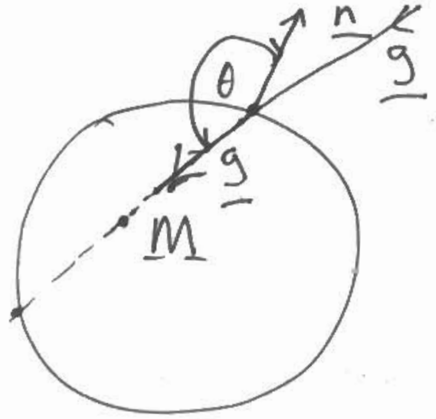

Fig (1)

It is seen that $\underline{g}$ is directed towards M.
Therefore:

$$\frac{\pi}{2} < \theta < \pi \quad -(25)$$

and $\cos\theta$ is negative.

This gives eq. (21). From eq. (8), integrating da over a sphere gives 4π. From geometry:

$$\cos\theta \, da = r^2 d\Omega \quad -(26)$$

as in Jackson, third ed., page 28, eq. (1.8).
So:

$$\underline{g} \cdot \underline{n} \, da = MG \, d\Omega \quad -(27)$$

Now integrate over the surface of a sphere to give:

$$\boxed{\oint_S \underline{g} \cdot \underline{n} \, da = 4\pi MG} \quad -(28)$$

for one mass M. For i masses:

$$\oint_S \underline{g} \cdot \underline{n} \, da = 4\pi G \sum_i M_i \quad -(29)$$

For a continuous charge density:

$$\oint \underline{g} \cdot \underline{n} = 4\pi G \int \rho(\underline{x})\, d^3x \quad -(30)$$

which is eq. (18), QED.

This gives the origin of the factor 4π. Therefore:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(31)$$

is adopted as the standard in ECE theory. The electrostatic equivalent of eq. (31) is:

$$\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0 \quad -(32)$$

because in S.I. units, the inverse square law is:

$$\underline{F} = \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \underline{r}, \quad -(33)$$

and there is an extra factor 4π in the denominator by convention. Also, the force law in electrostatics is written conventionally as positive, whereas in dynamics it is written conventionally as negative:

$$\underline{F} = -\frac{mMG}{r^2} \underline{r}. \quad -(34)$$

The electric field strength $\underline{E}$ is directed ~~towards~~ away from the source charge e_2 in electrostatics, but the acceleration due to gravity $\underline{g}$ is directed towards the source mass M in dynamics.

These are received conventions of physics.

In ECE theory both eqs. (31) and (32) are derived from:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(35)$$

which is rewritten as:

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \quad -(36)$$

For:

$$\kappa = 0 \quad -(37)$$

eq. (36) can be written as:

$$\boxed{\underline{\nabla} \cdot \underline{T} = j^0} \quad -(38)$$

where $\underset{\sim}{T}$ is a well defined orbital torsion vector.

The Newton and Coulomb laws are due to orbital torsion. This is a direct result of relativity with the correct geometry.

Newton Law

Define:

$$\boxed{\underline{g} = c^2 \underset{\sim}{T}} \quad -(39)$$

and

$$\boxed{j^0 = \frac{4\pi G}{c^2} \rho_m} \quad -(40)$$

The acceleration due to gravity is an orbital torsion vector within c^2. The charge term j^0 is mass density ρ_m within $4\pi G / c^2$. These definitions are of course given by experimental data. In the limit where the spin connection goes to zero:

$$j^0 \rightarrow R \qquad -(41)$$

where R is a well defined scalar curvature. In this limit we recover the Newton inverse square laws. The geometrical structure of the law is:

$$\boxed{\underline{\nabla} \cdot \underset{\sim}{T} = R} \qquad -(42)$$

and $\underset{\sim}{T}$ and R are of the same order of magnitude if $\underline{\nabla}$ is of the order of an inverse metre. Note carefully that no use is made of the Einstein constant k, because of the incorrectness of the Einstein field equation. In the weak field limit:

$$\boxed{\begin{aligned} R &= \frac{4\pi G}{c^2}\rho_m \\ R &= 0.93298 \times 10^{-26} \rho_m \end{aligned}} \qquad -(43)$$

For the earth, $\rho_m \sim 1$ kgm m^{-3}, so

$$\boxed{R(\text{Earth}) \sim 0.9 \times 10^{-26} \; m^{-2}} \qquad -(44)$$

This is very small but not identically zero, meaning that Newtonian dynamics are adequate for terrestrial problems, as is well known.

Coulomb Law

This is again defined by eq. (38), with:

$$\boxed{\underline{E} = V\,\underset{\sim}{T}} \quad - (45)$$

where V is in volts, and where the electric field strength $\underline{E}$ is in volts per metre. Therefore:

$$\boxed{j^{\circ} = \rho_e / \epsilon_0} \quad - (46)$$

and

$$\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0 \quad - (47)$$

where ρ_e is the charge density in coulombs per cubic metre, and ϵ_0 is the vacuum permittivity.

It can no longer be assumed that the spin connection goes to zero because for the free electromagnetic field,

$$R \wedge q = \omega \wedge T \quad - (48)$$

and ω is of the same order as q. For the free electromagnetic field:

$$\tilde{R} \wedge q = \omega \wedge \tilde{T} \quad - (49)$$

9) So the geometrical structure of the free electromagnetic field is:

$$d \wedge T = 0 \quad -(50)$$
$$d \wedge \tilde{T} = 0 \quad -(51)$$

When the field interacts with mass (the gravitational field) ~~the~~ then in general:

$$j = R \wedge q - \omega \wedge T \neq 0 \quad -(52)$$
$$\tilde{j} = \tilde{R} \wedge q - \omega \wedge \tilde{T} \neq 0 \quad -(53)$$

and <u>experimentally</u>:

$$\tilde{j} \gg j. \quad -(54)$$

In this case the controlling geometry is:

$$d \wedge T = j \quad -(55)$$
$$d \wedge \tilde{T} = \tilde{j} \quad -(56)$$

Experimentally:

$$j \rightarrow 0 \quad -(57)$$

and

$$\tilde{j} = \left(\tilde{R} \wedge q^{grav} - \omega \wedge \tilde{T} \right)_{\text{interaction}} \quad -(58)$$

i.e. the interaction of the electromagnetic field with mass gives rise to the Coulomb law and Ampère Maxwell law, because charge is always carried by mass.