

1) 112(7): Computer Algebra Results

For all correct exact solutions of the Einstein field equation:

$$R^{\kappa}_{\mu} \neq 0 \quad - (1)$$

and $R^{\kappa}_{\mu\nu\sigma} + R^{\kappa}_{\sigma\mu\nu} + R^{\kappa}_{\nu\sigma\mu} = 0 \quad - (2)$

when: $T_{\mu\nu} \neq 0 \quad - (3)$

For example, when $n=0$:

$$R^{\kappa}_{1^{10}} + R^{\kappa}_{2^{20}} + R^{\kappa}_{3^{30}} \neq 0 \quad - (4)$$

Taking individual Hodge duals of eq. (4):

$$R^{\kappa}_{1^{10}} = \tilde{R}^{\kappa}_{123}, \quad - (5)$$

$$R^{\kappa}_{2^{20}} = \tilde{R}^{\kappa}_{231}, \quad - (6)$$

$$R^{\kappa}_{3^{30}} = \tilde{R}^{\kappa}_{312}. \quad - (7)$$

So:

$$\tilde{R}^{\kappa}_{123} + \tilde{R}^{\kappa}_{231} + \tilde{R}^{\kappa}_{312} \neq 0 \quad - (8)$$

$$R^{\kappa}_{123} + R^{\kappa}_{231} + R^{\kappa}_{312} = 0 \quad - (9)$$

Using the Bianchi identity:

$$R^{\kappa}_{123} + R^{\kappa}_{231} + R^{\kappa}_{312} = D_1 T^{\kappa}_{23} + D_2 T^{\kappa}_{31} + D_3 T^{\kappa}_{12} = 0 \quad - (10)$$

and

$$\tilde{R}^{\kappa}_{123} + \tilde{R}^{\kappa}_{231} + \tilde{R}^{\kappa}_{312} = D_1 \tilde{T}^{\kappa}_{23} + D_2 \tilde{T}^{\kappa}_{31} + D_3 \tilde{T}^{\kappa}_{12} \neq 0 \quad - (11)$$

Eq. (11) is a contradiction because:

$$\tilde{T}^{\kappa}_{23} = \tilde{T}^{\kappa}_{31} = \tilde{T}^{\kappa}_{12} = 0 \quad - (12)$$

The EH equation violates the Bianchi identity.