III(8): Further Development of the Spherically Symmetric Metric.

Begin with the equation:

$$r = \int dr. \quad -(1)$$

This assumes a spherically symmetric space (r, θ, ϕ) and (X, Y, Z), and a zero constant of integration:

$$\mu = 0. \quad -(2)$$

The spherically symmetric line element is:

$$ds^2 = -mc^2dt^2 + n\,dr^2 + r^2 d\Omega^2 \quad -(3)$$

Assume that m is defined by:

$$mr = \int dr \quad -(4)$$

and

$$\frac{r}{n} = \int dr. \quad -(5)$$

Integrating eq. (4):

$$mr = r + \mu \quad -(6)$$

and integrating eq. (5):

$$\frac{r}{n} = r + \mu. \quad -(7)$$

So:

$$m = 1 + \frac{\mu}{r}, \quad n = \left(1 + \frac{\mu}{r}\right)^{-1}. \quad -(8)$$

It is known from satellite data that:

2)

$$ds^2 = -\left(1+\frac{\mu}{r}\right)c^2dt^2 + \left(1+\frac{\mu}{r}\right)^{-1}dr^2 + r^2d\Omega^2 \quad -(9)$$

with

$$\mu = \frac{2MG}{c^2} \quad -(10)$$

therefore the data indicate that m and n are indeed defined by eqns. (4) and (5). Furthermore it is known that special relativity is described by:

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2 \quad -(11)$$

i.e. by:

$$\mu \rightarrow 0 \quad -(12)$$

and therefore by eq. (1).

Therefore equations of type (1), (4) and (5) are essentially the geometrical reasons for observed orbits. They must therefore be part of a class of topology which has particular significance in physics.

Experimental Departures from Eqn. (10)

The only known experimental departures from eq. (10) are the orbits of binary pulsars and the Pioneer / Cassini anomalies. In paper 108 it was shown that these orbits are described by:

3)

$$\mu = \frac{2MG}{c^2} + \frac{a}{r} \quad -(13)$$

where a is a small constant perturbation. The latter causes the orbits of a binary pulsar to spiral inwards from the trajectories defined by eq. (10).

Using computer algebra (Dr. Horst Eckardt) it was found that the line element defined by eq. (13) obeys the Bianchi identity:

$$D \wedge T := R \wedge q \quad -(14)$$

because it produces:

$$T = 0, \quad R \wedge q = 0. \quad -(15)$$

However, eq. (13) produces:

$$\tilde{T} = 0, \quad \tilde{R} \wedge q \neq 0, \quad -(16)$$

so does not obey the Hodge dual identity:

$$D \wedge \tilde{T} := \tilde{R} \wedge q. \quad -(17)$$

The reason for this inconsistency is the use of the symmetric or Christoffel connection:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu} \quad -(18)$$

In order for eq. (13) to self-consistently the more obey both eqns. (14) and (17)

4) general connection:

$$\Gamma^{\kappa}_{\mu\nu} \neq \Gamma^{\kappa}_{\nu\mu} \quad - (19)$$

must be used.

It was found that a line element of type (13) produces:

$$R_{\mu\nu} \neq 0 \quad - (20)$$

but

$$R = 0. \quad - (21)$$

Here $R_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar

$$R = g^{\mu\nu} R_{\mu\nu}. \quad - (22)$$

If one were to attempt to describe these results with the Einstein field equation one would find several self-inconsistencies. For example:

1) the field equation would obey eq. (14) but not eq. (17). It would have to be written:

$$R_{\mu\nu} = k T_{\mu\nu} \neq 0. \quad - (23)$$

2) the contracted form of eq (23) would however be zero:

$$R = -kT = 0. \quad - (24)$$