1) III(2): Lense - Thirring Effect and Rotation of the So - Called Schwarzschild Metric

In the usual treatment of the Lense - Thirring effect an acceleration field is induced by a slowly rotating mass shell:

$$\underline{a} = -2d_1 \underline{\omega} \times \underline{v} - d_2 \left(\underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2(\underline{\omega} \cdot \underline{r}) \underline{\omega} \right) \quad -(1)$$

where:

$$d_1 = \frac{4mG}{3Rc^2}, \quad d_2 = \frac{4mG}{15Rc^2}. \quad -(2)$$

There are well known problems with Thirring's original derivation because he used a dust - like $T^{\mu\nu}$ which did not obey the Noether theorem. So he did not give a true solution of the EH field equation. Brill and Cohen in 1966 considered a rotational perturbation of the so - called "Schwarzschild solution".

Essentially therefore the Lense - Thirring effect is a general relativistic correction to the well known Coriolis and centripetal acceleration. The original concept is due to Einstein, and he intended to correct the linear velocity $\underline{v}$ in eq. (1) by application of general relativity. He proposed translational and rotational frame dragging.

Recently, Gravity Probe B has tested the relativistic correction. It has found that there are relativistic effects and this is claimed as evidence for the correctness of the EH theory. The Lense - Thirring effect is one of the so - called precision tests of general relativity.

2)

By now (papers 93 to 110) it is well known that the basic geometry of the EH equation is incorrect. The reason for this is that in the equation:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad — (3)$$

exact solutions of the EH equation give in general:

$$T^{\kappa\mu\nu} = 0, \; R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0. \quad — (4)$$

Only the so-called vacuum solution give:

$$T^{\kappa\mu\nu} = 0, \; R^{\kappa}{}_{\mu}{}^{\mu\nu} = 0, \quad — (5)$$

but in the presence of a non-zero energy-momentum density, eq. (4) is true in general.

This basic flaw was difficult to discover, and computer algebra was necessary to find it. The basic problem is the neglect of torsion in the geometry used by Einstein (and Hilbert) in 1915. ECE theory (2003 to present) re-instates the torsion using Cartan's geometry (1922). In paper 110 it was shown that rotation about the Z axis produces the infinitesimal generator of torsion, which is directly proportional to the well known infinitesimal rotation generator, so rotation always produces torsion. The self-consistent treatment of any effect in general relativity needs torsion.

3)

Therefore Cartan geometry must be used to describe the precision tests of general relativity. In paper 108 this was carried out for the orbit of a binary pulsar and the Pioneer | Cassini anomalies. The data currently available are very precise, but the basic geometry of the EH equation is incorrect.

In paper 110 the Thomas precession was described in terms of Cartan torsion by rotating the Minkowski line - element. In this note it is shown that the Cartan torsion can be used to describe the observed order of magnitude of the Lense - Thirring effect by rotating the so-called "Schwarzschild solution", using a rotating line element:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi'^2 \quad - (6)$$

where:

$$d\phi'^2 = d\phi^2 + 2\omega \, d\phi \, dt + \omega^2 dt^2 \quad - (7)$$

The frame rotates at an angular velocity ω, defined by:

$$\phi' = \phi + \omega t . \quad - (8)$$

Eq. (6) gives:

$$ds^2 = \left(\left(1 - \frac{r_s}{r}\right) - \frac{v^2}{c^2} \sin^2\theta\right)\left(c^2 dt^2 - 2r^2 \sin^2\theta \, \Omega \, d\phi \, dt\right) - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad - (9)$$

4) where:

$$v = r\omega \quad -(10)$$

and:

$$\Omega = \omega\left(1 - \frac{r_s}{r} - \frac{v^2}{c^2}\sin^2\theta\right)^{-1} \quad -(11)$$

is the relativistically corrected angular velocity. The rotational frame dragging effect is therefore:

$$\boxed{\underline{a} = -2\,\underline{\Omega}\times\underline{v} - \underline{\Omega}\times(\underline{\Omega}\times\underline{r})} \quad -(12)$$

In the situation of Fig (3) of notes III(1) it is

$$\underline{a} = \left(2\Omega v + \Omega^2 r\right)\underline{e}_1 \quad -(13)$$

In eq. (11):
$$r_s = \frac{2mG}{c^2}, \quad -(14)$$

and
$$\Omega^2 = \omega^2\left(1 - \frac{r_s}{r} - \frac{v^2}{c^2}\sin^2\theta\right)^{-2} \quad -(15)$$

If:
$$\left(\frac{r_s}{r} + \frac{v^2}{c^2}\sin^2\theta\right) \ll 1 \quad -(16)$$

then:
$$\Omega = \omega\left(1 - \frac{2mG}{c^2 r} - \frac{v^2}{c^2}\sin^2\theta + \ldots\right) \quad -(17)$$

$$\Omega^2 = \omega^2\left(1 - \frac{4mG}{c^2 r} - \frac{2v^2}{c^2}\sin^2\theta + \ldots\right) \quad -(18)$$

5)

Correction to the Coriolis Acceleration

This is:

$$d_1 = \frac{2mG}{c^2 r} + \frac{v^2}{c^2} \sin^2\theta \quad -(19)$$

Correction to the Centripetal Acceleration

This is:

$$d_2 = \frac{4mG}{c^2 r} + \frac{2v^2}{c^2} \sin^2\theta. \quad -(20)$$

It is seen that these are of the same order as the correction given in the calculation leading to eq. (2). For example the Coriolis corrections are the same if:

$$\frac{2mG}{c^2 r} + \frac{\omega^2 r^2}{c^2} \sin^2\theta_1 = \frac{4mG}{3Rc^2} \quad -(21)$$

and the centripetal correction are the same if:

$$\frac{4mG}{c^2 r} + \frac{2\omega^2 r^2}{c^2} \sin^2\theta_2 = \frac{4mG}{15Rc^2} \quad -(22)$$

in the limit:

$$r << R \quad -(23)$$

Eqs. (21) and (22) can be solved simultaneously for r and θ in terms of R. Alternatively eqs. (19) and (20) can be regarded as approximation, but one that self-consistently

6) give the correct non-zero Cartan torsion.

This calculation does not use a mass-shell as in the calculation that lead to the result (2). The conceptual problem with the so-called SM is that it assumes that there exists a vacuum solution of EH:

$$G_{\mu\nu} = 0 \quad -(24)$$

where:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \quad -(25)$$

This is pure geometry. There is no $T_{\mu\nu}$ present in eq. (24), and therefore no mass. A possible solution of eq. (24) is:

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 \quad -(26)$$

where α is to be determined. Eq. (26) is the vacuum solution of eq. (24) given by Schwarzschild in 1916, using the assumption of the Christoffel connection:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu}. \quad -(27)$$

In order to obtain Newtonian dynamics as a limit, it was assumed in general relativity based on EH that:

$$\alpha = \frac{2MG}{c^2} \quad -(28)$$

Schwarzschild however did not make this assumption

7) because it is self-inconsistent. On the one hand $T_{\mu\nu}$ is 0 in eq. (24), so M is zero, but in the solution of eq. (24) M is non-zero.

The major self inconsistency discovered by the AIAS group in paper 93 is that the assumption (27) produces eq. (4) which is inconsistent with the Hodge dual of the Bianchi identity. This is due to the neglect of torsion in EH geometry.

The third type of major self-inconsistency in the theory that leads to eq. (2) is that there is rotational motion present, but there is no torsion:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} = 0. \quad -(2a)$$

In this note the order of magnitude of the Lense-Thirring effect has been obtained from the torsion generated by rotating the so-called Schwarzschild solution.

The Lense-Thirring effect is due to the Cartan torsion.