1)

110(5): Cross Check on Derivation of Newtonian Dynamics from ECE Theory.

This cross-check self consistently derives Newtonian dynamics from the ECE Lemma (subject of paper 110) and the method of paper 105.

Newtonian Dynamics from the ECE Lemma

The ECE Lemma is:

$$\square q^a_\mu = R q^a_\mu . \quad -(1)$$

using the postulate:

$$R = -kT \quad -(2)$$

it becomes the ECE wave equation:

$$(\square + kT) q^a_\mu = 0 \quad -(3)$$

This is the only equation from which gravitational radiation can be deduced self consistently. The eigenfunction of gravitational radiation is the tetrad. The d'Alembertian operator is:

$$\square := \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(4)$$

Assuming that q^a_μ varies slowly with time then eq. (3) can be written as:

$$\nabla^2 q^a_\mu = kT q^a_\mu . \quad -(5)$$

Now consider the q^0_0 component of the tetrad, and expand it as follows:

$$q^0_0 = 1 + h^0_0 + \ldots \quad (6)$$

where:

$$h^0_0 << 1 . \quad -(7)$$

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In this approximation eq. (5) becomes:

$$\nabla^2 h^0_0 \doteq k T^0_0 \quad -(8)$$

where:

$$T^0_0 = \rho \quad -(9)$$

where ρ is the mass density.

So:

$$\nabla^2 h^0_0 = k\rho \quad -(10)$$

$$= \frac{4\pi G}{c^2}\rho \quad -(11)$$

This is the Poisson equation of Newtonian dynamics:

$$\nabla^2 \Phi = 4\pi G\rho \quad -(12)$$

if

$$\Phi = h^0_0 / c^2. \quad -(13)$$

Therefore Newtonian dynamics is the limit of a time-independent q^a_μ whose V^0_0 component is represented by the perturbation (6).

Cross-Check with the Method of Paper 105.

In paper 105 Newtonian dynamics were obtained from:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(14)$$

which is a tensorial equation obtained from the differential form equation:

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(15)$$

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In vector notation the following equation ~~were~~ was obtained from eq. (14):

$$\underline{\nabla} \cdot \underline{T} = R \quad - (16)$$

This is the gravitational equivalent of the Coulomb law. Here

$$\underline{T} = T^{010}\,\underline{i} + T^{020}\,\underline{j} + T^{030}\,\underline{k} \quad - (17)$$

and $$R = R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}. \quad - (18)$$

From eq. (16) the acceleration due to gravity was identified as:

$$\underline{g} = c^2 \left(T^{010}\,\underline{i} + T^{020}\,\underline{j} + T^{030}\,\underline{k} \right) \quad - (19).$$

and the mass density was identified as:

$$\rho = \frac{1}{k} \left(R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} \right). \quad - (20)$$

Therefore acceleration due to gravity is due to components of the Cartan torsion, and mass density is due to components of the Cartan curvature.

Therefore:

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 R} \quad - (21)$$

This is the gravitational equivalent of the Coulomb Law:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0} \quad - (22)$$

where ρ_e is charge density.

Thus:

$$\boxed{\begin{matrix}\underline{\nabla}\cdot\underline{g} = c^2 k\rho \\ \text{gravitation}\end{matrix}} \longleftrightarrow \boxed{\begin{matrix}\underline{\nabla}\cdot\underline{E} = \rho_e/\epsilon_0 \\ \text{electromagnetism}\end{matrix}} \quad -(23)$$

Both equations are obtained in ECE theory from Cartan geometry. For all fields the equation:

$$\underline{\nabla}\cdot\underline{g} = c^2 k \rho \quad -(24)$$

is true.

<u>It is now necessary to investigate how the Newtonian limit is obtained from eq. (24).</u>

In ECE theory it is known that:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} + \phi\,\underline{\omega} - \omega \underline{A} \quad -(25)$$

where ϕ is the scalar potential, $\underline{A}$ is the vector potential, $\underline{\omega}$ is the spin connection vector and ω is the spin connection scalar. By analogy:

$$\boxed{\underline{g} = -\underline{\nabla}\,\Phi - \frac{\partial \underline{\alpha}}{\partial t} + \Phi\,\underline{\omega} - \omega\,\underline{\alpha}} \quad -(26)$$

where $\underline{\alpha}$ is the gravitational equivalent of $\underline{A}$ in electrodynamics. It is well known that in Newtonian dynamics:

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$$\underline{g} = -\underline{\nabla}\,\Phi \quad -(27)$$

where the Newtonian gravitational potential is:

$$\Phi = -\frac{GM}{r}. \quad -(28)$$

The Newtonian potential energy is:

$$U = m\,\Phi \quad -(29)$$

and the Newtonian force is:

$$\underline{F} = -\underline{\nabla}\,U = -\frac{mMG}{r^2}. \quad -(30)$$

The static electric field in electrodynamics is obtained from eq. (25) with:

$$\underline{A} = \underline{0} \quad -(31)$$

so:

$$\underline{E} = \left(-\underline{\nabla} + \underline{\omega}\right)\phi. \quad -(32)$$

Therefore the static $\underline{g}$ of Newtonian gravitation is:

$$\underline{g} = \left(-\underline{\nabla} + \underline{\omega}\right)\Phi \quad -(33)$$

As was shown in paper 63, a spin connection of the type:

$$\underline{\omega} = -\frac{1}{r}\,\underline{e_r} \quad -(34)$$

produces:

$$\underline{E} = -\underline{\nabla}(2\phi) \quad -(35)$$

i.e. simply doubles the value of $\underline{E}$.

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Similarly, for gravitation, eq. (34) produces:

$$\underline{g} = - \underline{\nabla} (2 \Phi) \quad -(36)$$

Thus:

$$\boxed{\Phi_{\text{Newtonian}} := 2 \Phi} \quad -(37)$$

The Newtonian limit of (34) is therefore obtained when $\underline{\alpha}$ is zero and when the spin connection is defined by eq. (34). Similarly, the Coulombic limit is obtained when $\underline{A}$ is zero, and with the same type of spin connection.

From eqs. (13) and (37):

$$\boxed{\Phi_{\text{Newtonian}} = 2 \Omega^0 / c^2} \quad -(38)$$

Finally use the definition of the Cartan ~~tetrad~~ torsion:

$$T := D \wedge q \quad -(39)$$

In differential form notation, eq. (39) is:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(40)$$

Therefore:

$$T^0_{10} = \partial_1 q^0_0 - \partial_0 q^0_1 + \omega^0_{1b} q^b_0 - \omega^0_{0b} q^b_1 \quad -(41)$$

It is seen from eq. (41) that the torsion elements $T^0{}_{10}$, $T^0{}_{20}$ and $T^0{}_{30}$ are defined in terms of the $q^0{}_0$ element of the tetrad. Eqn. (26) is derived from eq. (41).

Therefore this shows that the tetrad element of eq. (6) is indeed the one that gives Newtonian gravitation. The ECE method has been cross checked for self-consistency because it has been derived self-consistently from complementary aspects of the same Cartan geometry. These are 1) the tetrad postulate that leads to the ECE wave equation, and 2) the Bianchi identity that leads to the ECE field equations.

Newtonian dynamics is a special case of ECE theory, which gives a lot more information.