108(7): Limiting Form of the Potential Energy in the Relativistic Kepler Problem.

The potential energy is :

$$V = mc^2\left(\frac{1}{2} - \frac{r_s}{r}\right) + \frac{mL^2}{2r^2} - mL^2\frac{r_s}{r^3} \quad -(1)$$

where

$$|r_s| = \frac{I}{R}. \quad -(2)$$

In the standard model :

$$r_s = \frac{2mG}{c^2} \quad -(3)$$

1) As $r_s \to 0$:

$$V \to \frac{1}{2}mc^2 + \frac{mL^2}{2r^2} \quad -(4)$$

where:

$$L = r^2\frac{d\phi}{d\tau}. \quad -(5)$$

This means that there is no force of attraction in the limit $r_s \to 0$. The two particles (a star) have collided. $\quad -(6)$

2) From eq. (1) :

$$r_s = \left(\frac{1}{2}mc^2 - V + \frac{mL^2}{2r^2}\right)\left(\frac{mc^2}{r} + \frac{mL^2}{r^3}\right)^{-1}$$

and as $r \to 0$:

$$r_s \to \frac{r}{2} \quad -(7)$$

3) As $r_s \to 0$, $V \to \frac{1}{2}mc^2 + \frac{mr^2}{2}\frac{d\phi}{d\tau} \quad -(8)$

2)

4) In the limit of circular orbits:

$$\frac{dV}{dr} \to 0 \qquad -(9)$$

$$r \to \frac{1}{2c^2 r_s}\left(L^2 \pm L\left(L^2 - 12c^2 r_s^2\right)^{1/2}\right) \qquad -(10)$$

and the Newtonian limit is identified as:

$$r_s \to 0 \qquad -(11)$$

i.e.

$$r \to \frac{L^2}{c^2 r_s} = \left(\frac{L^2}{c^2}\right) r_s \qquad -(12)$$

5) From eq. (6), the limit $r_s \to 0$ also implies:

$$\frac{mc^2}{r} + \frac{mL^2}{r^3} \to \infty \qquad -(13)$$

and since L is a constant of motion:

$$r \to 0 \qquad -(14)$$

Therefore, as $\boxed{r_s \to 0, \quad r \to 0}$ $\qquad -(15)$

6) From eq. (5), if L is a constant of motion and if $r \to 0$, then as $r \to 0$, $d\phi / d\tau \to \infty$. This again means that the two stars have collided.

7) Finally in the limit $V \to 0$,

$$mc^2\left(\frac{1}{2} - \frac{r_s}{r}\right) + \frac{mL^2}{2r^2} - mL^2\frac{r_s}{r^3} \to 0$$

$$-(16)$$

In the standard model the loss of potential energy is interpreted as the emission of gravitational radiation, but in ECE theory it can be interpreted by calculating r from the cubic equation (16) and expressing r in terms of the constant L and r_s. Eq (16) is the cubic:

$$\frac{1}{2}c^2r^3 - c^2r^2r_s + \frac{L^2}{2}r - L^2r_s = 0 \quad -(17)$$

so there are three roots. Thus as $V \to 0$, r may be expressed in terms of r_s and L. If the angular momentum L is very small in eq (17) then:

$$r \to 2r_s \quad -(18)$$

which is eq. (7). This is interpreted to mean that for a very small L, $r \to 2r_s \to 0$. As $r_s \to 0$ in eq. (17),

$$c^2r^2 + L^2 \to 0 \quad -(19)$$

and this is interpreted again as $r \to 0$, $L \to 0$, as $r_s \to 0$. In other words the two stars have collided.

<u>Roots of the Cubic (17)</u>.

The general cubic equation:

$$a_0x^3 + a_1x^2 + a_2x + a_3 = 0 \quad -(20)$$

may be transformed to:

4).

$$y^3 + Ay + B = 0 \quad -(21)$$

by using $\quad x = y - \frac{1}{3}\frac{a_1}{a_0} \quad -(22)$

To solve eq (21) use:

$$y = \lambda \cos\theta \quad -(23)$$

so: $\quad \lambda^3 \cos^3\theta + A\lambda\cos\theta + B = 0 \quad -(24)$

Compare eq. (24) with:

$$4\cos^3\theta - 3\cos\theta - \cos 3\theta = 0 \quad -(25)$$

i.e. $\quad \frac{\lambda^3}{4} = -\frac{A\lambda}{3} = -\frac{B}{\cos 3\theta} \quad -(26)$

Thus: $\quad \cos 3\theta = \frac{3B}{A\lambda} \quad -(27)$

$$\lambda^2 = -\frac{4A}{3} \quad -(28)$$

The roots of the cubic may be found this way or numerically, i.e. as V decreases, r may be expressed in terms of r_s and L, and the decrease in V may be explained as a decrease in $|r_s| = T / R$, and not as a loss of gravitational radiation.