1) 100(12) : Hodge Duality ii the Bianchi Identity

$$D \wedge T := R \wedge q = -q \wedge R$$

$$D_\rho T^\lambda_{\mu\nu} + D_\mu T^\lambda_{\nu\rho} + D_\nu T^\lambda_{\rho\mu} := -\left(R^\lambda_{\rho\mu\nu} + R^\lambda_{\mu\nu\rho} + R^\lambda_{\nu\rho\mu}\right)$$

$$D_0 T^\lambda_{12} + D_2 T^\lambda_{01} + D_1 T^\lambda_{20} := -\left(R^\lambda_{012} + R^\lambda_{201} + R^\lambda_{120}\right)$$

for example

$$\tilde{T}^{\lambda\rho\sigma} = \frac{1}{2}\bar{\epsilon}^{\rho\sigma\mu\nu} T^\lambda_{\mu\nu}, \quad \tilde{R}^{\lambda\ \rho\sigma}_{\ \nu} = \frac{1}{2}\bar{\epsilon}^{\rho\sigma\alpha\beta} R^\lambda_{\ \nu\alpha\beta},$$

$$\bar{\epsilon}^{\rho\sigma\alpha\beta} = \|g\|^{1/2} \epsilon^{\rho\sigma\alpha\beta},$$

where $|g|$ is the determinant of $g_{\mu\nu}$

$$T^\lambda_{12} = \frac{1}{2}\bar{\epsilon}_{1203}\tilde{T}^{\lambda 03} = \|g\|^{1/2}\tilde{T}^{\lambda 03}$$

$$T^\lambda_{01} = \|g\|^{1/2}\tilde{T}^{\lambda 23} \quad \text{etc.}$$

$$T^\lambda_{20} = \|g\|^{1/2}\tilde{T}^{\lambda 13}$$

$$D_0\left(\|g\|^{1/2}\tilde{T}^{\lambda 03}\right) + D_2\left(\|g\|^{1/2}\tilde{T}^{\lambda 23}\right) + D_1\left(\|g\|^{1/2}\tilde{T}^{\lambda 13}\right)$$
$$= \|g\|^{1/2}\left(\tilde{R}^{\lambda\ 03}_{\ 0} + \tilde{R}^{\lambda\ 23}_{\ 2} + \tilde{R}^{\lambda\ 13}_{\ 1}\right)$$

$$D_\mu g_{\rho\sigma} = D_\mu g^{\rho\sigma} = 0, \text{ metric compatibility}$$

$$D_\mu \tilde{T}^{\lambda\mu\nu} = -\tilde{R}^{\lambda\ \mu\nu}_{\ \mu}$$

2) Type Two Hodge Duality of the Bianchi Identity

$$D \wedge T := R \wedge q = -q \wedge R$$

$$R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu} = \text{cyclic sum in 4 terms of gamma connections}$$

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = g_{\mu\alpha} g_{\nu\beta} \tilde{R}^{\lambda\ \alpha\beta}{}_{\rho} \qquad \text{etc.}$$

$$\tilde{R}^{\lambda\ \alpha\beta}{}_{\rho} = \frac{1}{2} \|g\|^{1/2} \epsilon^{\alpha\beta\mu\nu} R^{\lambda}{}_{\rho\mu\nu}$$

and inverse Hodge duals.

$$R^{\lambda}{}_{012} + R^{\lambda}{}_{201} + R^{\lambda}{}_{120} = \|g\|^{1/2} \left(\tilde{R}^{\lambda\ 03}{}_{0} + \tilde{R}^{\lambda\ 23}{}_{2} + \tilde{R}^{\lambda\ 13}{}_{1} \right)$$

for example;

$$\tilde{R}^{\lambda}{}_{012} + \tilde{R}^{\lambda}{}_{201} + \tilde{R}^{\lambda}{}_{120} = \|g\|^{1/2} \left(R^{\lambda\ 03}{}_{0} + R^{\lambda\ 23}{}_{2} + R^{\lambda\ 13}{}_{1} \right)$$

for example.

$$D_0 \tilde{T}^{\lambda}{}_{12} + D_2 \tilde{T}^{\lambda}{}_{01} + D_1 \tilde{T}^{\lambda}{}_{20} = \|g\|^{1/2} \left(D_0 T^{\lambda 03} + D_2 T^{\lambda 23} + D_1 T^{\lambda 13} \right)$$

$$D \wedge \tilde{T} := \tilde{R} \wedge q = -q \wedge \tilde{R}$$

$$D_\mu T^{\kappa\mu\nu} = -R^{\kappa}{}_{\mu}{}^{\mu\nu}$$