

NEW CONCEPTS FROM THE EVANS UNIFIED FIELD THEORY. PART ONE: THE EVOLUTION OF CURVATURE, OSCILLATORY UNIVERSE WITHOUT SINGULARITY, CAUSAL QUANTUM MECHANICS, AND GENERAL FORCE AND FIELD EQUATIONS

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Received 7 February 2004

The Evans field equation is solved to give the equations governing the evolution of scalar curvature R and contracted energy-momentum T . These equations show that R and T are always analytical, oscillatory, functions without singularity and apply to all radiated and matter fields from the sub-atomic to the cosmological level. One of the implications is that all radiated and matter fields are both causal and quantized, contrary to the Heisenberg uncertainty principle. The wave equations governing this quantization are deduced from the Evans field equation. Another is that the universe is oscillatory without singularity, contrary to contemporary opinion based on singularity theorems. The Evans field equation is more fundamental than, and leads to, the Einstein field equation as a particular example, and so modifies and generalizes the contemporary Big Bang model. The general force and conservation equations of radiated and matter fields are deduced systematically from the Evans field equation. These include the field equations of electrodynamics, dark matter, and the unified or hybrid field.

Key words: Evans field equation, equations of R , oscillatory universe, general field and force equations, causal quantization.

1. INTRODUCTION

The Evans unified field theory [1-10] is based on the well-known geometrical concept of the tetrad [11], but uses and develops the tetrad, in several novel ways. All physics is reduced essentially to the tetrads,

and thus to geometry. Unification of the radiated and matter fields of nature is achieved by the synthesis of new equations which are all based on the properties of the tetrad in differential geometry. The basic field equation of nature in this theory is the Evans field equation [1-10]

$$Rq_{\mu}^a = -kTq_{\mu}^a, \quad (1)$$

which can be developed systematically in many directions. In Eq. (1), R is scalar curvature [1-12], T the contracted energy-momentum tensor, and k the Einstein constant. The tetrad is denoted by q_{μ}^a and in differential geometry is a vector-valued one-form [11]. In the Evans unified field theory the tetrad is the potential field and is also governed by the Evans wave equation [1-10]

$$\square q_{\mu}^a = -kTq_{\mu}^a, \quad (2)$$

derived from the well-known tetrad postulate [11]:

$$D^{\nu}q_{\mu}^a = 0. \quad (3)$$

Figure (1) is a schematic of how the Evans field equation reduces to known equations of physics, and Fig. (2) is a similar schematic for the Evans wave equation.

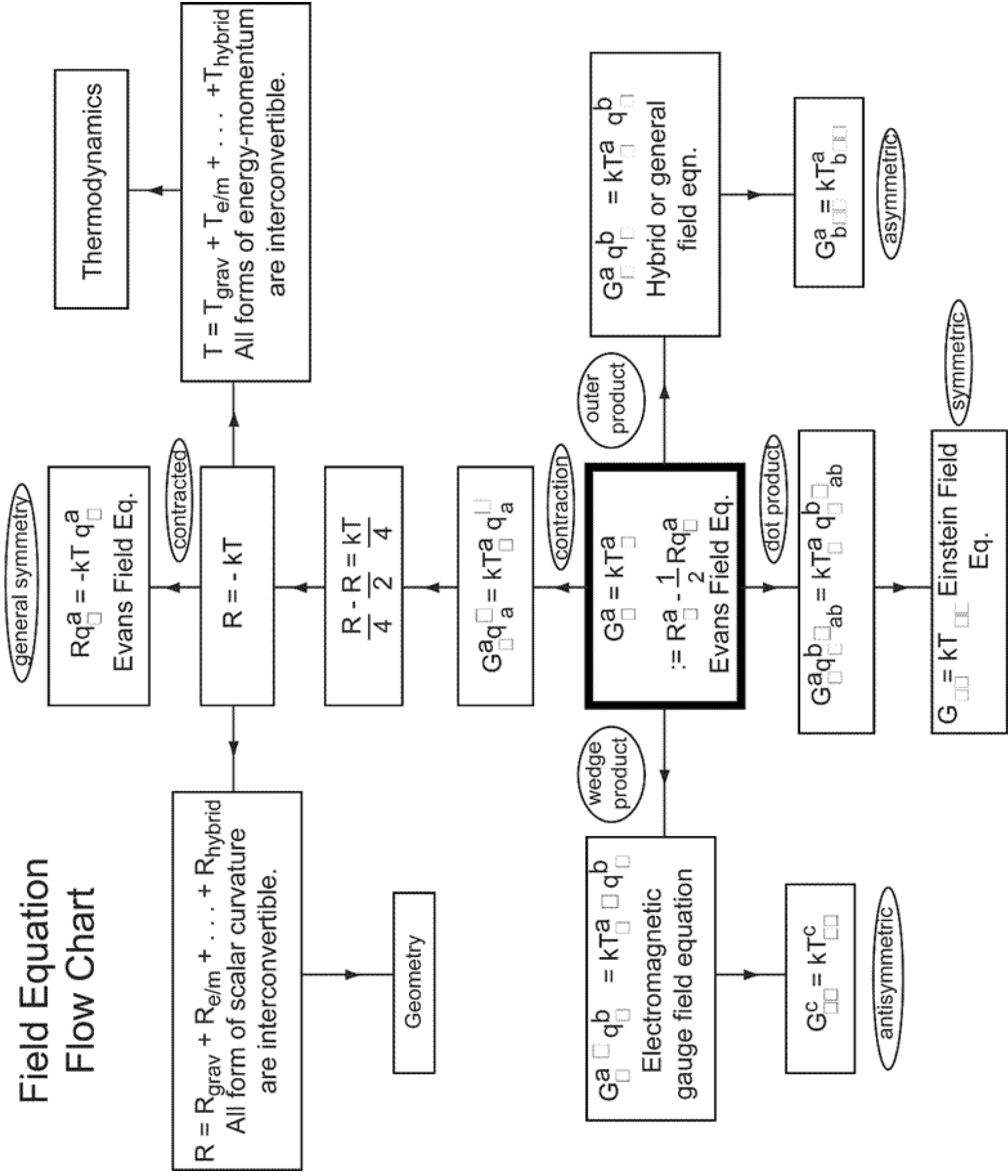
In this paper, the first of a series dealing with new concepts from the Evans unified field theory, it is shown that the Evans field equation, when combined with the tetrad postulate, produces the following equations for the evolution of R and T :

$$\frac{1}{R}\partial^{\mu}R = \pm R\partial^{\mu}\left(\frac{1}{R}\right), \quad (4)$$

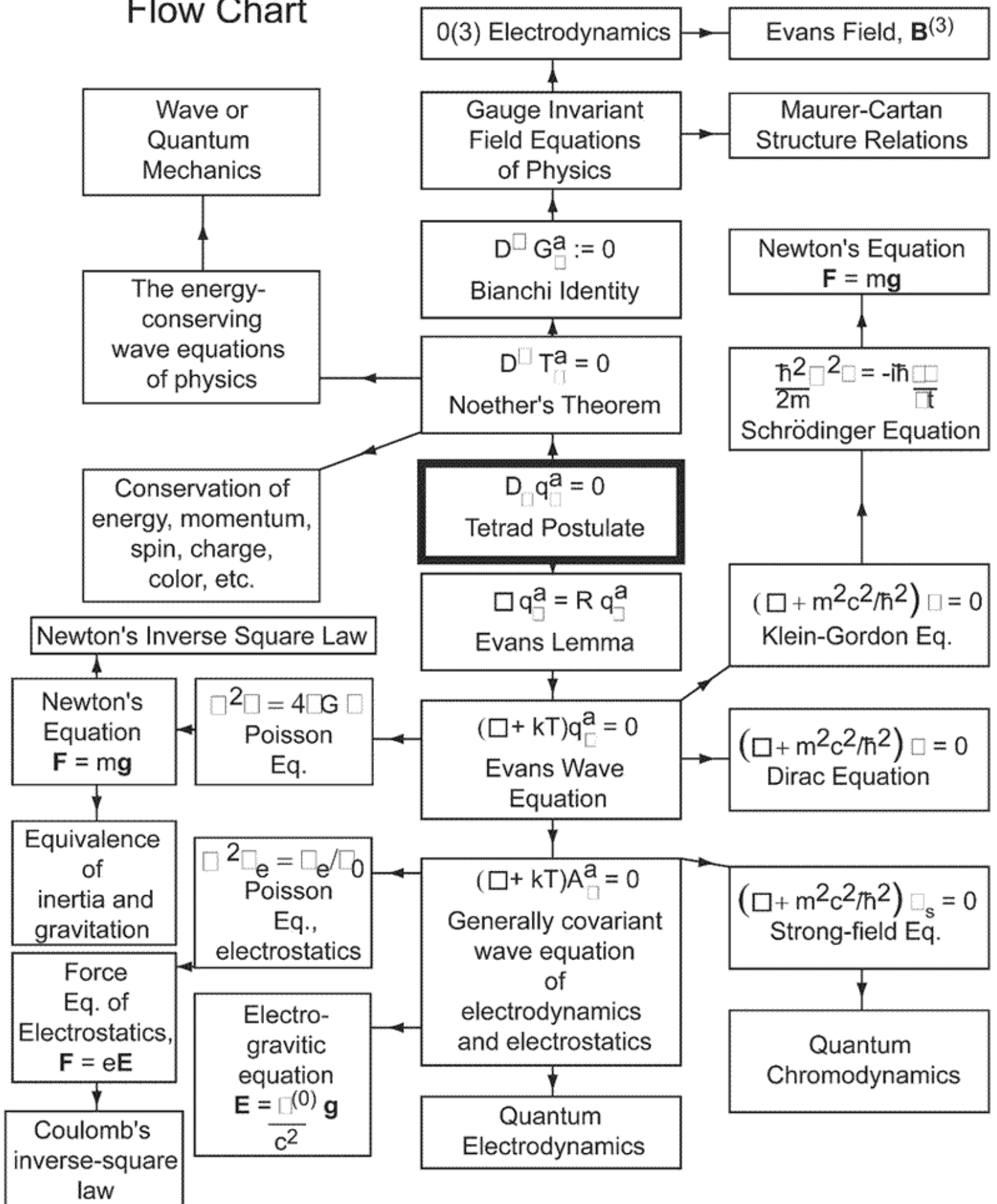
$$\frac{1}{T}\partial^{\mu}T = \pm T\partial^{\mu}\left(\frac{1}{T}\right). \quad (5)$$

This is shown in Sec. 2. In Sec. 3 the structure of the most general gauge field and force equations of nature is deduced systematically from Eq. (1). The well-known Einstein field equation for gravitation is one example, out of several new possible structures or classes, of equations to emerge from the Evans field equation. If we accept general relativity, it is therefore likely that these new equations contain a great deal of hitherto undiscovered and unexplored physics. In other words, we proceed rigorously on the basic assumption that all physics is causal and generally covariant, as originally proposed by Einstein [12]. These papers [1-10] can therefore be viewed as completing the theory of general relativity, and as extending it to all radiated and matter fields in nature. This is what is meant by “unified field theory.”

Field Equation Flow Chart



Wave Equation Flow Chart



2. THE EVOLUTION EQUATIONS OF R AND T

The field equation (1) is a balance of the identity

$$D^\mu(Rq_\mu^a) = 0 \quad (6)$$

of differential geometry and the conservation equation

$$D^\mu(Tq_\mu^a) = 0 \quad (7)$$

and is therefore a geometrization of physics in terms of the tetrad. The usual form of the Bianchi identity in the general-relativistic theory of gravitation

$$D^\mu G_{\mu\nu} = 0, \quad (8)$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (9)$$

is a special case of Eq. (6), and the well-known Noether theorem

$$D^\mu T_{\mu\nu} = 0 \quad (10)$$

is a special case of Eq. (7). The Einstein field equation balances Eqs. (8) and (10) to give

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = kT_{\mu\nu} \quad (11)$$

and can be deduced [1-10] as a special case of the Evans field equation. Here

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (12)$$

is the well-known Einstein field, where $R_{\mu\nu}$ is the Ricci tensor and $g_{\mu\nu}$ is the symmetric metric tensor of Einstein's original theory [12]. Finally, $T_{\mu\nu}$ is the well-known symmetric canonical energy-momentum tensor.

These familiar Einsteinian tensors are now known to be special cases of more general tetrad matrices of the Evans field theory and are defined by dot products of tetrads [1-11] as follows:

$$R_{\mu\nu} = R_\mu^a q_\nu^b \eta_{ab}, \quad (13)$$

$$T_{\mu\nu} = T_\mu^a q_\nu^b \eta_{ab}, \quad (14)$$

$$g_{\mu\nu} = q_\mu^a q_\nu^b \eta_{ab}. \quad (15)$$

Here η_{ab} is the (diagonal) metric of the Euclidean orthonormal space labelled by the a index of the tetrad [1-11]. The more general field and force equations of nature, introduced systematically in Sec. 3, follow

from the fundamental fact that one can define wedge (or cross) and outer products of tetrads as well as dot products. The wedge products give rise to antisymmetric torsion fields such as the electromagnetic field dual to antisymmetric curvature fields such as the Riemann curvature field [1-11], and the outer products gives rise to fields in nature which are hitherto ill-understood or unexplored and must be classified theoretically on the basis of symmetry. This is one purpose of this series of papers. One of these fields may be that of dark matter [13]. Another type of field is the unified or hybrid field which may become observable in the tiny and hitherto ill-understood effects of electromagnetism on gravitation [14,15]. Most generally there exist in the Evans unified field theory symmetric, antisymmetric, and asymmetric fields which originate in the well-known fact [16] that any square (in general asymmetric) matrix can be resolved into the sum of symmetric and antisymmetric components. The geometrization of physics inherent in the basic Evans field equation means that each component has a physical significance, i.e., each component is a type of radiated or matter field in nature, some are known, others are unexplored, but all are understood consistently and are thus unified philosophically.

To understand nature, study geometry: the essence of general relativity.

In order to derive Eqs. (4) and (5), start from the Evans field Eq. (1) and use the following relations:

$$G_{\mu}^a = -\frac{1}{4}Rq_{\mu}^a, \quad (16)$$

$$T_{\mu}^a = \frac{1}{4}Tq_{\mu}^a. \quad (17)$$

Equations (16) and (17) are derived from the definitions [1-12] of R and T introduced originally by Einstein [12]:

$$R = g^{\mu\nu} R_{\mu\nu}, \quad T = g^{\mu\nu} T_{\mu\nu}. \quad (18)$$

Using the Einstein convention [12]

$$g^{\mu\nu} g_{\mu\nu} = 4 \quad (19)$$

and the Cartan convention [11]

$$q_{\mu}^a q_a^{\mu} = 1, \quad (20)$$

together with the definitions (13) and (14), we obtain:

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ &= q_a^{\mu} q_b^{\nu} \eta^{ab} R_{\mu}^a q_{\nu}^b \eta_{ab} \\ &= (\eta^{ab} \eta_{ab})(q_b^{\nu} q_{\nu}^b)(q_a^{\mu} R_{\mu}^a) = 4q_a^{\mu} R_{\mu}^a. \end{aligned} \quad (21)$$

Multiply either side of Eq. (21) by q_μ^a to obtain

$$R_\mu^a = \frac{1}{4}Rq_\mu^a, \quad (22)$$

$$G_\mu^a = R_\mu^a - \frac{1}{2}Rq_\mu^a = -\frac{1}{4}Rq_\mu^a, \quad (23)$$

which is Eq. (16). Similarly, we obtain Eq. (17).

Substitution of Eqs. (16) and (17) in the Evans field Eq. (1) gives

$$G_\mu^a = kT_\mu^a. \quad (24)$$

We first show as follows that Eq. (24) leads to the Einstein field equation as a particular case by writing Eq. (24) in the form

$$\frac{1}{4}Rq_\mu^a - \frac{1}{2}Rq_\mu^a = \frac{1}{4}kTq_\mu^a. \quad (25)$$

Multiply both sides of Eq. (25) by $q_\nu^b \eta_{ab}$ to get the Einstein field Eq. (11). The latter is therefore a structure that is derivable straightforwardly from the more general Evans field Eqs. (1) or (24) by forming dot products of tetrads. The latter reveal the inner or deeper structure of the well-known Einstein field equation. It follows that the most general form of the Bianchi identity of geometry is

$$D^\mu G_\mu^a = 0, \quad (26)$$

and the most general conservation law of physics is consequently

$$D^\mu T_\mu^a = 0. \quad (27)$$

Consider now a special case of the tetrad postulate (3):

$$D^\mu q_\mu^a = 0, \quad (28)$$

a special case which follows from Eq. (3) on using

$$D^\mu q_\mu^a = D^0 q_0^a + D^1 q_1^a + D^2 q_2^a + D^3 q_3^a = 0. \quad (29)$$

The Evans identity (26) can therefore be developed using Eq. (28) as:

$$D^\mu (Rq_\mu^a) = RD^\mu q_\mu^a + q_\mu^a D^\mu R = q_\mu^a D^\mu R = 0. \quad (30)$$

Similarly the Evans conservation law (27) can be developed as

$$D^\mu (Tq_\mu^a) = q_\mu^a D^\mu T = 0, \quad (31)$$

where $R = -kT$ for all radiated and matter fields.

On using the well-known geometrical result [11] that the covariant derivative acting on any scalar quantity is the ordinary derivative:

$$D^\mu R = \partial^\mu R, \quad D^\mu T = \partial^\mu T, \quad (32)$$

Equations (30) and (31) become

$$q_\mu^a \partial^\mu R = 0, \quad q_\mu^a \partial^\mu T = 0, \quad (33)$$

and the Evans field equation becomes the identity

$$q_\mu^a \partial^\mu (R + kT) = 0, \quad (34)$$

where

$$q_\mu^a \partial^\mu R = q_\mu^a \partial^\mu T = 0. \quad (35)$$

Equation (34) is similar in structure to the Evans wave equation (2), but Eq. (34) is an identity because R is $-kT$. The Evans identity (30) shows that $\partial^\mu R$ is orthogonal to the tetrad q_μ^a in the non-Minkowski base manifold indexed μ . Similarly, the Evans conservation law (31) shows that $\partial^\mu T$ is orthogonal to the tetrad. These results give deeper insight into the meaning of the Bianchi identity and the Noether theorem.

From Eq. (22),

$$q_\mu^a = \frac{4}{R} R_\mu^a, \quad (36)$$

and, using Eq. (28),

$$D^\mu \left(\frac{4}{R} R_\mu^a \right) = 0, \quad (37)$$

i.e.,

$$R_\mu^a D^\mu \left(\frac{4}{R} \right) = 0. \quad (38)$$

Therefore, Eq. (38) shows that

$$R q_\mu^a D^\mu \left(\frac{1}{R} \right) = 0. \quad (39)$$

Equations (30) and (39) show that

$$\partial^\mu R \propto \pm R \partial^\mu \left(\frac{1}{R} \right), \quad (40)$$

where the coefficient of proportionality must in general be scalar curvature R . Thus we arrive at Eq. (4). Replacing R by $-kT$ gives Eq. (5), and differentiating Eq. (4) leads to

$$\square R = \pm R^2 \square \left(\frac{1}{R} \right). \quad (41)$$

Equations (4) and (41) must have analytical solutions in general, i.e., R must be continuously differentiable. If we consider

$$\frac{1}{R} \partial^\mu R = -R \partial^\mu \left(\frac{1}{R} \right) \quad (42)$$

or, specifically, the time component

$$\frac{1}{R} \frac{\partial R}{\partial t} = -R \frac{\partial}{\partial t} \left(\frac{1}{R} \right), \quad (43)$$

then a solution of Eq. (43) is seen to be

$$R = R_0 e^{i\omega t}, \quad (44)$$

with a real part

$$\text{Re}(R) = R_0 \cos \omega t. \quad (45)$$

The cosine function is bounded by plus or minus unity and never goes to infinity. Therefore there can no singularity in the scalar curvature R . It follows from Eq. (18) that there is never a singularity in the metric $g_{\mu\nu}$ or Ricci tensor $R_{\mu\nu}$. In other words, the universe evolves without a singularity, and it follows that the well-known singularity theorems built around the Einstein field equation do not have any physical meaning. These singularity theorems are complicated misinterpretations. In other words, general relativity must always be a field theory that is everywhere analytical [17]. Similarly, the older Newton theory must be everywhere analytical. There are no singularities in nature. Equation (45) shows that the universe can contract to a dense state, but then re-expands and re-contracts. Apparently we are currently in a state of evolution where the universe is on the whole expanding. This does not mean that every individual part of the universe is expanding. Some parts may be contracting or may be stable with respect to the laboratory observer.

Equations (4) and (41) suggest that all radiated and matter fields evolve through a wave equation. This inference leads to causal wave mechanics. The Evans wave equation (2) is an equation of wave mechanics in which the eigenfunction is the tetrad. The evolution of the tetrad is causal, in the sense that there is no Heisenberg uncertainty principle in general relativity, and there is no need for such a

principle to describe nature. The uncertainty principle is subjective; it essentially asserts that nature is unknowable and that measurement is subjective. Any assertion about the unknowable, however, is inevitably subjective in itself, is unmeasurable by definition, and is therefore outside the domain of natural philosophy. General relativity is fundamentally incompatible with this principle, because general relativity asserts that nature is knowable and objectively measurable, given the equations that govern it. These are now known to be the equations of the Evans unified field theory. Figures (1) and (2) summarize how well-known and tested equations of classical and quantum mechanics emerge [1-10] from the Evans unified field theory. The latter unifies all radiated and matter fields and also unifies general relativity and quantum mechanics. Wave equations may also be constructed in which the eigenfunction is R or T , proving in another way that nature is knowable, because R and T are governed by general relativity. In the rest of this section we illustrate the construction of this class of wave equations from the original Einstein field theory itself. This exercise can be repeated to give a more general class of such wave equations based on the Evans unified field theory.

The starting point for this class of wave equation are the Einsteinian definitions [12]

$$R = R_{\mu\nu}g^{\mu\nu}, \quad (46)$$

$$T = T_{\mu\nu}g^{\mu\nu}. \quad (47)$$

Multiplication on both sides by $g_{\mu\nu}$ gives

$$Rg_{\mu\nu} = 4R_{\mu\nu}, \quad Tg_{\mu\nu} = 4T_{\mu\nu}, \quad (48)$$

i.e.,

$$R_{\mu\nu} = \frac{1}{4}Rg_{\mu\nu}, \quad T_{\mu\nu} = \frac{1}{4}Tg_{\mu\nu}. \quad (49)$$

The restricted or conventional Noether theorem [11] therefore reads

$$D_{\mu}(Tg^{\mu\nu}) = g^{\mu\nu}D_{\mu}T + TD_{\mu}g^{\mu\nu} = 0. \quad (50)$$

Multiplying this equation by $g_{\mu\nu}$, we get

$$D_{\rho}R = \alpha_{\rho}R, \quad (51)$$

where

$$\alpha_{\rho} = -\frac{1}{4}g_{\rho\nu}D_{\mu}g^{\mu\nu}. \quad (52)$$

Equation (51) is a first-order differential equation. Similarly,

$$D_{\rho}T = \alpha_{\rho}T. \quad (53)$$

Use of Eqs. (32) gives

$$\partial_\rho R = \alpha_\rho R, \quad \partial_\rho T = \alpha_\rho T \quad (54)$$

which are first-order equations in the ordinary rather than the covariant derivative.

The wave equations follow straightforwardly by differentiation:

$$\square R = \partial^\rho(\alpha_\rho R), \quad \square T = \partial^\rho(\alpha_\rho T). \quad (55)$$

On using

$$\partial^\rho(\alpha_\rho R) = \alpha^\rho \partial^\rho R + R \partial^\rho \alpha_\rho, \quad (56)$$

Equation (55) becomes the second-order differential, or wave, equation

$$\square R = (\alpha_\rho \alpha^\rho + \partial^\rho \alpha_\rho) R, \quad (57)$$

where

$$\alpha_\rho \alpha^\rho = \frac{1}{16} (g_{\rho\nu} D_\mu g^{\mu\nu}) (g^{\rho\nu} D^\mu g_{\mu\nu}). \quad (58)$$

The wave equation can be written as

$$(\square + \beta) R = 0, \quad (59)$$

where

$$\beta = -(\partial^\rho \alpha_\rho + \alpha^\rho \alpha_\rho). \quad (60)$$

Similarly,

$$(\square + \beta) T = 0. \quad (61)$$

Equations (59) and (61) have the structure (see Fig. (2)) of the main wave equations of physics; but, along with the Evans wave equation [1-10], they are also equations of general relativity and therefore causal. They show that R and T are quantized for all radiated and matter fields of nature. We describe this procedure as “causal quantization,” to distinguish it from Heisenberg’s quantization, which is subjective as argued and should have no place within objective, and objectively measurable, natural philosophy. The Evans unified field theory imparts a deterministic structure to nature and resolves the twentieth century debate between the Copenhagen school and the deterministic school in physics, coming down firmly on the latter’s side. The Evans unified field theory also suggests the existence of unexplored areas of physics and develops the standard model into a generally covariant field theory of all radiated and matter fields, while recovering (see Figs. (1) and (2)) the previously known and tested equations of physics. Some of them, for example the Maxwell-Heaviside equations, are developed into structures such as $O(3)$ electrodynamics [18], which has been fully tested experimentally and is compatible with general relativity.

3. GENERAL WAVE, FIELD AND FORCE EQUATIONS OF THE EVANS THEORY

The wave and field equations of this section are generalizations to unified field theory of the well-known wave and gauge field equations of electrodynamics [19]. Consider the Evans field equation in the form

$$Gq_{\mu}^a = kTq_{\mu}^a. \quad (62)$$

The unified potential field is the tetrad or vector-valued one-form q_{μ}^a , which is in general an asymmetric square matrix. The latter can always be written as the sum of symmetric and antisymmetric component square matrices, components that are physically meaningful potential fields of nature:

$$q_{\mu}^a = q_{\mu}^{a(S)} + q_{\mu}^{a(A)}. \quad (63)$$

In the Evans unified field theory the gravitational potential field is identified [1-10] as the tetrad q_{μ}^a and the electromagnetic potential field as $A^{(0)}q_{\mu}^a$, where $A^{(0)}$ is measured in $V \cdot s/m$. The unit of magnetic flux, i.e., the weber (or $V \cdot s$) belongs to \hbar/e , the magnetic fluxon, and both \hbar and e are manifestations of the principle of least curvature [1-10] of the Evans unified field theory. Both the gravitational and the electromagnetic potential fields can in general have symmetric and antisymmetric components:

$$A_{\mu}^a = A^{(0)}q_{\mu}^a = A_{\mu}^{a(S)} + A_{\mu}^{a(A)}, \quad (64)$$

and all four components appearing in Eqs. (63) and (64) are objectively measurable fields of nature. Geometry shows that there can be two types of gravitational potential fields: $q_{\mu}^{a(S)}$ and $q_{\mu}^{a(A)}$, and two types of electromagnetic potential field, $A_{\mu}^{a(S)}$ and $A_{\mu}^{a(A)}$. These four types of field are governed by four Evans field equations:

$$R_1q_{\mu}^{a(S)} = -kT_1q_{\mu}^{a(S)}, \quad (65)$$

$$R_2q_{\mu}^{a(A)} = -kT_2q_{\mu}^{a(A)}, \quad (66)$$

$$R_3A_{\mu}^{a(S)} = -kT_3A_{\mu}^{a(S)}, \quad (67)$$

$$R_4A_{\mu}^{a(A)} = -kT_4A_{\mu}^{a(A)}, \quad (68)$$

in which appear four types of canonical energy momentum tensor: symmetric gravitational, antisymmetric gravitational, symmetric electromagnetic, and antisymmetric electromagnetic. In Einstein's generally

covariant theory of gravitation [12], only one type of canonical energy-momentum tensor appears, the symmetric gravitational. In the weak-field limit, the latter gives Newtonian dynamics, in which the force field is centrally directed along the line between two point masses in Newton's inverse square law. The latter emerges from Einstein's theory of 1915 [12] as a flat spacetime limit of Riemannian geometry with curvature but no torsion. There is no sense of torsion or spin in Newtonian dynamics and no sense of torsion or spin in Einstein's generally covariant theory of gravitation [12]. In electrostatics, the force field corresponding to the Coulomb inverse square law is also central, and there is no torsion present. In electrodynamics however, there exists the magnetic field, signifying spin, and electrodynamics in the Evans unified field theory [1-10] is a generally covariant theory with torsion as well as curvature. We conclude that the symmetric part of the tetrad $q_\mu^{a(S)}$ represents the central, gravitational potential field, and the symmetric $A_\mu^{a(S)}$ represents the central, electrostatic potential field. The antisymmetric $A_\mu^{a(A)}$ represents the rotating and translating electrodynamic potential field.

The antisymmetric $q_\mu^{a(A)}$ represents a type of spinning potential field which is C positive, where C is charge conjugation symmetry [20]. This fundamental potential field of nature is not present in the Einsteinian or Newtonian theories of gravitation as argued and is a field that is governed by the Evans equation (66). It may be the potential field of dark matter [13], which is observed to constitute the great majority of mass in the vicinity of spiral galaxies. Significantly, the latter are thought to be formed by spinning motion, responsible for their characteristic spiral shape. The field $q_\mu^{a(A)}$ is not centrally directed and so does not manifest itself in the Newtonian inverse square law in the weak-field limit. (Similarly, the antisymmetric electrodynamic $A_\mu^{a(A)}$ does not reduce to the Coulomb inverse square law, which must be obtained [1-10] from the symmetric electrostatic $A_\mu^{a(S)}$.) The antisymmetric $q_\mu^{a(A)}$ is also the root cause of the well-known Coriolis and centripetal accelerations, which conventionally require a rotating frame not present in Newtonian dynamics. The rotating frame is built into the Evans unified field theory as spacetime torsion.

All of these fields emerge systematically from the tetrad q_μ^a by splitting it into its symmetric and antisymmetric components and by multiplying them by a C negative coefficient whose unit is the volt. The original asymmetric tetrad is the unified potential field of nature. The C negative manifestation of the unified field is $\zeta^{(0)}q_\mu^a$, where $\zeta^{(0)}$ must be determined experimentally. The coefficient $\zeta^{(0)}$ determines for example the way in which an electrostatic field affects the gravitational field. All forms of energy-momentum are interconvertible, implying

that

$$T_{\mu}^a = T_{\mu}^{a(S)} + T_{\mu}^{a(A)}. \quad (69)$$

The interaction field $\zeta^{(0)}q_{\mu}^a$ and its concomitant T_{μ}^a may for example be measurable in the influence of an electrostatic field on the gravitational field [14,15]. If so, the total interaction between two charged particles would be the sum of the Newton and Coulomb inverse square laws and a hitherto unknown interaction component which must be looked for with high precision balance experiments [14]. For example, there may be a tiny effect of an electrostatic field on a perfect insulator in one arm of a high precision (e.g., picogram resolution) balance. There may also be a tiny effect of mass (perfect insulators) used to unbalance a high precision device such as a Wheatstone bridge. In other words, the fundamental term $\zeta^{(0)}$ is not present in Einstein's original theory of general relativity, because that deals only with gravitation. So $\zeta^{(0)}$ is an example of a new concept of the Evans theory, the subject of this series of papers. The concept of $\zeta^{(0)}$ has its fundamental origin in thermodynamics: All types of energy-momentum are interconvertible, and so all types of potential field are interconvertible. The mechanism of the interconversion must be found by experiment. Reproducible and repeatable effects of an electric field on gravitation, for example, can be understood within the Evans unified field theory, but not within the standard model.

We may always define the unified field by

$$G_{\mu}^a = G^{(0)} \left(R_{\mu}^a - \frac{1}{2} R q_{\mu}^a \right) \quad (70)$$

and the unified energy-momentum by

$$G_{\mu}^a = G^{(0)} k T_{\mu}^a. \quad (71)$$

The unified (i.e., most general form of the) Bianchi identity is Eq. (26), and the unified conservation theorem is Eq. (27). By covariant differentiation, we obtain

$$D_{\mu}(D^{\mu}G_{\mu}^a) = (D_{\rho}D^{\rho})G_{\mu}^a = 0, \quad (72)$$

and therefore [1-10] arrive at the unified wave equations

$$(\square + kT)G_{\mu}^a = (\square + kT)T_{\mu}^a = 0, \quad (73)$$

whose eigenfunctions are the unified field G_{μ}^a and the unified T_{μ}^a . Self-consistently, these wave equations can be obtained straightforwardly

from the Evans wave Eq. (2) on using Eqs. (16) or (17). If $\zeta^{(0)}$ exists, it is also governed by a wave equation

$$(\square + kT)(\zeta^{(0)} q_\mu^a) = 0. \quad (74)$$

The Evans lemma [1-10]

$$\square q_\mu^a = Rq_\mu^a \quad (75)$$

gives rise to a class of identities:

$$\square G_\mu^a = RG_\mu^a, \quad (76)$$

$$\square T_\mu^a = RT_\mu^a, \quad (77)$$

so that the Bianchi identity is generalized to the wave equation

$$\square G_\mu^a = RG_\mu^a = -\frac{1}{4}R^2 q_\mu^a \quad (78)$$

and the Noether theorem to the wave equation

$$\square T_\mu^a = RT_\mu^a = \frac{1}{4}RT q_\mu^a. \quad (79)$$

The most general asymmetric gauge field is

$$G_{\mu\nu}^{ab} = \frac{G}{4} q_\mu^a q_\nu^b \quad (80)$$

and most general $T_{\mu\nu}^{ab}$ tensor is

$$T_{\mu\nu}^{ab} = \frac{T}{4} q_\mu^a q_\nu^b. \quad (81)$$

These can also be written as sums of symmetric and antisymmetric components:

$$G_{\mu\nu}^{ab} = G_{\mu\nu}^{ab(S)} + G_{\mu\nu}^{ab(A)}, \quad (82)$$

$$T_{\mu\nu}^{ab} = T_{\mu\nu}^{ab(S)} + T_{\mu\nu}^{ab(A)}. \quad (83)$$

The homogeneous field equation is then defined by the Jacobi identity for any antisymmetric matrix,

$$D^\mu G_{\mu\nu}^{ab(A)} = 0, \quad (84)$$

and the general inhomogeneous field equation is defined by

$$D^\mu \tilde{G}_{\mu\nu}^{ab(A)} = J_\nu^{ab}, \quad (85)$$

where $\tilde{G}_\mu^{ab(A)}$ is the dual of $G_{\mu\nu}^{ab}$:

$$\tilde{G}_{\mu\nu}^{ab} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma ab(A)} \quad (86)$$

and where J_ν^{ab} is a general charge-current density. It is also possible to define a general force equation from the asymmetric matrix $T_{\mu\nu}^{ab}$:

$$D^\mu T_{\mu\nu}^{ab} = -f_\nu^{ab}. \quad (87)$$

Note that Eq. (87) is valid only for a subsystem [21]. For a closed system the net force of Eq. (87) may be zero. Finally, the most general form of the Lorentz force equation may be written as

$$f_\nu^{ab}(\text{Lorentz}) = G_{\mu\nu}^{ab} J^{\mu ab}, \quad (88)$$

showing that the Lorentz force equation is also an equation of general relativity.

Acknowledgments. The Ted Annis Foundation, ADAS, and Craddock Inc. are thanked for funding, and the staff of AIAS for many interesting discussions.

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