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Interferometry in Higher Symmetry Forms of Electrodynamics and Physical Optics

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Abstract

It is shown that higher symmetry (O(3)) electrodynamical considerations lead to a straightforward explanation of the Sagnac effect in terms of the topological phase and Aharonov Bohm effect. The method is extended to interferometry and physical optics in general, using the Sagnac, Michelson and Young interferometers as examples.

1. Introduction

There are several indications that the Maxwell equations are an incomplete [1–4] description of electrodynamics and optics. In 1975 Wu and Yang [15] suggested the introduction of a non-Abelian, topological phase factor which multiplies the usual electromagnetic phase, and this is related to the well known topological phases [6] which are observable yet do not exist in Maxwell's equations, more accurately, Heaviside's reduction of Maxwell's equations to vector form from the original twenty quaternionic equations devised by J. C. Maxwell himself [3,4]. In this note it is demonstrated that the well known Sagnac effect with platform at rest and in motion is a straightforward result of a higher symmetry form of Maxwell's equations suggested by the topological and Wu Yang phases. The symmetry used is the non-Abelian O(3) symmetry of the rotation group. Recently this higher symmetry electrodynamics has been developed extensively [7–12] and applied to several phenomena which distinguish it from the U(1) symmetry of electrodynamics in which the familiar Maxwell equations are usually written.

In their O(3) form the Maxwell equations look like:

$$\left.\begin{aligned} D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} &:= o \\ D_\mu \boldsymbol{H}^{\mu\nu} &:= \boldsymbol{J}^\nu \end{aligned}\right\} \tag{1}$$

where the usual partial four-derivatives of the ordinary Maxwell equations are replaced by covariant derivatives (D_μ) and where the usual field tensors and four current are written as vector symbols, because they are defined in an internal gauge space of O(3) symmetry, a vector space. The equations are written in Minkowski spacetime, so the Greek subscripts and superscripts have their usual significance [1–4] in special relativity. Therefore O(3) electrodynamics is a theory of special relativity, but improves significantly on the usual U(1) electrodynamics, in which Maxwell's equations are the usual ones:

$$\left.\begin{aligned} \partial_\mu \tilde{G}^{\mu\nu} &:= o \\ \partial_\mu H^{\mu\nu} &= J^\nu \end{aligned}\right\} \tag{2}$$

with ordinary four derivatives, field tensors and four-vectors.

2. The Sagnac effect

There are several explanations available of the Sagnac effect, because U(1) electrodynamics ("U(1)" for short) has considerable difficulty in explaining it [4,13]. When a beam of light is split into two components, sent around two paths, anticlockwise (A) and clockwise (C), and recombined at a beamsplitter, an interferogram is observed, despite the fact that there is no phase difference between identical paths A and C from the U(1) Maxwell equations. When the platform on which the interferometer rests is rotated about an axis perpendicular to its plane the fringes of the interferogram are shifted – the Sagnac effect, for which several, sometimes conflicting, explanations are available in U(1) [4,13]. None of these is successful in explaining the interferogram with platform at rest.

In O(3) electrodynamics however the explanation is a straightforward application of the Wu Yang phase [1–5] difference between A and C:

$$\Delta\phi = P\exp\left(\mathrm{i}\,\frac{e}{\hbar}\oint_{\mathrm{A-C}} A_\mu\,\mathrm{d}x^\mu\right) \tag{3}$$

where P denotes the path dependence of the phase, and where $e/\hbar$ is the ratio of the elementary charge to the Planck constant. The integration over the four vector is path dependent. In order to recover the phase shift with platform at rest we use the de Broglie relation for the free photon:

$$p = \hbar\kappa = eA^{(\mathrm{o})} \tag{4}$$

where κ is the wave-vector magnitude and $A^{(o)}$ the scalar magnitude of A_μ. Application of the non-Abefian Stokes Theorem relevant to O(3) electrodynamics [1–12] to the Wu Yang phase equation (3) produces equal and opposite magnetic flux densities for loops A and C on the static platform:

$$\left.\begin{aligned} \boldsymbol{B}^{(3)}(\mathrm{A}) &= \kappa A^{(o)}\boldsymbol{k} \\ \boldsymbol{B}^{(3)}(\mathrm{C}) &= -\kappa A^{(o)}\boldsymbol{k} \end{aligned}\right\} \tag{5}$$

The phase shift with platform at rest is therefore topological in origin (i.e. is an example of an observable topological phase shift) and so is the optical equivalent of the Aharonov Bohm effect [14]. It signals the existence of a topological magnetic monopole and concomitant magnetic flux density, $\boldsymbol{B}^{(3)}$. It is given by:

$$\Delta\phi = \cos\left(2\frac{\omega^2}{c^2}Ar \pm 2\pi n\right), \tag{6}$$

where n is an integer and Ar the loop area. The phase shift occurs therefore in a non simply connected covering space [14]. This explanation depends on the existence [7–12] of the longitudinally directed four potential peculiar to O(3) electrodynamics and the absense from U(1) electrodynamics.

The rotation of the platform of the Sagnac interferometer is the equivalent in O(3) electrodynamics of a gauge transformation, a physical, Lorentz covariant, rotation [7–12, 14] which produces the Doppler shift:

$$\omega \to \omega \pm \Omega \tag{7}$$

where Ω is the angular frequency at which the platform is rotated. Using this result in eqn. (6) and in the limit $\omega^2 >> 2\omega\Omega >> \Omega^2$ produces the well known expression for the Sagnac effect (ring laser gyro effect):

$$\Delta\Delta\phi = \cos\left(\frac{4\omega\Omega Ar}{c^2} \pm 2\pi n\right) \tag{8}$$

We have easily explained the Sagnac effect using some well known features of the higher Maxwell equations (1) implied by the Wy Yang phase (3).

3. Interferometry

This is a general result in interferometry, and therefore signals the superiority of O(3) over U(1) symmetry electrodynamics. The well known Michelson interferogram, for example, is recovered from the Wu Yang phase equation (3) and the de Broglie equivalence (4). The changes of phase in arms X and Y of the Mchelson interferometer are:

$$\left.\begin{aligned} \Delta\phi_x &= P\exp\left(\mathrm{i}\frac{\kappa}{A^{(o)}}\oint A_X\,\mathrm{d}X\right) \\ \Delta\phi_y &= P\exp\left(\mathrm{i}\frac{\kappa}{A^{(o)}}\oint A_Y\,\mathrm{d}Y\right) \end{aligned}\right. \tag{9}$$

and in general these changes of phase are unequal, because the arms are unequal [15]. The, Michelson interferogram appears and is again, a topological phase effect. In U(1) we always have the following result:

$$\Delta\phi_x - \Delta\phi_y = 0 \tag{10}$$

and there can be no explanation of the interferogram unless an arbitrary phase is introduced without explanation.

Note that A_x and A_y do not exist in U(1) because both are longitudinally directed in fire space, a feature specific to O(3) [7–12].

As a final example we recover immediately the well known Young interferogram (two slit experiment) from the Wu Yang phase with, again, longitudinally directed (O(3) specific)) free space potentials:

$$\begin{aligned} \Delta\phi &= P\exp\left(\mathrm{i}\frac{\kappa}{A^{(o)}}\oint \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{r}\right) \\ &= P\exp\left(\mathrm{i}\frac{2\pi\Delta r}{\lambda}\right); \\ \mathrm{Re}(\Delta\phi) &= P\cos\left(2\pi\left(\frac{\Delta r}{\lambda} \pm n\right)\right). \end{aligned} \tag{11}$$

Vigier [16] has recently reviewed and discussed the Sagnac effect in the context of finite photon mass special and general relativity, as first suggested by de Broglie. He provides a satisfactory explanation of why the fringe shift in the Sagnac effect is the same for the observer on and off the platform. The explanation given in this paper can be related straightforwardly to Vigier's theory because non-Abelian electrodynamics is a gauge covariant theory which allows for the existence of the preferred frame needed for finite photon mass in special relativity.

3. Discussion

It has been shown that various kinds of interferometry cannot be explained by the standard model, which relies on a U(1) sector symmetry for electrodynamics, but can be explained by O(3) electrodynamics. In this discussion the difference between the two theories (U(1) and O(3)) are explained in more detail.

There have been many attempts to explain the Sagnac, effect [16–21], both kinematic and electrodynamic [3, 4]. The unmodified Heaviside Maxwell ((U(I)) theory [22] gives a null result, both for platform at rest and in motion. The experimental result on the other hand is well described by Fleming [23], and quoted as follows: "If a beam of light (photons) is split by means of a combined beam-splitter/interferometer, and sent in opposite directions around the circumference of a stationary disc using mirrors or optical fibres, an interference pattern is observed on the interferometer. The disc is capable of being rotated and the apparatus is fixed in the laboratory [23]. If the disc is now rotated the interference fringe is shifted on the interferometer relative to the stationary disc position. If the disc is now rotated in the other direction the fringe moves to the other side of the stationary disc position. The effect is seen irrespective of whether the observer rotates with the disc on its periphery, or is stationary in the laboratory".

In the free space Heaviside Maxwell theory the electromagnetic phase is te scalar quantity

$$\phi = \omega t - \kappa Z \tag{12}$$

where ω is the angular frequency of the light at instant t and κ the wave-number at position Z. Under motion reversal symmetry ϕ is invariant:

$$\hat{T}(\phi) = \phi. \tag{13}$$

Since motion reversal symmetry generates the anticlockwise loop of the Sagnac interferometer from the clockwise loop the phase difference with platform at rest in U(l) theory is zero, contrary to observation [23]. This null phase difference is furthermore Lorentz and frame invariant, so the U(I) sector of the standard model cannot explain the Sagnac effect with platform at rest.

When the platform is rotated the observed fringe shift does not appear in the Heaviside Maxwell theory because these equations are invariant to rotation in free space. The null result $\Delta\phi = 0$ does not change because nothing physical is added to or subtracted from ϕ. In the theory when the platform is rotated, and the Maxwell Heaviside equations are also metric independent. As early as 1917, Pegram [24] showed that there is a cross relation between electric and magnetic fields which is denied by Lorentz transformation [2,3]. The true equations of the electromagnetic field in vacuo are not Lorentz invariant. Such a cross relation was confirmed recently by the observation of magneto-chiral birefringence [25] which depends on the existence of the cross product $\boldsymbol{E} \times \boldsymbol{B}^*$, and which for plane waves in the basis ((1), (2), (3)) can be written as $\boldsymbol{E}^{(1)} \times \boldsymbol{B}^{(2)}$ [8–12]. This cross product can be related to the conjugate product of vector potentials in the vacuum:

$$\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} = -\frac{\mathrm{i}c}{\omega^2}\boldsymbol{B}^{(1)} \times \boldsymbol{E}^{(2)} \tag{14}$$

where c is the speed of light. Under the rules of a U(1) gauge transformation however, $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is not invariant, which is self-contradictory. In the definition of the field tensor in U(1) gauge field theory, furthermore [12], the cross product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is rigorously zero. Yet it is also an observable of the inverse Faraday effect [8–12] and of the third Stokes parameter, through:

$$S_3 = -\mathrm{i}\,|\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}|, \tag{15}$$

the third Stokes parameter being

$$S_3 = -\mathrm{i}|\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}| = -\mathrm{i}\omega^2|\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}| = -\mathrm{i}c^2|\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}| \tag{16}$$

Post [2,3] has attempted to explain the Sagnac effect in terms of $\boldsymbol{E}^{(1)} \times \boldsymbol{B}^{(2)}$, but as the above argument shows, this leads to a consideration of $\boldsymbol{A}^{(1)} \times \boldsymbol{B}^{(2)}$, which does not exist by definition in the U(1) theory of electromagnetism. Nevertheless, Post was on the right track to the higher symmetry gauge field theory that self-consistently contains $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, and explains the Sagnac effect straightforwardly.

To self check eqn. (13) we define the four potential in vacuo as:

$$A^{\mu} := (A_{\mathrm{o}}, cA) \tag{17}$$

which has the units of J/C. It follows that:

$$\hat{T}(A^{\mu}) = A^{\mu}; \quad \hat{P}(A^{\mu}) = A^{\mu} \tag{18}$$

where $\hat{T}$ and $\hat{P}$ are the motion and parity inversion operators. The d'Alembert equation for the A and C loops of the Sagnac interferometer with platform at rest is therefore the same:

$$\Box A^{\mu}(A) = \Box A^{\mu}(C) = 0 \tag{19}$$

and the solutions are the same. There is no phase difference, checking eqn. (13). In Michelson interferometry:

$$\hat{P}(\Box A^{\mu} = 0) = (\Box A^{\mu} = 0) \tag{20}$$

and the Heaviside Maxwell equations do not describe Michelson interferometry, because the d'Alembert equation in vacuo, is unchanged by reflection at the mirror. Recombination at the beam divider of the Michelson interferometer therefore results in no change of phase, and no interferogram, contrary to observation [26].

Further problems occur for the U(1) theory of interferometry because A^{μ} is unphysical in U(1) theory [3,4], and a random quantity may be added to the phase ϕ of the plane wave under U(1) gauge transformation. The phase ϕ also becomes unphysical, which is obviously contrary to the everyday observations of various kinds of interferometry as described in this paper. The existence of interferometry is therefore a counter example to the Heaviside Maxwell theory.

The O(3) theory on the other hand is able to describe the Sagnac, Michelson and Young effects straightforwardly. The fundamental reason for this is the existence in the O(3) of a longitudinal and physical radiated magnetic field, defined classically by:

$$\boldsymbol{B}^{(3)*} := -\mathrm{i}\frac{\kappa}{A^{(\mathrm{o})}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{21}$$

This definition of $\boldsymbol{B}^{(3)}$ occurs in the fundamental field tensor and may be integrated as follows to give a theorem relating the dynamical and topological phases [2,3]. The latter are well established experimentally [3] and are due to an integral over $\boldsymbol{B}^{(3)}$. Thus $\boldsymbol{B}^{(3)}$ is clearly a physical observable whenever we observe interferometry. It is the fundamental reason for the existence of all interferometric phenomena of light with light and is therefore a novel fundamental quantity in optics and spectroscopy.

Using the relation:

$$B^{(\mathrm{o})} = \kappa A^{(\mathrm{o})} \tag{22}$$

in eqn. (21) and multiplying both sides by the area $Ar := \pi R^2$, we obtain:

$$\pi\kappa A^{(\mathrm{o})}\boldsymbol{R}\cdot\boldsymbol{R} = \boldsymbol{B}^{(3)}\cdot Ar\,\boldsymbol{k}. \tag{23}$$

Using the identity:

$$\pi R^2 = \int 2\pi\boldsymbol{R}\cdot\mathrm{d}\boldsymbol{R} = \int\!\!\int \mathrm{d}\boldsymbol{Ar} \tag{24}$$

the area integral over $\boldsymbol{B}^{(3)}$ can be equated to a line integral as on the left hand side of the following equation:

$$\oint 2\pi\boldsymbol{R}\cdot\mathrm{d}\boldsymbol{R} = \int\!\!\int \boldsymbol{B}^{(3)}\cdot\mathrm{d}\boldsymbol{Ar}. \tag{25}$$

The ordinary integral can be converted to a line integral if the contributions perpendicular to the direction of the $\boldsymbol{R}$ vector vanish, as in the case of an electromagnetic field. Therefore the correct expression for phase in interferometry in O(3) electrodynamics becomes:

$$\gamma = 2\pi \oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R} = \frac{\kappa}{A^{(0)}} \int\int \boldsymbol{B}^{(3)} \cdot \mathrm{d}\boldsymbol{Ar} \tag{26}$$

which is a component of an O(3) Stokes Theorem. (The use of a U(l) Stokes theorem is incorrect, so the papers by Comay [11] and Hunter [12] are completely incorrect because the $\boldsymbol{B}^{(3)}$ field does not exist in U(1) electrodynamics). The line integral is sketched as follows:

and as for all line integrals, reverses sign from OA to AO. Eqn. (26) equates the topological and dynamical phase in O(3). The effect of parity and motion reversal on the O(3) dynamical phase are both negative:

$$\hat{T}\left(\oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R}\right) = -\oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R}; \quad \hat{P}\left(\oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R}\right) = -\oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R} \tag{27}$$

because of this property of a line integral. Therefore an interferogram appears with platform at rest in the Sagnac effect, and an interferograin appears in the Michelson interferometer. In each case the dynamical phase difference is:

$$\Delta\gamma = 2\pi \oint \boldsymbol{\kappa} \cdot \mathrm{d}\boldsymbol{R}. \tag{28}$$

The difference in the Sagnac effect is due to motion reversal symmetry, and due to parity inversion symmetry in the Michelson effect. There is no explanation for these effects in the Heaviside Maxwell electrodynamics.

As discussed already, when the platform is rotated in the Sagnac effect an O(3) gauge transformation produces the result:

$$\omega \rightarrow \omega \pm \Omega. \tag{29}$$

Using this in the topological phase defined by $\boldsymbol{B}^{(3)}$:

$$\gamma = \frac{\kappa}{A^{(0)}} \int\int \boldsymbol{B}^{(3)} \cdot \mathrm{d}\boldsymbol{Ar} \tag{30}$$

gives a phase shift for the Sagnac effect of

$$\Delta\gamma = 4\frac{\omega\Omega Ar}{c^2} \tag{31}$$

which is the original result obtained by Sagnac and accurate [23] to one part in 10^{20}. In a recent experiment by Bilger *et al.* [23]. The gauge transformation (29) is due to the fact that in O(3) electrodynamics gauge transformation is covariant, and the potentials are physical on the classical level. The theory of this paper is of course on the classical level.

As discussed already the well known interferogram of the Michelson interferometer [15] which appears in every Fourier transform infra red spectrometer, is due to the dynamical phase, but can also be due to the topological phase. The latter has been observed now in all kinds of interferometry [2,3] and is again due to the $\boldsymbol{B}^{(3)}$ field.

Conclusion

It may be claimed logically that the $\boldsymbol{B}^{(3)}$ field is the first successful explanation of interferometry, a phenomenon that has been known for four hundred years. This is a significant milestone in optics and shows that the Heaviside Maxwell theory of electrodynamics, is limited in its applicability and is incomplete. The use of a U(1) sector for the electromagnetic field in unified field theory is similarly limited in validity. This is the major conclusion of this paper, based on a simple and accurate explanation in classical optics of the Sagnac, Michelson and Young effects, an explanation which is framed within special relativity and gauge theory. These results follow straightforwardly once it is realized that the internal space of the O(3) gauge theory can be the physical space ((1), (2), (3)), one based on the existence of circular polarization. Therefore this is a major advance in the understanding of optics, and in particular, interferometry.

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