

FERMION RESONANCE

**ULTRA HIGH FREQUENCY FERMION RESONANCE INDUCED BY
CIRCULARLY POLARIZED RADIATION:
THE RESONANCE INVERSE FARADAY EFFECT****ABSTRACT**

The techniques of nuclear magnetic resonance (NMR); electron spin resonance (ESR) and magnetic resonance imaging (MRI) are among the most widely used in analytical science and medicine. They are limited in resolving power by the magnetic flux density that can be generated by a permanent magnet, usually a very expensive and cumbersome superconducting magnet. In this paper, it is demonstrated in detail that NMR, ESR, and MRI occur in principle without having to use a permanent magnet, which is replaced by an inexpensive microwave or laser beam. If implemented, this technique would revolutionize the subject because it is capable of very high resolution using inexpensive apparatus. A broad overview is given first, at an introductory level, and the advanced technical details are relegated to later sections for those interested.

It is demonstrated that circularly polarized radiation induces fermion resonance without the use of a permanent magnet, a technique which we name radiatively induced fermion resonance (RFR). The resonance frequency is proportional to the power density of the circularly polarized radiation and inversely proportional to its frequency squared. It is the resonance equivalent of the empirically observable inverse Faraday effect and therefore there is indirect empirical evidence for its existence. The existence of the phenomenon is demonstrated on the following levels: classical non-relativistic; classical relativistic; quantum non-relativistic (time independent and time dependent Schrödinger equations with intrinsic spin); quantum relativistic (Dirac equation); and in quantum electrodynamics. Numerical computations of the effect are given on the quantum electrodynamic level. Suggestions are given for its experimental investigation with microwave and electron beams. All levels of theory show that the fermion resonance can be tuned to a much higher frequency than available with the most powerful of contemporary superconducting magnets. The chemical shift of the effect is worked out and depends on a novel molecular property tensor and so RFR has its own individual, site specific fingerprint. It is therefore potentially at least as useful to practical chemical physics as NMR and ESR. Potentially, it also gives a fermion resonance spectrum at a much higher resolution than conventional NMR or ESR. These combined advantages make it, potentially, a powerful new technique of wide utility in chemical physics.

INTRODUCTION

The technique of nuclear magnetic resonance (NMR), electron spin resonance (ESR) and magnetic resonance imaging (MRI) are widely used in contemporary analytical science and medicine, and all rely on one basic principle: fermion resonance. In simple terms, fermion resonance is the absorption at a particular frequency of radio or microwave radiation by inducing a change in the spin state of a fermion, which may be an electron (ESR) or proton (NMR). In magnetic resonance imaging, a field gradient is used to produce an image of an organ, an image which may be used in the analysis of many types of ailments. In conventional ESR and NMR, the resonance is detected by a circularly polarized radio or microwave frequency beam, and the energy states of the electron and proton are separated by a very tiny amount by using a very powerful permanent magnet, usually an expensive superconducting magnet. This is magnetically induced resonance, whose resolving power and usefulness is limited by the strength of the magnetic field which can be generated by the permanent magnet.

In this paper, it is shown that these limitations can be removed by replacing the powerful permanent magnet by a circularly polarized beam of electromagnetic radiation. The paper first gives an overview on an introductory level, followed by technical details for those interested.

All polarizations of light and electromagnetic radiation can be built from circular polarization, which can be left or right handed, and can be thought of in terms of a helix, the path drawn out by the tip of an electric or magnetic component of an electromagnetic field as its travel through the vacuum. Linear polarization consists of 50% left circularly polarized radiation and 50% right circularly polarized radiation, and on the fundamental level, it is thought that the photon is circularly polarized, i.e. is handed, or chiral. One of the interesting consequences of this property of linear polarization is that the plane of polarization of the linearly polarized light is tilted as it travels through a magnetic field. This is an effect first demonstrated in the early nineteenth century by Michael Faraday, and is named after him. Over a hundred years later, it was realized by the Bloembergen group at Harvard, in particular Peter Pershan, that the Faraday effect could be reversed, a circularly polarized laser pulse can induce magnetization in all material. This was named the inverse Faraday effect. In 1996, it was realized by Evans, Vigier, Roy and Jeffers that the inverse Faraday effect could be used to induce fermion resonance in all material by replacing the usual permanent magnetic field with a circularly polarized microwave field. In this paper, an overview of the effect is first given, followed by the technical details, which are relegated to later sections for the interest of specialists.

The replacement of the hugely expensive and cumbersome superconducting magnet by an ordinary microwave beam results in radiatively induced fermion resonance (RFR). This paper shows in full technical detail that RFR has a much more powerful resolution capability than ordinary NMR or ESR. RFR is therefore likely to be of interest also in MRI. RFR has this advantage at a greatly reduced cost, the multi-million dollar magnet is replaced by an ordinary circularly polarized microwave beam. It is shown that the resolving power of RFR is proportional to the circularly polarized microwave beam's power density, or intensity (watts per meter squared) and inversely proportional to its frequency (hertz). It may be possible to develop mobile MRI units for use in emergency, and mobile NMR and ESR units for the analytical laboratory and industrial on-site analysis. The reason is that the superconducting magnet, which weighs several tons, is replaced by a low power microwave beam. The technique of RFR has many applications as conventional NMR and ESR, which are used in about 40% of physical chemistry, for example, and as many applications as MRI, which is to be found in every medical facility of significance. In addition to the advantages described already, RFR has a much higher resolving power than NMR and ESR, and also has its own distinctive spectral fingerprint, known as "chemical shift".

It is shown in the technical sections of this paper, intended for specialists, that these results emerge at all levels of physics, from every known equation, from the classical level to the quantum relativistic. The technical discussion section suggests ways of initiating the experimental development of RFR, and suggests prototype experiments using microwave beam interacting with an electron or other beams. The significance of a successful development would be akin to the development of the horseless carriage. There follows progressively more difficult technical material in support of the above overview, with a technical introduction.

In this paper, it is demonstrated that the well known and empirically observable inverse Faraday effect {1-6} has a resonance equivalent, which we name radiatively induced fermion resonance (RFR). The empirically observable existence of the inverse Faraday effect means that RFR must also exist empirically. In RFR, circularly polarized radiation induces fermion resonance (for example in a proton or electron), without the use of a permanent magnet. The effect is derived theoretically at several levels from the classical non-relativistic level to quantum electrodynamics in which numerical computations of the effect are given.

In Section 2, the existence of RFR is demonstrated on the classical non-relativistic level using the minimal prescription with a complex vector potential, and also through elementary considerations of the Stokes parameters of radiation. The radiatively induced fermion resonance frequency (f_{res}) is shown to be

proportional to the power density of the radiation (I) and inversely proportional to the square of its angular frequency ($\omega = 2\pi f$, where f is the frequency in hertz). The resonance frequency (f_{res}) can therefore be tuned by adjusting I and f of the circularly polarized pump beam. The resonance frequency can be tuned to much higher than any available with conventional superconducting magnets (as in conventional NMR or ESR). Therefore the resolution of RFR is much higher than that of NMR and ESR and the chemical shift in RFR is governed by a novel molecular property tensor which is derived in section (6). Therefore RFR has its own distinct, site specific, spectral fingerprint, as well as a much higher spectral resolution than conventional NMR or ESR. The properties make it potentially very useful and important practical method in chemical physics.

The theoretical mechanism responsible for RFR is classical, and occurs in the non-relativistic limit. It therefore occurs on the relativistic classical level and in quantum mechanics. In Section (3), it is derived on the classical relativistic level using the Einstein equation. In Section (4), it is derived on the non-relativistic quantum level using the time independent and time dependent Schrödinger equations with intrinsic spin as described by Sakurai {7}. In Section (5), it is derived on the relativistic quantum level. In Section (6), the chemical shift in RFR is derived using a novel molecular property tensor which shows that the effect is site specific, as in NMR and ESR. This is the most useful analytical property of RFR in analogy with NMR or ESR, where the chemical shift is the most useful property. Finally, in Section (7), the RFR phenomenon is demonstrated in quantum electrodynamics, and numerical results given from a model computation. A discussion closes the paper with suggestions for experimental verification with a carefully designed two beam prototype experiment, the interaction of a circularly polarized microwave beam with an electron beam.

NON-RELATIVISTIC CLASSICAL LEVEL

In order to derive RFR in classical, non-relativistic, physics, we consider the interaction of a classical electron with the classical electromagnetic field using the minimal prescription with complex vector potential A (a plane wave). First consider the Newtonian kinetic energy of the classical electron:

$$H_{KE} = \frac{1}{2m} \mathbf{p} \cdot \mathbf{p} \quad (1)$$

where \mathbf{p} is the linear momentum of the electron and m its mass. Let the electron interact with a classical electromagnetic field through the minimal prescription, in which the vector potential A is in general complex valued. The interaction kinetic energy is therefore the real part of:

$$H_{KE} = \frac{1}{2m} (\mathbf{p} - e\mathbf{A}) \cdot (\mathbf{p} - e\mathbf{A}^*) \quad (2)$$

where A and A^* are complex conjugate transverse plane waves. It is well known that the conjugate product $A \times A^*$ is responsible for the empirically observable inverse Faraday effect {1-6} and that $A \times A^*$ is proportional to the third Stokes parameter that describes circular polarization {8}. On the classical level, the energy of interaction becomes:

$$H_{KE} = \frac{1}{2m} \mathbf{p} \cdot \mathbf{p} - \frac{e}{2m} \text{Re}(A \cdot \mathbf{p}) - \frac{e}{2m} \text{Re}(\mathbf{p} \cdot A^*) + \frac{e^2}{2m} A \cdot A^* \quad (3)$$

a well known result which is described in numerous texts {9}.

In order to derive the RFR term, we use Pauli matrices as the basis of the calculation as described by Sakurai {7} in his eqn. (3.18). The interaction between the classical electron and the classical electromagnetic field is then described on the classical level by:

$$H_{KE} = -\frac{1}{2m} \sigma \cdot (p - eA) \sigma \cdot (p - eA^*) \quad (4)$$

This interaction energy consists of four separate terms which are described as follows.

1) Magnetic Dipole Term

where m_0 is the magnetic dipole moment of the electron or proton and $\text{Re } B$ is the real magnetic component of the electromagnetic field.

$$H_1 = -\frac{e}{2m} p \cdot (A + A^*) \equiv \frac{e}{2m} m_0 \cdot \text{Re } B \quad (5)$$

2) Spin-flip term

$$H_2 = -i \frac{e}{2m} \sigma \cdot p \times (A^* - A) \quad (6)$$

which for an electron or proton moving in the Z axis can be expressed as:

$$H_2 = -e \frac{A^{(0)}}{\sqrt{2}} p_z \sigma_z \cdot (j \cos \phi + i \sin \phi) \quad (7)$$

where

$$\phi = \omega t - \kappa Z = \omega \left(t - \frac{Z}{c} \right) \quad (8)$$

If initially $\phi = 0$, the Pauli matrix (or "spin") σ points in the Y axis; when $\phi = \pi/2$, it points in the X axis; when $\phi = \pi$, in the $-Y$ axis; when $\phi = 3\pi/2$, in the $-X$ axis; and when $\phi = 2\pi$, back in the Y axis. So this confirms that H_2 is the spin-flip term used in all Fourier transform ESR and NMR instruments.

3) Polarizability Term

This is

$$H_3 = \frac{e^2}{2m} A \cdot A^* = \frac{e^2}{2m} A^{(0)2} \quad (9)$$

and is the basis of susceptibility theory {10}.

4) The RFR Term

The RFR term is

$$H_4 = i \frac{e^2}{2m} \sigma \cdot A \times A^* = -\frac{e^2}{2m} A^{(0)2} \sigma \cdot k \quad (10)$$

All four terms have been observed empirically. Terms (1) to (3) are well known and term (4) was observed by Pershan et al. {1} in the paramagnetic inverse Faraday effect. Thus eqn. (4) contains the spin-flip and RFR term in addition to the other familiar and observable {1-6, 8-10} terms.

Therefore the RFR term exists on the classical and non-relativistic level and so exists in quantum mechanics according to the correspondence principle.

The existence of the RFR term can be confirmed by writing the four Stokes parameters {11} as:

$$\begin{aligned} S_0 &= A_X A_X^* + A_Y A_Y^* \\ S_1 &= A_X A_X^* - A_Y A_Y^* \\ S_2 &= -(A_X A_Y^* + A_Y A_X^*) \\ S_3 &= -i(A_X A_Y^* - A_Y A_X^*) \end{aligned} \quad (11)$$

For circularly polarized light {11}:

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 = S_3^2 \quad (12)$$

so

$$S_0 = \pm S_3. \quad (13)$$

Therefore the existence of $\mathbf{A} \cdot \mathbf{A}^*$ which is proportional to S_0 implies the existence of $\pm i\mathbf{A} \times \mathbf{A}^*$, which is proportional to S_3 . If, following Landau and Lifshitz {12}, we define the light intensity tensor as

$$\rho_{\alpha\beta} = \frac{A_\alpha A_\beta^*}{A^{(0)2}} \quad (14)$$

then from eqns. (11) and (13), in circular polarization:

$$\rho_{\alpha\beta} = \frac{1}{2A^{(0)2}} \begin{bmatrix} S_0 & iS_3 \\ -iS_3 & S_0 \end{bmatrix} \quad (15)$$

If we now define the Pauli matrices {7}:

$$\sigma_X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma_Y \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; \quad \sigma_Z \equiv \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}; \quad (16)$$

they obey the commutation relations:

$$\left[\frac{\sigma_X}{2}, \frac{\sigma_Y}{2} \right] = i \frac{\sigma_Z}{2} \text{ et cyclicum,} \quad (17)$$

and so the light intensity tensor can be written as:

$$\rho_{\alpha\beta} = \frac{1}{2A^{(0)2}} \left(S_0 - i\sigma_Z \cdot \mathbf{A} \times \mathbf{A}^* \right). \quad (18)$$

This shows that the RFR term occurs in the fundamental definition of the light intensity tensor itself for circularly polarized electromagnetic radiation.

The Inverse Frequency Squared Dependence.

The RFR term shows that the intrinsic spin of a fermion forms an interaction energy with the optical conjugate product $\mathbf{A} \times \mathbf{A}^*$. It can be shown as follows that the conjugate product $\mathbf{A} \times \mathbf{A}^*$ is proportional to the

power density I of the field and inversely proportional to the square of its angular frequency $\omega = 2\pi f$. In S.I. units, the fundamental equation linking A to the magnetic field B is, in classical electrodynamics:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (19)$$

So if A is a vacuum plane wave, so is B (and its electric counterpart E , the electric field strength in volts per meter). If the plane wave A is a solution of the vacuum d'Alembert equation, it may be written as:

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}(ii + j)e^{i\phi} \quad (20)$$

and so the plane wave B is, from eqn. (19):

$$\mathbf{B}^{(1)} = \mathbf{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}}(ii + j)e^{i\phi} \quad (21)$$

Using the classical Maxwell equation, the plane wave E is:

$$\mathbf{E}^{(1)} = \mathbf{E}^{(2)*} = \frac{E^{(0)}}{\sqrt{2}}(i - ij)e^{i\phi}. \quad (22)$$

Here, ϕ is the electromagnetic phase, $A^{(0)}$, $B^{(0)}$, and $E^{(0)}$ are scalar amplitudes, and i and j are unit cartesian vectors in X and Y , perpendicular to the propagation axis Z of the plane wave. The following relations between conjugate products then follow using elementary algebra:

$$\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{c^2}{\omega^2} \mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = \frac{1}{\omega^2} \mathbf{E}^{(1)} \times \mathbf{E}^{(2)} \quad (23)$$

and show that the product $\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}$ is proportional to $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ divided by the square of the angular frequency. Expressing $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ in terms of beam intensity of power density $\{I\}$ (I in W/m^2):

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = i \frac{\mu_0}{c} I e^{(3)*} \quad (24)$$

where μ_0 is the vacuum permeability in S.I. units.

The basis of the RFR technique is that a probe photon at a resonance angular frequency ω_{res} , can be absorbed under resonance condition:

$$\hbar\omega_{res} = \frac{e^2 c^2 B^{(0)2}}{2m\omega^2} (1 - (-1)) \quad (25)$$

defined by the transition from the negative to the positive states of the Pauli matrix σ_z . This process is precisely analogous to ordinary optical absorption, in which terms NMR is a radio frequency spectroscopy. The RFR probe resonance frequency is therefore:

$$f_{res} = \frac{\omega_{res}}{2\pi} = \left(\frac{e^2 \mu_0 c}{2\pi \hbar m} \right) \frac{I}{\omega^2} \quad (26)$$

and is inversely proportional to the square of the angular frequency ω of the circularly polarized pump electromagnetic field which takes the place of the superconducting permanent magnet of ordinary NMR or ESR {14, 15}.

For ^1H proton resonance, the result (26) is adjusted empirically in our development for the experimentally different g factors of the proton and electron, respectively 5.5857 and 2.002. A more complete theory must rest on the internal structure of the proton, and similarly for other nuclei whose g factors differ from that of the proton. In principle, the theory developed here gives rise to a means of investigating nuclear properties using readily available microwave or radio-frequency generators instead of superconducting magnets.

For proton resonance, the RFR equation is therefore:

$$\omega_{\text{res}} = \left(\frac{5.5857e^2\mu_0c}{2.002\hbar m} \right) \frac{I}{\omega^2} = 1.532 \times 10^{25} \frac{I}{\omega^2} \quad (27)$$

and some data from this equation are given in Table 1, which shows that RFR proton resonance frequencies can be far higher than those attainable in ordinary NMR. Therefore the concomitant resolution of RFR is also far higher in theory and the instrumental resolution substantially improved in theory. Additionally, RFR has its own spectral fingerprint because the molecular property tensor determining the chemical shift is unique (Section (6)).

If RFR is applied to the electron, the same overall advantage is obtained, the equivalent of eqn. (27) is:

$$\omega_{\text{res}} = 1.007 \times 10^{28} \frac{I}{\omega^2} \quad (28)$$

Table 1 RFR Frequencies from eqn. (27) for the Proton for $I = 10 \text{ W/cm}^2$	
Pump Frequency	Resonance Frequency
5,000 cm^{-1} (visible)	0.28 Hz
500 cm^{-1} (infrared)	28.0 Hz
1.8 GHz	1.8 GHz (auto-resonance)
1.0 GHz (microwave)	6.18 GHz
0.1 GHz (r.f.)	20.6 cm^{-1} (far infrared)
10.0 MHz (r.f.)	2,060 cm^{-1} (infrared)
1.0 MHz (r.f.)	206,000 cm^{-1} (ultraviolet)

These conclusions can be obtained on the non-relativistic classical level and it is possible in theory to practice proton and electron spin resonance without permanent magnets at much higher resolution, and with a unique spectral fingerprint determined by a novel, site specific, molecular property tensor (Section (6)).

Practical design suggestions are given in the discussion section.

RELATIVISTIC CLASSICAL LEVEL

In this section, it is demonstrated that the RFR exists on the relativistic level. In order to do this, we use the Einstein equation {16}:

$$p^\mu p_\mu = m^2 c^2 \quad (29)$$

where p^μ and p_μ are energy momentum four vectors, and where m is the mass of the fermion, c being the velocity of light. In order to demonstrate the existence of the RFR term, eqn. (29) must be written in terms of the well known gamma matrices {17}:

$$\gamma^\mu p_\mu \gamma^\mu p_\mu = m^2 c^2 \quad (30)$$

and in the classical electromagnetic field is introduced through the minimal prescription:

$$\gamma^\mu (p_\mu - eA_\mu) \gamma^\mu (p_\mu - eA_\mu^*) = m^2 c^2. \quad (31)$$

In the compact Feynman slash notation {17}, eqn. (30) becomes:

$$\not{p}\not{p} = m^2 c^2 \quad (32)$$

and eqn. (31) becomes:

$$(\not{p} - e\not{A})(\not{p} - e\not{A}^*) = m^2 c^2. \quad (33)$$

This is the classical relativistic expression for the interaction of an electron, or proton, with the classical electromagnetic field. The quantized equivalent of eqn. (33) is the Van der Waerden equation as described by Sakurai {7}(eqn. (3.24)). The RFR term in relativistic classical physics is contained within the term $e^2 \not{A}\not{A}^*$. This result is demonstrated by expanding the terms as follows:

$$e^2 \not{A}\not{A}^* = e^2 \gamma^\mu A_\mu \gamma^\mu A_\mu^* = e^2 (\gamma^0 A_0 - \boldsymbol{\gamma} \cdot \mathbf{A}) (\gamma^0 A_0^* - \boldsymbol{\gamma} \cdot \mathbf{A}^*). \quad (34)$$

We now use the well known relations between the gamma matrices and Pauli matrices {17}:

$$\begin{aligned} (\boldsymbol{\gamma} \cdot \mathbf{p})(\boldsymbol{\gamma} \cdot \mathbf{p}) &= \begin{bmatrix} \mathbf{0} & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{p} & \mathbf{0} \\ \mathbf{0} & \mathbf{p} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{p} & \mathbf{0} \\ \mathbf{0} & \mathbf{p} \end{bmatrix} \\ &= \begin{bmatrix} (\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) & \mathbf{0} \\ \mathbf{0} & (\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) \end{bmatrix} \end{aligned} \quad (35)$$

to find:

$$e^2 \not{A}\not{A}^* = e^2 (A_0 A_0^* - \mathbf{A} \cdot \mathbf{A}^* - i\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^*). \quad (36)$$

This includes the RFR term:

$$T_{RFR} = -ie^2 \boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^* \quad (37)$$

which therefore exists in relativistic classical physics.

NON-RELATIVISTIC QUANTUM LEVEL

Time Independent Schrödinger-Pauli Equation.

Both the time independent and the time dependent Schrödinger equations can be used to demonstrate the existence of the RFR term in quantum mechanics, a demonstration which is consistent with the correspondence principle, because the RFR term is classical in nature. As demonstrated by Sakurai {7}, the time independent Schrödinger-Pauli equation can be used to demonstrate **ordinary** ESR and NMR in the non-relativistic quantum limit. We adopt this method to demonstrate the RFR term in non-relativistic quantum mechanics with the time independent Schrödinger-Pauli equation:

$$\hat{H}\psi = En\psi \tag{38}$$

with the Hamiltonian operator:

$$\hat{H} = \frac{1}{2m}(\sigma \cdot p)(\sigma \cdot p) + V_0, \tag{39}$$

where V is a potential energy term which does not affect the result. The method is first checked for self-consistency using a real valued potential function A corresponding to a static magnetic field, then the same equation is used to demonstrate the existence of the RFR term.

In a static magnetic field, the minimal prescription shows that the time independent Schrödinger-Pauli equation of a fermion in a classical electromagnetic field is:

$$\hat{H} \rightarrow \frac{1}{2m}(\sigma \cdot (p + eA))(\sigma \cdot (p + eA)) + V_0. \tag{40}$$

The ESR or NMR term is obtained from

$$\begin{aligned} \hat{H}\psi &= i\frac{e}{2m}(\sigma \cdot p \times A + \sigma \cdot A \times p)\psi + \dots \\ &= \frac{e\hbar}{2m}\sigma \cdot (\nabla \times (A\psi) + A \times \nabla\psi) + \dots \\ &= \frac{e\hbar}{2m}\sigma \cdot ((\nabla \times A)\psi + (\nabla\psi) \times A + A \times (\nabla\psi)) + \dots \\ &= \frac{e\hbar}{2m}\sigma \cdot B\psi + \dots \end{aligned} \tag{41}$$

This is the famous “half-integral spin” term first usually derived from the Dirac equation. However, following Sakurai {7}, this term exists in the **non-relativistic** limit of quantum mechanics from the time independent Schrödinger-Pauli equation, as just demonstrated. There is no need to use the time dependent Schrödinger-Pauli equation to show the existence of this term. However, it is a purely quantum term, because it depends on the well known operator equivalent $p \rightarrow -i\hbar\nabla$. It is the spin Zeeman effect and in perturbation theory gives the non-zero ground state energy:

$$En = \frac{e\hbar}{2m}\langle 0|\sigma \cdot B|0\rangle \neq 0 \tag{42}$$

and is the basis of magnet based ESR and NMR.

To obtain the RFR term, the same method is used for complex valued A . This gives an extra classical term, or expectation value, which can be written as:

$$En = \frac{ie^2}{2m} \sigma \cdot A \times A^* \quad (43)$$

Perturbation theory gives the extra ground state term:

$$En = \frac{ie^2}{2m} \langle 0 | \sigma \cdot A \times A^* | 0 \rangle, \quad (44)$$

which is again classical and real valued. It has the inverse square frequency dependence described in Section 2 and exists on the non-relativistic quantum level. This result is consistent with the correspondence principle. If we define the magnetic field {18}:

$$B^{(3)} \equiv -i \frac{e}{\hbar} A \times A^* \quad (45)$$

the RFR term is described succinctly as:

$$En = -\frac{e\hbar}{2m} \langle 0 | \sigma \cdot B^{(3)} | 0 \rangle \quad (46)$$

and the RFR term is the fundamental fermion to one photon interaction. From Table 1, it is seen to be potentially of great utility because of its advantages in resolution over ordinary NMR and ESR.

Time Dependent Perturbation Theory.

Consider a two level system consisting of a fermion in up and down spin states. The time dependent Schrödinger equation is used to consider the effect of the RFR term on this fermion:

$$H\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad (47)$$

$$H = H^{(0)} + H^{(1)}(t) \quad (48)$$

$$\Psi(t) = \Psi_n e^{-iE_n t} \quad (49)$$

The unperturbed two level system has energies E_1 and E_2 and eigenfunctions ψ_1 and ψ_2 . These are solutions of

$$H^{(0)}\psi_n = E_n \psi_n \quad (50)$$

In the presence of the time dependent perturbation $H^{(1)}(t)$, the state of the system is described by a linear combination of basis functions:

$$\Psi(t) = a_1(t)\Psi_1(t) + a_2(t)\Psi_2(t). \quad (51)$$

The system evolves under the influence of the perturbation, so a_1 and a_2 are also time dependent. If it starts as state 1, it may evolve to state 2. The probability at any instant that the system is in state 2 is $a_2(t)a_2^*(t)$ and the probability that it remains in state one is:

$$a_1(t)a_1^*(t) = 1 - a_2(t)a_2^*(t) \quad (52)$$

Therefore

$$\begin{aligned} H\Psi &= a_1H^{(0)}\Psi_1 + a_1H^{(1)}(t)\Psi_1 + a_2H^{(0)}\Psi_2 + a_2H^{(1)}(t)\Psi_2 \\ &= i\hbar\frac{\partial}{\partial t}(a_1\Psi_1 + a_2\Psi_2) \\ &= i\hbar a_1\frac{\partial\Psi_1}{\partial t} + i\hbar\frac{\partial a_1}{\partial t}\Psi_1 + i\hbar a_2\frac{\partial\Psi_2}{\partial t} + i\hbar\frac{\partial a_2}{\partial t}\Psi_2. \end{aligned} \quad (53)$$

Each basis function satisfies

$$H^{(0)}\Psi_n = i\hbar\frac{\partial\Psi_n}{\partial t} \quad (54)$$

$$\therefore a_1H^{(1)}(t)\Psi_1 + a_2H^{(1)}(t)\Psi_2 = i\hbar\dot{a}_1\Psi_1 + i\hbar\dot{a}_2\Psi_2. \quad (55)$$

This equation is:

$$a_1H^{(1)}(t)\Psi_1e^{-iE_1t/\hbar} + a_2H^{(1)}(t)\Psi_2e^{-iE_2t/\hbar} = i\hbar\dot{a}_1\Psi_1e^{-iE_1t/\hbar} + i\hbar\dot{a}_2\Psi_2e^{-iE_2t/\hbar}. \quad (56)$$

Now multiply through by ψ_1^* and integrate over all space. Since ψ_1 and ψ_2 are orthonormal,

$$a_1H_{11}^{(1)}(t)e^{-iE_1t/\hbar} + a_2H_{12}^{(1)}(t)e^{-iE_2t/\hbar} = i\hbar\dot{a}_1e^{-iE_1t/\hbar}. \quad (57)$$

Similarly multiply through by ψ_2^*

$$a_1H_{21}^{(1)}(t)e^{-iE_1t/\hbar} + a_2H_{22}^{(1)}(t)e^{-iE_2t/\hbar} = i\hbar\dot{a}_2e^{-iE_2t/\hbar}. \quad (58)$$

Here

$$H_{ij}^{(1)}(t) \equiv \int \psi_i^* H^{(1)}(t) \psi_j d\tau \quad (59)$$

and ψ_1 and ψ_2 are time dependent parts of the wavefunction of states 1 and 2 of the unperturbed fermion. Thus:

$$H_{11}^{(1)}(t) \equiv \int \psi_1^* H^{(1)}(t) \psi_1 d\tau \equiv \langle 1 | H^{(1)}(t) | 1 \rangle \quad (60)$$

and so on.

At this point, we input the RFR Hamiltonian:

$$H^{(1)}(t) = i\frac{e^2}{2m}\sigma \cdot A \times A^* \quad (61)$$

so the existence of $H_{11}^{(1)}(t)$ and $H_{12}^{(1)}(t)$ and so on depends on the properties of σ between fermion states.

Define

$$\mathbf{S} \equiv \frac{1}{2} \hbar \boldsymbol{\sigma} \quad (62)$$

and

$$\begin{aligned} \alpha &\equiv \left| \frac{1}{2}, \frac{1}{2} \right\rangle \equiv \text{state 1} \\ \beta &\equiv \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \equiv \text{state 2} \end{aligned} \quad (63)$$

then

$$S_z \alpha = \frac{1}{2} \hbar \alpha; \quad S_z \beta = -\frac{1}{2} \hbar \beta \quad (64)$$

and

$$\begin{aligned} \langle \alpha | S_z | \alpha \rangle &= \frac{1}{2} \hbar = \frac{1}{2} \hbar \int \alpha^* \alpha d\tau \\ \langle \alpha | S_z | \beta \rangle &= 0 = -\frac{1}{2} \hbar \int \alpha^* \beta d\tau. \end{aligned} \quad (65)$$

Now define

$$\mathbf{B}^{(3)*} \equiv -i \frac{e}{\hbar} \mathbf{A} \times \mathbf{A}^* \quad (66)$$

and

$$H^{(1)}(t) = -\frac{e}{m} \mathbf{S} \cdot \mathbf{B}^{(3)} = -\frac{e}{m} S_z B_z^{(3)} \quad (67)$$

So eqn. (9) and (10) become

$$\begin{aligned} a_1 H_{11}^{(1)}(t) &= i \hbar \dot{a}_1 \\ a_2 H_{22}^{(1)}(t) &= i \hbar \dot{a}_2 \end{aligned} \quad (68)$$

because

$$\begin{aligned} H_{11}^{(1)}(t) &= -\frac{e \hbar}{2m} B_z^{(3)}; & H_{12}^{(1)}(t) &= 0; \\ H_{22}^{(2)}(t) &= \frac{e \hbar}{2m} B_z^{(3)}; & H_{21}^{(1)}(t) &= 0. \end{aligned} \quad (69)$$

Eqns. (68) are decoupled differential equations of the form

$$\dot{a}_1 = i \frac{e B_z^{(3)}}{2m} a_1; \quad \dot{a}_2 = -i \frac{e B_z^{(3)}}{2m} a_2; \quad (70)$$

where

$$B_z^{(3)} \equiv \frac{e}{\hbar} A^{(0)2}. \quad (71)$$

There is the constraint

$$a_1 a_1^* + a_2 a_2^* = 1. \quad (72)$$

A particular solution of eqns. (70) and (72) is

$$a_1 = \frac{1}{\sqrt{2}} e^{i \frac{e t B_z^{(3)}}{2m}}; \quad a_2 = \frac{1}{\sqrt{2}} e^{-i \frac{e t B_z^{(3)}}{2m}}. \quad (73)$$

The perturbed wave function is therefore

$$\Psi = \frac{\Psi_1}{\sqrt{2}} e^{i \frac{e t B_z^{(3)}}{2m}} + \frac{\Psi_2}{\sqrt{2}} e^{-i \frac{e t B_z^{(3)}}{2m}} \quad (74)$$

and

$$\begin{aligned} p_1 &= a_1 a_1^* = 0.5 \\ p_2 &= a_2 a_2^* = 0.5 \end{aligned} \quad (75)$$

The probability of finding the system in one state or the other remains constant at 50%, and:

$$\Psi = \frac{\Psi}{\sqrt{2}} e^{i \omega_{res} t} + \frac{\Psi}{\sqrt{2}} e^{-i \omega_{res} t} \quad (76)$$

where

$$\omega_{res} = \frac{e B_z^{(3)}}{2m}, \quad (77)$$

is the **RFR resonance frequency** defined by:

$$\hbar \omega_{res} = H^{(1)}(t). \quad (78)$$

The final result is

$$\Psi = \frac{\Psi_1}{\sqrt{2}} e^{i \omega_{res} t} + \frac{\Psi_2}{\sqrt{2}} e^{-i \omega_{res} t} \quad (79)$$

where

$$H \Psi = i \hbar \frac{\partial \Psi}{\partial t} \quad (80)$$

which is a combination of states with energies $\pm \hbar \omega_{res}$. The RFR term prepares or dresses the fermion in a combination of α and β spin states in a fashion exactly analogous to NMR and ESR.

RELATIVISTIC QUANTUM LEVEL.

The Einstein equation becomes the van der Waerden equation {7} with the usual operator rules between momentum and coordinate space:

$$\begin{aligned} p^\mu &\rightarrow i \hbar \partial^\mu \\ p_\mu &\rightarrow i \hbar \partial_\mu \end{aligned} \quad (81)$$

to give:

$$(i\gamma^\mu \partial_\mu)(i\gamma^\mu \partial_\mu)\psi_W = \frac{m^2 c^2}{\hbar^2}\psi_W \quad (82)$$

where ψ_W is a two component wave function as described by Sakurai {7}, eqn. (3.24). The classical electromagnetic field is introduced into eqn. (82) using the covariance derivatives $\partial_\mu + ieA_\mu$ and $\partial_\mu - ieA_\mu$, with which procedure we regain the term $e^2 \mathbf{A} \mathbf{A}^*$ on the quantum relativistic level. Finally the Dirac equation is regained from the van der Waerden equation using standard methods {7} and the two equations are equivalent, so the RFR term appears at the quantum relativistic level. The RFR term was first derived using the Dirac equation {18}.

CHEMICAL SHIFT TERM IN RFR.

The technique of RFR is site specific, and the chemical shift is given by a unique molecular property tensor, so RFR has its own spectral fingerprint in theory as well as a greatly increased spectral resolution. This result can be shown using time independent perturbation theory at second order with the energy:

$$E_n = \sum_n \frac{\langle 0|H|n\rangle \langle n|H|0\rangle}{\hbar\omega_{0n}} \quad (83)$$

and perturbation Hamiltonian {19}:

$$H = \frac{1}{2m}(\mathbf{p} + e(\mathbf{A} + \mathbf{A}_N))^2 + V \quad (84)$$

where V is a potential energy and where

$$\mathbf{A}_N = \frac{\mu_0}{4\pi r^3} \mathbf{m}_N \times \mathbf{r} \quad (85)$$

is the vector potential due to the nuclear magnetic dipole moment \mathbf{m}_N . Here μ_0 is the vacuum permeability in S.I. units and \mathbf{r} the distance between the nucleus and electron. The relevant perturbation term is {19}:

$$E_n = i \frac{e^3}{m^2 \hbar \omega_{0n}} \sum_n \langle 0|\mathbf{p} \cdot \mathbf{A}|n\rangle \langle n|\mathbf{A}_N \cdot \mathbf{A}^*|0\rangle + \text{c.c.} \quad (86)$$

We use the following relation {19} between the linear momentum \mathbf{p} and the transition electronic electric dipole moment $\boldsymbol{\mu}$:

$$\langle 0|\mathbf{p}|n\rangle = \frac{m}{e} \omega_{0n} \langle 0|\boldsymbol{\mu}|n\rangle. \quad (87)$$

Using some straightforward vector algebra and the definition:

$$\mathbf{m}_N \equiv g_N \frac{e}{4m} \hbar \boldsymbol{\sigma} \quad (88)$$

converts term (86) into:

$$EN = \zeta \left(i \frac{e^2}{2m} \boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^* \right) \quad (89)$$

where

$$\zeta = \frac{g_N e \mu_0}{8\pi m} \sum_n \langle 0 | \mu | n \rangle \langle n | \frac{\mathbf{r}}{r^3} | 0 \rangle \quad (90)$$

is the RFR shielding coefficient or chemical shift. Here g_N is the nuclear g factor of the proton and m the proton mass. The coefficient ζ is unitless and has an order of magnitude 10^{-6} . It is of the same order of magnitude as in NMR and is site specific, meaning that RFR is expected to give a unique spectral fingerprint because the molecular property tensor defining ζ is a novel one:

$$\beta = \sum \langle 0 | \mu | n \rangle \langle 0 | \frac{\mathbf{r}}{r^3} | 0 \rangle. \quad (91)$$

This molecular property tensor does not appear in NMR. The site specific RFR spectrum is proportional to I/ω^2 because $iA \times A^*$ has this basic property.

DISCUSSION

To detect RFR experimentally, it would be necessary to work initially on the interaction of a fermion beam with an electromagnetic beam. All levels of one fermion theory given in this paper could then be tested under conditions which most closely approximate them. A successful demonstration of RFR would require careful engineering in the matter of beam interaction. The inverse Faraday effect has been demonstrated experimentally {20} at the microwave frequencies necessary for successful demonstration of RFR. The simplest demonstration is autoresonance, where the circularly polarized pump frequency (ω) is adjusted to be the same as the RFR frequency (ω_{res}):

$$\omega_{res} = \omega. \quad (92)$$

Under this condition, the pump beam is absorbed at resonance because the pump frequency matches the resonance frequency exactly. Eqn. (28) simplifies to:

$$\omega_{res}^3 = 1.007 \times 10^{28} I \quad (93)$$

Therefore we can either tune ω_{res} for a given I or vice-versa using interacting fermion and electromagnetic beams. Since auto-resonance must appear in the GHz if the pump frequency is in this (microwave) range, it might be convenient to modify the set up described by Deschamps et alia {20} in their demonstration of the inverse Faraday effect through Faraday induction rather than through resonance, as in this paper. They used a pulsed microwave signal at 3.0 GHz from a klystron delivering megawatts of power over 12 microseconds with a repetition rate of 10 Hz. The TE_{11} mode was circularly polarized with a polarizer placed inside a circular waveguide of 7.5 cm of diameter. A plasma was created by the very intense microwave pulse. To detect RFR experimentally, the same standard of engineering would have to be reached with an electromagnetic beam interacting with an electron beam, rather than a plasma. The intensity of the microwave radiation would be much lower as governed by eqn. (b) for autoresonance. As in the design used by Deschamps et alia {20}, the section of the waveguide surrounding the tube would have to be made of nylon internally coated with a 20 micron layer of copper. The incoming electron beam would have to be guided carefully into the circular waveguide used to circularly polarize the microwave radiation. The engineering design has to be at least as accurate as in the experiment of Deschamps et alia {20} which detected the inverse Faraday effect. Eqn. (b) predicts that resonance occurs at 3.0 GHz if I is tuned to 0.0665 watts/cm² for an electron beam. For a circular waveguide of 7.0 cm diameter, this requires only 2.94 watts of circularly polarized microwave power from the klystron at 3.0 GHz.

The above estimate is based on a one fermion theory, and the resonance frequency in a fermion beam is different, due to fermion-fermion interaction in the beam. Therefore it is strongly advisable that I can be tuned over a considerable range round the theoretical 2.94 watts to allow for unforeseen discrepancies. Once the main resonance frequency is detected, refinements can follow, and the electron beam replaced for example

by a proton beam or atomic beam. The experimental demonstration should aim to detect the expected I/ω^2 dependence, and in atomic beam or molecular beam samples, the expected site specificity due to the novel molecular property tensor ζ .

If this series of beam experiments is developed successfully, a major new technique in analytical chemical physics will emerge. It is to be emphasized that RFR is the resonance equivalent of the well known inverse Faraday effect first detected at microwave frequencies by Deschamps et alia using magnetization of a plasma, a method that requires much greater microwave pump power. Therefore, in this sense, the phenomenon underlying RFR has been detected empirically.

ACKNOWLEDGMENTS

Many colleagues worldwide are acknowledged for many interesting discussions. The constituent institutes of AIAS (Alpha Foundation's Institute for Advanced Study) are thanked for funding.

REFERENCES

- {1} A. Piekara and S. Kielich, *Arch. Sci.*, **11**, 304 (1958); *Acta Physica Polonica*, **32**, 405 (1967).
- {2} P.S. Pershan, *Phys. Rev.*, **130**, 919 (1963).
- {3} J.P. van der Ziel, P.S. Pershan and L.D. Malmstrom, *Phys. Rev. Lett.*, **15**, 190 (1965).
- {4} P.S. Pershan, J.P. van der Ziel and L.D. Malmstrom, *Phys. Rev.*, **143**, 574 (1966).
- {5} P.W. Atkins and M.H. Miller, *Mol. Phys.*, **75**, 491, 503 (1968).
- {6} M.W. Evans and S. Kielich (eds.), "Modern Nonlinear Optics" (Wiley, New York, 1967, paperback).
- {7} J.J. Sakurai, "Advanced Quantum Mechanics" (Addison Wesley, New York, 1967, 11th printing), chap. 3.
- {8} L.D. Baron, "Molecular Light Scattering and Optical Activity" (Cambridge Univ. Press, Cambridge, 1982).
- {9} E.R. Pike and S. Sarkav, "The Quantum Theory of Radiation", (Oxford, 1995), example 3.4.
- {10} C. Itzykson and J.B. Zuber, "Quantum Field Theory" (McGraw-Hill, New York, 1980).
- {11} J.D. Jackson, "Classical Electrodynamics" (Wiley, New York, 1962).
- {12} L.D. Landau and E.M. Lifshitz, "The Classical Theory of Fields" (Pergamon, Oxford, 1975).
- {13} V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevski, "Relativistic Quantum Field Theory" (Pergamon, Oxford, 1971).
- {14} R.R. Ernst, G. Bodenhausen and A. Wokaun, "Principles of Nuclear Magnetic Resonance in One and Two Dimensions" (Oxford Univ. Press, Oxford, 1987).
- {15} P.W. Atkins, "Molecular Quantum Mechanics" (Oxford Univ. Press, Oxford, 1983).
- {16} L.H. Ryder, "Quantum Field Theory" (Cambridge Univ. Press, 1987, 2nd Ed.).
- {17} J.D. Bjorken and S.D. Drell, "Relativistic Quantum Mechanics" (McGraw-Hill, New York, 1964).
- {18} M.W. Evans, J.P. Vigiier, S. Roy and S. Jeffers, "The Enigmatic Photon" (Kluwer Academic, Dordrecht, 1994 to 1999), in five volumes.
- {19} R.A. Harris and I. Tinoco, *J. Chem. Phys.* **101**, 9289 (1994).
- {20} J. Deschamps, M. Fitaire and M. Lagoutte, *Phys. Rev. Lett.*, **25**, 1330 (1970); *Rev. Appl. Phys.*, **7**, 155 (1972)