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Paper 11

The Derivation of the Majorana Form of Maxwell's Equations from the B Cyclic Theorem

It is demonstrated that the *B* Cyclic theorem (equivalence principle) of the new electrodynamics gives Majorana's form of Maxwell's equations in the vacuum. This demonstration provides a link between the new and received views of vacuum electrodynamics, showing that the equations of motion can be derived from the equivalence principle, assuming only the correspondence principle of quantum mechanics. Therefore the *B* Cyclic theorem is quantized to give the Maxwell equations in Majorana's form.

11.1 Introduction

In the past few years it has become clear that a major advance in electrodynamics has occurred. The electromagnetic field is now thought to have longitudinal components in the vacuum [1—7], one of which, conveniently referred to as the $\boldsymbol{B}^{(3)}$ field, being phase free and observable empirically, for example in magneto-optics. This longitudinally directed magnetic field forms part of the structure of the B Cyclic theorem [1—3], which inter-relates transverse and longitudinal components in the vacuum. (There is also an equivalent theorem in the presence of sources and matter.) The purpose of this Letter is to show that the Maxwell equations in the form given by Majorana [8—10] can be derived from the B Cyclic theorem, showing that Maxwell's equations themselves must give longitudinal solutions which are inter-related to the usual transverse electromagnetic waves through a novel principle of equivalence between space-time and the electromagnetic field.

11.2 Derivation of the Maxwell Equations from the *B* Cyclic Theorem

The equivalence principle of the new electrodynamics, the B Cyclic theorem, inter-relates the transverse and longitudinal components of the vacuum electromagnetic field as follows,

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \qquad \text{et cyclicum}, \qquad (2.11.1)$$

where $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ is the transverse component (e.g. a plane wave) and where

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)}, \qquad (2.11.2)$$

is the longitudinal, phase free, component in the basis ((1), (2), (3)), a complex basis of the rotation sub-group *O(3)* of the Poincaré group. For the present purposes it proves convenient to reduce Eq. (2.11.1) to cyclics in the vector potential $\boldsymbol{A}$, defined in *S.I.* units by

$$\boldsymbol{B} = \nabla \times (i\boldsymbol{A}), \qquad (2.11.3a)$$

$$\boldsymbol{E} = c\nabla \times \boldsymbol{A}, \qquad (2.11.3b)$$

where $\boldsymbol{B}$ is magnetic flux density, $\boldsymbol{E}$ is electric field strength, and c the speed of light in vacuo. Equation (2.11.3) uses the Hertz-Stratton representation of $\boldsymbol{E}$ as the curl of an axial (*i.e.*, rotational) $\boldsymbol{A}$, for example the plane wave,

$$\boldsymbol{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j})e^{i\phi}, \qquad \phi = \omega t - \kappa Z. \qquad (2.11.4)$$

Here $\boldsymbol{e}^{(1)} = (\boldsymbol{i} - i\boldsymbol{j})/\sqrt{2}$; $A^{(0)}$ is a scalar amplitude, and ϕ is the electromagnetic phase, where ω is the angular frequency at instant t, and κ the wavevector at Z. Using Eq. (2.11.3) and (2.11.4) gives the standard result [1—7],

$$\boldsymbol{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi}, \qquad \boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j})e^{i\phi}, \qquad (2.11.5)$$

from which it is inferred that $\boldsymbol{A}$ is axial and $i\boldsymbol{A}$ is polar. Self consistently, therefore, the complex polar vector $\boldsymbol{E}$ is the curl of the complex axial vector $\boldsymbol{A}$; and the complex axial vector $\boldsymbol{B}$ is the curl of the complex polar vector $i\boldsymbol{A}$. Since $\boldsymbol{A}$ is axial, it is described by the $\boldsymbol{A}$ cyclics:

$$\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} = iA^{(0)}\boldsymbol{A}^{(3)*}, \qquad (2.11.6a)$$

$$\boldsymbol{A}^{(2)} \times \boldsymbol{A}^{(3)} = iA^{(0)}\boldsymbol{A}^{(1)*}, \qquad (2.11.6b)$$

$$\boldsymbol{A}^{(3)} \times \boldsymbol{A}^{(1)} = iA^{(0)}\boldsymbol{A}^{(2)*}, \qquad (2.11.6c)$$

where

$$\begin{aligned} B^{(0)} &= \kappa A^{(0)}, & E^{(0)} &= -ic\kappa A^{(0)}, \\ \boldsymbol{B}^{(1)} &= \kappa \boldsymbol{A}^{(1)}, & \boldsymbol{E}^{(1)} &= -ic\kappa \boldsymbol{A}^{(1)}, \\ \boldsymbol{B}^{(2)} &= \kappa \boldsymbol{A}^{(2)}, & \boldsymbol{E}^{(2)} &= -ic\kappa \boldsymbol{A}^{(2)}, \\ \boldsymbol{B}^{(3)} &= \kappa \boldsymbol{A}^{(3)}, & \boldsymbol{E}^{(3)} &= -ic\kappa \boldsymbol{A}^{(3)}. \end{aligned} \tag{2.11.7}$$

›w multiply both sides of Eq. (2.11.6a) by i and transform to Cartesian ordinates to give

$$\left(iA^{(0)}\right)A_Z^* + i\left(\left(iA_X\right)A_Y^* - \left(iA_Y\right)A_X^*\right) = 0, \tag{2.11.8}$$

d it is seen that this equation has the Cartesian structure,

$$p^{(0)}\psi_Z + i\left(p_X\psi_Y - p_Y\psi_X\right) = 0, \tag{2.11.9}$$

ıere $\boldsymbol{p}$ is a polar vector, ψ an axial vector, and $p^{(0)}$ the scalar magnitude $\boldsymbol{p}$. Equation (2.11.9) is one of the Maxwell equations as derived by ajorana [8—10]. In the vacuum, the axial vector potential is defined by

$$\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*} = \frac{c}{\kappa}\left(c\boldsymbol{B}^{(1)} + i\boldsymbol{E}^{(1)}\right), \tag{2.11.10}$$

d the polar vector potential iA is identified through the correspondence nciple with the del operator,

$$\frac{1}{e}\boldsymbol{p} = iA \rightarrow -i\frac{\hbar}{e}\hat{\nabla}. \tag{2.11.11}$$

e $iA^{(0)}$ function is identified with a time differential operator through the ne correspondence principle, i.e.,

$$\frac{1}{e}p^{(0)} = iA^{(0)} \rightarrow i\frac{\hbar}{ec}\frac{\hat{\partial}}{\partial t}. \tag{2.11.12}$$

Similarly the other two equations of the A cyclics reduce to the other two Majorana-Maxwell equations as follows,

$$\boldsymbol{A}^{(2)} \times \boldsymbol{A}^{(3)} = iA^{(0)}\boldsymbol{A}^{(1)*} \rightarrow p^{(0)}\psi_X + i\left(p_Y\psi_Z - p_Z\psi_Y\right) = 0, \tag{2.11.13}$$

and

$$\boldsymbol{A}^{(3)} \times \boldsymbol{A}^{(1)} = iA^{(0)}\boldsymbol{A}^{(2)*} \rightarrow p^{(0)}\psi_Y + i\left(p_Z\psi_X - p_X\psi_Z\right) = 0, \tag{2.11.14}$$

11.3 Discussion

The cyclic equations of the new electrodynamics reduce to the Majorana form of the Maxwell equations using the correspondence principle in the form (2.11.11) and (2.11.12), which is also a form of the minimal prescription for the free field [1—7], i.e., the proportionality between linear momentum and the vector potential, the latter having the dimensions of linear momentum multiplied by charge. The novel cyclic field equations represent an equivalence principle between rotation generators of *O(3)* and the electromagnetic field. Using the correspondence principle, the equivalence principle reduces to Maxwell's equations in the form given by Majorana, in which complex field combinations take the role of wavefunctions. The Maxwell equations have therefore been *derived* from a more fundamental cyclical structure, and have therefore been derived in a form which has *O(3)* symmetry. This is the Majorana form of Maxwell's equations in vacuo.

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Paper 12

The Evans-Vigier Field $\boldsymbol{B}^{(3)}$ Interpreted as a De Broglie Pilot Field

A straightforward consideration of the antisymmetric part of the tensor of free space light intensity leads to the result $\boldsymbol{B}^{(3)} / B^{(0)} = \boldsymbol{J}^{(3)} / \hbar$, where $\boldsymbol{B}^{(3)}$ is the Evans-Vigier field [1—10], a phase free magnetic flux density of amplitude $B^{(0)}$ carried in free space by the electromagnetic wave component, and where $\boldsymbol{J}^{(3)} = \hbar \boldsymbol{e}^{(3)}$, with $\boldsymbol{e}^{(3)}$ being a unit axial vector in the propagation axis. The field $\boldsymbol{B}^{(3)}$ therefore pilots the photon angular momentum, $\boldsymbol{J}^{(3)}$. The consequences are discussed of the wave-particle duality inherent in this result, using diffraction patterns due to $\boldsymbol{B}^{(3)}$ in a double slit interferometer.

Key words: $\boldsymbol{B}^{(3)}$ Field, de Broglie pilot field.

12.1 Introduction

It has been inferred recently [1—10] that the conventional view of free space electromagnetism is incomplete, because the classical wave interpretation produces a novel phase free magnetic flux density in the vacuum, the Evans-Vigier field $\boldsymbol{B}^{(3)}$. The latter exists in free space because there exists the *electromagnetic torque*