2-13-95 9:33

THE INVERSE FARADAY EFFECT, FERMION SPIN, AND $B^{(3)}$

M. W. Evans[1,2], and Sisir Roy[3]

[2]*Department of Physics and Astronomy*

York University

4700 Keele Street

Toronto, Ontario M3J 1P3 Canada

[3]*Division of Physics and Applied Mathematics*

Indian Statistical Institute

203 Barrackpore Trunk Road

Calcutta 700 035, India

ABSTRACT

The inverse Faraday effect is demonstrated from the Dirac equation of a fermion in a circularly polarized electromagnetic field, and is shown to be due to the Evans-Vigier field, $\boldsymbol{B}^{(3)}$, whose source in vacuo is the beam conjugate product. Fermion spin resonance in a microwave or radio frequency beam is demonstrated to be highly sensitive to the inverse Faraday effect, (IFE), which is related to photon mass through the de Broglie Guidance Theorem. Under well defined experimental conditions, the IFE demonstrates the existence of photon mass, and provides an upper bound which is compatible with cosmological

[1]Address for correspondence: *50 Rhyddwen Road, Craig Cefn Parc, Swansea SA6 5RA, Wales, Great Britain*

measurements.

1. INTRODUCTION

The inverse Faraday effect (IFE) is well known as a magneto-optical phenomenon [1-5] which was first suggested by Pershan [6] and first demonstrated by van der Ziel *et al.* [7], and by Deschamps *et al.* [8]. It has also been demonstrated experimentally in independent ways by Barrett and co workers [9] and is therefore well established. It is one of a class of magneto-optic phenomena [10] due to the electromagnetic conjugate product [1-5], and its theory has been developed extensively [11-13]. Wagnière has recently proposed a related phenomenon of inverse magneto-chiral birefringence [14], and other related phenomena have been proposed and reviewed by Evans [15].

In this Letter it is shown that there is an important connection between the IFE and on the one hand novel fermion resonance phenomena generated by a circularly polarized microwave or radio frequency field; and on the other hand between the IFE and the probable existence of photon mass [16]. The first type of connection is forged in Sec. 2 directly from the Dirac equation of an electron (or a proton) in a circularly polarized electromagnetic field, and it is shown that resonance can be induced between two energy levels generated by the interaction of a Pauli spinor and the conjugate product. By expressing the latter as the Evans Vigier field, $\boldsymbol{B}^{(3)}$, [17-20], it is shown that the form of this beam-fermion interaction is identical with that between fermion half integral spin and a uniform magnetic field. In Sec. 3 this result is used to show that the inverse Faraday effect can be detected with great sensitivity using the mature technology of fermion resonance such as Zeeman spectroscopy, ESR and NMR. We refer to this phenomenon as the *resonance IFE* or RIFE. Section 4 forges a connection between RIFE, the $\boldsymbol{B}^{(3)}$ field, and finite photon mass, and shows that magneto-optic phenomena lead to an upper bound on photon mass which is in order of magnitude agreement with cosmological measurements of the same thing.

2. BEAM-FERMION INTERACTION AND THE IFE

Consider as an idealization a single fermion such as an electron or proton in a circularly polarized electromagnetic field [21]. The standard Dirac equation can be used to describe the beam-fermion interaction in a frame of reference where the momentum ($\boldsymbol{p}$) of the fermion is approximately zero and its energy (En) is approximately its rest energy [21]. In this frame it is convenient to write the Dirac equation with its Hermitian transpose [22],

$$\begin{aligned}\left(En + e\phi - m_0c^2\right)u &= ec\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{A}^{(1)}v,\\ \left(En + e\phi + m_0c^2\right)v &= ec\boldsymbol{\sigma}^{2}\cdot\boldsymbol{A}^{(1)}u,\end{aligned} \qquad (1a)$$

$$\begin{aligned}u^{+}\left(En + e\phi - m_0c^2\right) &= ecv^{+}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{A}^{(2)},\\ v^{+}\left(En + e\phi + m_0c^2\right) &= ecu^{+}\boldsymbol{\sigma}^{1}\cdot\boldsymbol{A}^{(2)}.\end{aligned} \qquad (1b)$$

Here $\boldsymbol{A}$ and ϕ are vector and scalar potentials of the field defined as usual by

$$A_\mu = \left(\boldsymbol{A},\, i\frac{\phi}{c}\right), \qquad (2)$$

where c is the velocity of light in vacuo. The beam-fermion interaction is described in a circular basis ((1), (2), (3)) [17—21] by terms such as $\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{A}^{(1)}$, where the Pauli spinors are defined [21] in the same basis. Thus $\boldsymbol{A}^{(1)}$ is the complex conjugate of $\boldsymbol{A}^{(2)}$ and $\boldsymbol{A}^{(1)}\times\boldsymbol{A}^{(2)}$ is the conjugate product [1—5]. The latter defines the Evans-Vigier field [17],

$$\boldsymbol{B}^{(3)} = \boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)}\times\boldsymbol{A}^{(2)}, \qquad (3)$$

where e is both the elementary charge and an O(3) gauge scaling factor [17]. Here $\hbar$ is the Dirac constant as usual. In Eqs. (1) finally m_0 is the mass of the fermion and u and v are Dirac two-spinors, parity interconvertible components of the Dirac four-spinor. The superscript "+" in Eqs. (1) denote *Hermitian transpose* and the two-spinors in the second pair of equations are operated upon to the left.

It can be shown [21] that the beam-fermion interaction energy from the Dirac equation (1) is, in the Coulomb gauge,

$$En = \pm \frac{e^2}{2m_0} \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{A}^{(2)} \boldsymbol{\sigma}^{(2)} \cdot \boldsymbol{A}^{(1)}, \tag{4}$$

showing the well-known positive and negative energy eigenvalues of Dirac's equation of motion, representing respectively the interaction with the electromagnetic field of the fermion and anti-fermion. Taking the positive energy of interaction, spinor algebra in the basis ((1), (2), (3)) shows that

$$\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{A}^{(2)} \boldsymbol{\sigma}^{(2)} \cdot \boldsymbol{A}^{(1)} = \boldsymbol{A}^{(1)} \cdot \boldsymbol{A}^{(2)} + i\boldsymbol{\sigma}^{(3)} \cdot \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{5}$$

the second term of which shows the presence of an interaction between the conjugate product, $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, and the Pauli spinor, $\boldsymbol{\sigma}^{(3)} = \boldsymbol{\sigma}_Z$. Using Eq. (3), this term is

$$En_2 = -\frac{e}{m_0} \frac{\hbar}{2} \boldsymbol{\sigma}^{(3)} \cdot \boldsymbol{B}^{(3)}, \tag{6}$$

i.e., is the same form *precisely* as that between the well-known half integral intrinsic spin angular momentum of a fermion and a uniform magnetic field. In this case the magnetic field is the Evans-Vigier field [17] of electromagnetic radiation in vacuo.

3. RESONANCE CONDITION AND RIFE

From the relativistic principle of least action [23] it is well known that the magnetic field of the Lorentz force equation is *defined* as the curl of a vector potential,

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \tag{7}$$

so if we take $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ of the electromagnetic field in vacuo to be plane waves, then $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ are also plane waves,

$$\begin{aligned} \boldsymbol{B}^{(1)} &= \boldsymbol{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{ii} + \boldsymbol{j})e^{i\phi}, \\ \boldsymbol{A}^{(1)} &= \boldsymbol{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{ii} + \boldsymbol{j})e^{i\phi}, \end{aligned} \tag{8}$$

and solutions of d'Alembert's equation [17]. The conjugate products are related by

$$\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)*} = \frac{c^2}{\omega^2}\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = \frac{1}{\omega^2}\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}, \tag{9}$$

$\underline{A}^{(2)}$

where $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$ is the electric part of the electromagnetic field. The energy of interaction (6) is therefore expressible as

$$En_2 = \mp \frac{\chi'}{2} B^{(0)2}, \tag{10}$$

where χ' is a fermion susceptibility,

$$\chi' = \frac{e^2 c^2}{m_0 \omega^2}. \tag{11}$$

Resonance can be induced between the two energy levels represented by the Pauli spinor $\sigma^{(3)}$ at the angular frequency ω_{res} defined by

$$\omega_{res} = \frac{1}{\hbar}\left(\frac{\chi'}{2} - \left(-\frac{\chi'}{2}\right)\right). \tag{12}$$

If I_0 is beam power density (W m^{-2}) and μ_0 is the vacuum permeability then,

$$\omega_{res} = \left(\frac{e^2 \mu_0 c}{\hbar m_0}\right)\frac{I_0}{\omega^2}, \tag{13}$$

and is proportional to I_0, being inversely proportional to ω^2. For an electron in a circularly polarized 3 GHz microwave beam of moderate intensity, e.g. 100 watts per sq. cm., resonance from Eq. (13) occurs at 1501 cm^{-1} in the mid infra red, and so should be observable easily in theory with a Fourier transform spectrometer. Recall that this is a fundamental prediction of the Dirac equation in the Coulomb gauge.

To relate this result to the inverse Faraday effect we need only use the fact [24] that the rotational energy transferred from beam to electron is $\omega|\boldsymbol{J}^{(3)}|$, where $\boldsymbol{J}^{(3)}$ is the expectation value of an angular momentum. In the elastic limit [21], this is twice the intrinsic electronic angular momentum eigenvalue, which is $\pm\hbar/2$. In promoting resonance from the energy level $(-\chi'/2)B^{(0)2}$ to the energy level $(\chi'/2)B^{(0)2}$ this angular momentum is taken from the electromagnetic field by the electron. For one electron, angular momentum is (classically) proportional to magnetization through

$$\boldsymbol{M}^{(3)} = \frac{e}{m_0}\boldsymbol{J}^{(3)}, \tag{14}$$

and so the resonance jump from the lower energy level of the electron to the higher is a phenomenon involving a *change of magnetization,* and therefore a resonance inverse Faraday effect [25], (RIFE), of a simple type. The detailed theory requires the usual considerations of statistical mechanics etc., i.e., of relative populations of states with energies $(\pm\chi'/2)\boldsymbol{B}^{(0)2}$ in an electron gas, beam, or plasma, but the one electron theory clearly shows the presence of the RIFE effect, which, being a resonance phenomenon, should be sensitive to experimental detection. In contrast, the early methods [2,3] of detecting the inverse Faraday effect are insensitive, relying as they do on ordinary Faraday induction in response to a pulse of high intensity, visible frequency, light. The only resonance based experiment to date appears to be that of Barrett *et al.* [9] using resonance Raman technique. Additionally, theoretical work on resonance IFE phenomena from non-relativistic, semi-classical theory has been completed by Woźniak *et al.* [25].

4. RIFE AND PHOTON MASS

It has been shown that RIFE is an optically induced electron (or proton) spin resonance phenomenon [21], and the conditions under which it is observed are defined by Eq. (13), with its all-important inverse ω^2 dependence [21]. Low frequency radiation such as microwaves or radio frequency waves (rather than visible frequency lasers) should therefore be used to detect the inverse Faraday effect with high sensitivity through the characteristic infra red resonance line. The RIFE phenomenon has its roots, furthermore, in the Evans-Vigier field $\boldsymbol{B}^{(3)}$ defined by Eq. (3). It is predictable that a magnetic field ($\boldsymbol{B}^{(3)}$) should cause magnetization, picked up as a resonance IFE. The link between $\boldsymbol{B}^{(3)}$ and photon mass has been developed [17] since the discovery of $\boldsymbol{B}^{(3)}$ in 1992 [20].

Recently, Roy *et al.* [26,27] have shown that there is a link between the IFE and photon mass, forged primarily through the magnetizing $\boldsymbol{B}^{(3)}$ field when the latter interacts with matter such as fermions. The second essential element in this theory is the de Broglie Guidance Theorem [17],

$$\hbar\omega_0 = m_\gamma c^2, \tag{15}$$

where ω_0 is a photon rest frame frequency and m_γ the rest mass of the photon. Equation (15) indicates that a photon absorbed in RIFE can be expressed in a rest frame through the rest energy $m_\gamma c^2$. The photon then becomes a relativistic boson which must be considered as for any other particle with mass, i.e., it has a preferred rest frame, and the velocity of light, c, is no longer the velocity of the photon in all Lorentz frames, but a postulated constant of Einstein's second principle [17]. The Wigner theory of massless particles [28] shows that they are two dimensional with two helicities, but as soon as mass is associated with them, however tiny in magnitude, they develop three dimensionality. This is precisely what is indicated by the existence of $\boldsymbol{B}^{(3)}$, and therefore, through Eq. (3) by the existence of the conjugate product and therefore of magneto-optic phenomena in nature [1—21].

Roy and Evans [27] have recently proposed a development of gauge theory, based on the Dirac condition [29], which can be expressed as,

$$|A| \underset{FAPP}{\rightarrow} c\phi, \qquad A_\mu A^\mu \underset{FAPP}{\rightarrow} 0, \tag{16}$$

so that the vacuum A_μ becomes light-like and completely covariant. Here "FAPP" denotes *for all practical purposes* such as laboratory experiments. (On a cosmological scale, nothing can be rigorously light-like if photon mass is accepted.) The gauge condition (16) means that gauge invariance of type two [17] becomes compatible with finite photon mass, and related physics such as the Proca equation. Through the use of condition (16), finite photon mass theory is brought under the umbrella of contemporary field theory [30]. Roy *et al.* [26,27] have shown that the IFE puts an upper bound on photon mass which can be deduced in the laboratory ($\sim 10^{-44}$ kgm.) and that this bound is in satisfactory agreement with cosmological data [26] from several sources. This was achieved through a frequency dependent photon mass function [26,27].

We conclude that the exploration of $\boldsymbol{B}^{(3)}$ effects through RIFE and field-fermion resonance phenomena is linked in an interesting way to the existence of finite photon mass. The latter, furthermore, is not incompatible with gauge invariance of type two provided the gauge condition (16) is used.

ACKNOWLEDGEMENTS

One of us (MWE) thanks York University, Toronto, Canada, for a one year invitation to work in the Department of Physics and Astronomy there. Prof. J.-P. Vigier is thanked for his interest in this work, and the University of North Carolina is acknowledged for project support.

REFERENCES

[1] R. Zawodny, in M. W. Evans and S. Kielich, eds., *Modern Nonlinear Optics*, Vols. 85(1), 85(2), 85(3) of *Advances in Chemical Physics*, I. Prigogine and S. A. Rice, eds., (Wiley Interscience, New York, 1993).

[2] A. Piekara and S. Kielich, *J. Phys. Rad.* **18**, 490 (1957); *Arch. Sci.* **11**, 304 (1958); *Acta Phys. Polon.* **17**, 209 (1958); *J. Chem. Phys.* **29**, 1292 (1958).

[3] S. Kielich, *Acta Phys. Polon.* **32**, 405 (1967); *J. Colloid Interface Sci.* **30**, 159 (1969).

[4] P. W. Atkins and M. H. Miller, *Mol. Phys.* **15**, 491, 503 (1968); J. P. van der Ziel and N. Bloembergen, *Phys. Rev.* **138**, 1287 (1965).

[5] S. Woźniak, G. Wagnière, and R. Zawodny, *Phys. Lett. A* **154**, 259 (1991).

[6] P. S. Pershan, *Phys. Rev.* **130**, 919 (1963).

[7] J. P. van der Ziel, P. S. Pershan, and L. D. Malmstrom, *Phys. Rev. Lett.* **15**, 190 (1965); *Phys. Rev.* **143**, 574 (1966).

[8] J. Deschamps, M. Fitaire, and M. Lagoutte, *Phys. Rev. Lett.* **25**, 1330 (1970); *Rev. Appl. Phys.* **7**, 155 (1972).

[9] T. W. Barrett, *J. Chem. Soc., Chem. Comm.* p.9 (1982); *Thin Solid Films* **102**, 231 (1983); "Inverse Faraday Effect in Haemoglobin Detected by Raman Spectroscopy." *NADC-85074-60* (1985); T. W. Barrett, H. Wohltjen and A. Snow, *Nature* **301**, 694 (1983).

[10] G. Stedman in Ref. 1, Vol. 85(2).

[11] P. W. Atkins, *Molecular Quantum Mechanics*, 2nd edn. (Oxford University Press, Oxford, 1983).

[12] E. Courtens, *Phys. Rev. Lett.* **21**, 3 (1968).

[13] S. Kielich and R. Zawodny, *Opt. Commun.* **4**, 132 (1971).

[14] G. Wagnière, *Phys. Rev. A* **40**, 2437 (1989).

[15] M. W. Evans in I. Prigogine and S. A. Rice, eds., *Advances in Chemical Physics*, Vol. 81 (Wiley Interscience, New York, 1992).

[16] J.-P. Vigier, "Present Experimental Status of the Einstein-de Broglie Theory of Light," in *Proceedings, 4th International Symposium on Foundations of Quantum Mechanics* M. Tsukada *et al.*, eds. (Japanese Journal of Applied Physics, Tokoyo, 1993); A. S. Goldhaber and M. M. Nieto, *Rev. Mod. Phys.* **43**, 277 (1971).

[17] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Volume 1: The Field* $\boldsymbol{B}^{(3)}$ (Kluwer Academic Publishers, Dordrecht, 1994); ibid., *The Enigmatic Photon, Volume 2: Non-Abelian Electrodynamics* (Kluwer Academic Publishers, Dordrecht, 1995).

[18] A. A. Hasanein and M. W. Evans, *The Photomagneton in Quantum Field Theory* (World Scientific, Singapore, 1994), Vol. 1 of *Quantum Chemistry*

[19] M. W. Evans, in Ref. 1, Vol. 85(2).

[20] M. W. Evans, *Physica B* **182**, 227, 237 (1992); ibid., **183**, 103 (1993); **190**, 310 (1993); *Physica A*, in press; M. W. Evans, *Found. Phys.* **24**, 1519, 1671 (1994); ibid., ~~in press~~, Vigier Honorarium; M. W. Evans, *Found. Phys. Lett.* **7**, 67 (1994), and in press.

[21] M. W. Evans, *Physica A*, submitted.

[22] L. H. Ryder, *Quantum Field Theory*, 2nd edn. (Cambridge University Press, Cambridge, 1987).

[23] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th. edn. (Pergamon, Oxford, 1975).

[24] B. Talin, V. P. Kaftandjan, and L. Klein, *Phys. Rev. A* 11, 648 (1975).

[25] S. Woźniak, M. W. Evans and G. Wagnière, *Mol. Phys.* **75**, 81, 99 (1992).

[26] S. Roy *et al.*, *Int. J. Theor. Phys.*, **32**, 1025 (1993).

[27] S. Roy and M. W. Evans, *Found. Phys.*, submitted.

[28] E. P. Wigner, *Ann. Math.*, **40**, 149 (1939).

[29] P. A. M. Dirac, *Nature* **168**, 906 (1951); **169**, 702 (1952).

[30] T. W. Barrett, in A. Lakhtakia, ed., *Essays on the Formal Aspects of Electromagnetic Theory* (World Scientific, Singapore, 1993), pp. 6-86.