THE $\underline{B}^{(3)}$ FIELD IN MAXWELL CARTAN ELECTRODYNAMICS

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ABSTRACT

The necessary and sufficient condition is defined in Maxwell / Cartan electrodynamics for the existence of the $\underline{B}^{(3)}$ field, using the methods of differential geometry. The $\underline{B}^{(3)}$ field is non-zero when the three form A^dA is non-zero and is zero when A^dA is zero. Here A is the potential one-form which defines the Faraday two-form F = dA where d is the exterior derivative operator of differential geometry.

KEYWORDS : $\underline{B}^{(3)}$ field; Maxwell Cartan electrodynamics.

Historical Interest:
Early paper on Cartan geometry,
circa 1995/1996

1. INTRODUCTION

It is well known that Maxwell's equations can be expressed elegantly in the ~~lapguage~~ language of differential geometry {1}. In this letter it is shown that the recently inferred {2-10} $\underline{B}^{(3)}$ field of classical electrodynamics is defined by the three-form A^dA of Maxwell Cartan electrodynamics {1}. Here A is the classical potential one-form from which the Faraday two-form F is defined through F = dA, where d is the exterior derivative operator of differential geometry. The differential forms A, F and A^dA are objects defined by integration of chains, to which they are dual {1}, and the operator d produces an n+1-form from an n-form. In this elegant language the homogeneous and inhomogeneous Maxwell equations are dF = 0 and d*F = J respectively, where *F is the dual-form of F and J is the current density three-form.

The usual view of classical electrodynamics, given for example by Ryder {1}, is that F is a curvature two-form and that the Poincaré Lemma dF = 0 is a Bianchi identity. The U(1) group constraint is a device used in the standard model [1] to ensure that the ordered domain picks up a refined topology. In other words the U(1) constraint is an assumption that reduces the generality of Maxwell Cartan electrodynamics.

In Section 2 it is argued that the existence of A^dA (= A^F) implies that there exist at least three irreducible components of the magnetic part of the radiation field in all domains that support A^dA < > 0. If A^dA = 0 there can be only two such components. Thus the existence of $\underline{B}^{(3)}$ type solutions {2-10} is determined by A^dA < > 0 in a well defined domain. The three-form A^dA furthermore, exists in classical electrodynamics, and can be used {11-14} to define the concept of relativistic helicity. The latter is conserved if and only if the volume element four-form dA^dA = 0.

It is argued on this basis that $\underline{B}^{(3)}$ is non-linear in A, as postulated {2-10}, being determined by A^dA < > 0 or A^dA = 0. Thus $\underline{B}^{(3)}$ is not a static magnetic field as postulated {2-10} because a static magnetic field is linear in A, being included in the Faraday two-form F = dA. Since A^dA is a three-form, it obeys Stokes' formula {1}, which connects a p-form to a (p+1)-chain. Thus $\underline{B}^{(3)}$ as postulated {2-10} obeys Stokes' Theorem under well-defined existence conditions. Thus $\underline{B}^{(3)}$ is a property of non-linear optics, as postulated {2-10}, because A^dA is non-linear in A. In the presence of matter, dA^dA < > 0, and helicity is no longer conserved. The latter is conserved in general if and only if dA^dA = 0 identically {15}. This can be true even if A^dA < > 0. Thus, the interaction of $\underline{B}^{(3)}$ with matter can be described in terms of A^dA and dA^dA.

In Section 3, it is argued that the various critical claims {16} that $\underline{B}^{(3)}$ is incompatible with Maxwell's equations are fundamentally incorrect in differential geometry, because these claims mean that A^dA =? 0 in all domains, whereas A^dA < > 0 is compatible with F = dA and dF = 0 in well defined domains which may include vacuum domains. In other words F = dA; dF = 0 is not a sufficient condition for A^dA =? 0 in all domains. Furthermore, if helicity is conserved, dA^dA = 0 even if A^dA < > 0; so the Maxwell equation for non-zero $\underline{B}^{(3)}$ constant in magnitude and direction in vacuo is $\underline{\nabla} \times \underline{B}^{(3)} = \underline{0}$ as postulated {2-10}. There is no "Faraday induction" due to $\underline{B}^{(3)}$ in vacuo as observed experimentally {17}. Thus, the erroneous description of $\underline{B}^{(3)}$ as a static magnetic field is a critical misconception {16}.

(3)

2. THE LINK BETWEEN B AND A^dA.

It has been known for over sixty years that Maxwell's equations are metric independent tensor equations which are naturally covariant with respect to all diffeomorphisms. The latter include the Lorentz and Galilean maps, or any other map. Thus, Lorentz covariance is trivial for Maxwell's equations when written in terms of the covariant field intensities E and B and the contravariant tensor densities D and H. The physical solutions of Maxwell's equations are signals. These are singular solutions, the set o fspace-time points that define field discontinuities and topological defects. These singular solutions are linearly preserved only by the Lorentz transformations, and the singular solutions give the Lorentz transformations their physical importance.

It is much less well known that these singular solutions are preserved also by the non-linear Moebius transformation of projective geometry {18}. This transformation can explain physical phenomena inaccessible to the linear Lorentz transformation, for example phenomena in birefringent domains {19}. The Moebius transformation also accounts for T violation {20}, and needs solutions in terms of quaternions or spinors.

The imposition on this general structure of a specific metric or of a connection on a domain is a severe topological constraint. To deduce the equations dF = 0; d*F = J from F = dA; where A := A(X, Y, Z, t) is an ordered set of potentials depends only on an ordering of C2 functions. The deduction does not require the topological imposition of a metric or a connection on the base space. The Maxwell equations arise from F = dA and the Poincaré Lemma, ddA = dF = 0.

This deduction requires no metric, no connection, no covariant differentiation.

The generality of the theory is lost, however, if one subsumes a vector basis for teh base space; and if this vector basis is assumed to satisfy a group constraint of differential closure, such as the U(1), O(3), or any other group symmetry. Thus, the use of the U(1) group constraint in the standard model {1} is subjective. The U(1) group property of the basis set in the standard model effectively induces a connection and refines the most general topology. Thus, criticisms of $\underline{B}^{(3)}$ based on the U(1) group constraint are subjective {16}, and thus of no scientific value.

With these preliminaries in mind, the three-form A^dA becomes a generally applicable object in the topology of a well defined domain, including a vacuum domain. One can also construct the four-form dA^dA with equally general validity. The spatial parts of A^dA imply a flow in the longitudinal direction of the magnetic field component {21}. and this is the rigorous topological signature for the existence of the $\underline{B}^{(3)}$ field {2-10} and other longitudinally directed solutions of Maxwell's equations {2-10}. If A^dA <> 0 the potentials do not satisfy the Frobenius conditions of unique solubility. In such cases there does not exist a global set of isochronous points intersecting flow trajectories transversely. Therefore there must exist discontinuities and topological defects in spacetime. (These can be identified as dislocations and disclinations in crystals and fluids.) The manifold for F is not compact in four dimensional electromagnetic theory when derived from potentials and when A^dA <> 0. It is open, or compact with boundary. This is the perfect setting for a thermodynamics of irreversible processes that are open, or perhaps closed, but not isolated. The Cartan three-form A^dA, built on any one-form of action, does not satisfy the Frobenius integrability theorem. It is an object of topological torsion, A^F, in four dimensions, recently {11-14} referred to as relativistic helicity. The existence of A^F means that if a group structure is imposed upon the space then the induced connection cannot be symmetric in the lower indices. The connection is said to have Cartan torsion, and the base space cannot be riemannian. It is a generalization of a Finsler space. Cartan (or topological) torsion can exist on any space of ordered variables, such as {X, Y, Z, t}. Its non-zero value indicates that the one-form of action, (the four potential) cannot be represented by less than three variables. Hence real and complex representations are not permitted. The topological argument asserts that the one-form can be constructed in an n-dimensional space, where $n > 3$. There exists a submersive map from the higher to the lower space.

Kiehn {22} has shown that there exists a topological three-form $A\hat{}H$, where H is the (N-2)-form of field excitations, $H = H(D, H)$. When closed topologically, this object has a closed three-integral with the physical dimensions of spin. It is topologically dual to the topological torsion. The divergences of $A\hat{}H$ and $A\hat{}F$ lead to the two Poincaré invariants $\underline{E} \cdot \underline{E} - c\ \underline{B} \cdot \underline{B}$ and $\underline{E} \cdot \underline{B}$ respectively. Kiehn {23} has also demonstrated the applicability and classical meaning of the three-form $A\hat{}dA$. In hydrodynamics, $A\hat{}dA$ is the basis for transition to turbulence: streamline flow implies $A\hat{}dA = 0$, representing a connected topology; turbulence implies $A\hat{}dA <> 0$ {24}; representing a disconnected topology. The $A\hat{}dA$ three-form has been examined briefly {25} in electrodynamics; and general relativity {26}. It has been shown {27} that $A\hat{}dA <> 0$ leads to observable topological defects that occur in water. The existence of longitudinal vorticity can be demonstrated experimentally {28} and is a classical phenomenon.

The topological closure condition $d(A\hat{}dA) = 0$ is a conservation law for relativistic helicity {2-10}. Under this condition, the manifold of spacetime is a constant manifold and admits a unique hamiltonian (i.e. reversible) evolutionary process. Under the condition:

$$d(A \wedge dA) = 4\,\mathrm{Div}(A \wedge F) = 2(\underline{E} \cdot \underline{B})\,dx \wedge dy \wedge dz \wedge dt \quad -(1)$$

cap
x, y, z

the manifold cannot be compact without boundary. There does not exist a unique hamiltonian evolutionary path on such a domain. The only unique conformal evolutionary path on such a domain that leaves the action integral proportional to itself is thermodynamically irreversible. This important result {29} links thermodynamic irreversibility, topology and dynamics, without the use of statistics. [29]

Vortex lines exhibiting non-zero helicity have recently been observed experimentally, thus confirming the above analysis in all detail. The analogous data in electrodynamics arises from magneto-optics {2-10} which show the existence of non-zero relativistic helicity and non-zero $\underline{B}^{(3)}$ in field / matter interaction non-linear in A. The existence of A^dA means that $\underline{B}^{(3)}$ is always non-zero in the presence of matter, and tautologically, all non-linear magneto-optical effects are observed in the presence of matter, i.e. through the effect of a field on matter. Thus, the rigorous topological argument based on Maxwell Cartan electrodynamics shows that $\underline{B}^{(3)}$ is always observed in magneto-optics. Its existence in the vacuum requires a vacuum domain capable of supporting A^dA <> 0. In all probability, this is a generalization of Finsler spacetime. It seems that a riemannian vacuum will not support $\underline{B}^{(3)}$.

The Navier Stokes equations in hydrodynamics impose a constraint of isotropy on the dissipative effects, a constraint that need not exist in an electrodynamic system. However, for such constrained cases, the result would be that the helicity of the $\underline{B}^{(3)}$ field is an indicator of a four dimensional symplectic dissipative electromagnetic system, for which $\underline{E} \times \underline{A}$ is not necessarily zero. In order for $\underline{E} \times \underline{A}$ to be non-zero there must exist a non-zero torsion in the electromagnetic field, and this is the topological signature of the $\underline{B}^{(3)}$ field as postulated from magneto-optical data {2-10}. Thus $\underline{B}^{(3)}$ is the signature of an electromagnetic theory being examined on non-riemannian Finsler spaces, spaces with "in built torsion". The fact that the relativistic helicity is not zero leads in hydrodynamics to the ABC flow, an exact solution of the Navier Stokes equations that exhibits chaos.

What is thought to be observed in magneto-optical effects such as the non-linear inverse Faraday effect {2-10} is the conjugate product $\underline{A} \times \underline{A}^*$ in complex vector notation. This is related to $\underline{B}^{(3)}$ through the B Cyclic Theorem {2-10}:

$$\underline{B}^{(1)} \times \underline{B}^{(2)} = iB^{(0)}\underline{B}^{(3)*} \qquad (2)$$

which is a cyclic triplet (an angular momentum operator relation within $B^{(0)}$, the magnitude of $\underline{B}^{(3)}$ {2-10}). In the last of the division algebras (known as E8), the cyclic triplet sy~~mm~~mbols allow two algebraic structure realizations. One of these is non-abelian but associative; the other is abelian and non-associative {30}. The former seems to take on the well known structure of fermions, and the spinor equivalent of quaternions. The latter seems to represent bosons. Logic is a division algebra, and the latter need not be associative. Frobenius proved that the only associative division algebras, and hence the only associative logical systems, are the real and complex numbers and the quaternions. No other associative division algebra exists. However, based on the abovementioned analysis of the cyclic triplet, of which the B cyclic theorem is an example, there may exist an abelian, but non-associative, logical system based on the boson representation. This may find very general application in electromagnetic theory, and is yet another outcome of $\underline{B}^{(3)}$ theory, because the photon is thought to be a boson, not a fermion.

The example of the ABC flow in hydrodynamics has been used in a MAPLE code {31} to construct the Poincaré form of the topological torsion vector. This is an example of a generalization of the original $\underline{B}^{(3)}$ theory {2-10}, formulated in complex vector notation. This example shows the existence of magnetic helicity without the requirement of any group constraint. Thus, magnetic helicity and $\underline{B}^{(3)}$ is a very general property of Maxwell Cartan electrodynamics, one that is disallowed by the standard model and U(1) based critics { 16 }. Thus $\underline{B}^{(3)}$ is a clear counter-example to the standard model, a subjective construct, not necessarily a description of nature. The MAPLE code also appears to show that the rigorous form of the B Cyclic Theorem is one that must be based on quaternions, not complex vectors. The quaternionic representation allows longitudinal solutions of $\underline{B}^{(3)}$ to exist in all well defined domains. None of these are allowed by the standard model, which is therefore invalidated by $\underline{B}^{(3)}$ theory based on Maxwell Cartan electrodynamics.

We see that the standard model violates Maxwell Cartan electrodynamics, a major result of $\underline{B}^{(3)}$ theory; a result which was not known prior to this work on $\underline{B}^{(3)}$ in Maxwell Cartan electrodynamics. Similarly, the $\underline{B}^{(3)}$ theory invalidates the standard model in unified field theory, which also uses the U(1) group constraint. An entirely new approach to the problem of the unified field is needed, one based on topology. This conclusion explains much of the deeply illogical resistence of the critics { 16 } to the self-evident existence of A^dA and of $\underline{B}^{(3)}$ type solutions now being discovered and developed, solutions more general and more rigorous than the early ones proposed in the literature {2-10}. Recall that clear empirical evidence for A^dA <> 0 is available in classical physics (electrodynamics and hydrodynamics). We can therefore see clearly now that $\underline{B}^{(3)}$ is a major turning point in physics in the last thirty years, since the claims of a standard model first appeared in the literature.

This standard model is seriously flawed within its electromagnetic sector, habitually claimed to be a specialized topology derived from the entirely subjective U(1) group constraint. It is therefore flawed throughout, because U(1) is used throughout for the electromagnetic sector, for example in forming an electroweak theory. As we have argued, there is no group constraint applied in general Maxwell Cartan theory, which in certain vacua, allow longitudinal solutions to co-exist with the transverse solutions of the received view {1}. In the presence of matter (e.g. one fermion) the Maxwell Cartan theory produces A^dA < > 0, and so the $\underline{B}^{(3)}$ class of solutions always exists. Thus, this class of solutions is always observable through the effect of the electromagnetic field on matter. This is the only known way of observing an electromagnetic field. The standard model, on the other hand, asserts that under these and all other conditions, A^dA =? 0. The analogy of the ABC flow in hydrodynamics is a clear counter-example to the standard model, and an example of the $\underline{B}^{(3)}$ class of solutions. There will never be a standard model in physics, because physics is always a progression of free thought moderated by comparison with data. This same progression shows that the early stage of $\underline{B}^{(3)}$ theory {2-10}, based on complex vectors in ((1), (2), (3)), is giving way to a more mature phase in which various types of solutions are being found, solutions which can be identified within the same class; the $\underline{B}^{(3)}$ class of solutions in electrodynamics: a class whose rigorous topological signature is A^dA < > 0. The existence of this class is in conflict with the standard model across the whole of contemporarary physics: and empirical data supports the existence of the various known solutions within this class. This is logical progress.

The existence of the A^dA three-form, and therefore of the novel $\underline{B}^{(3)}$ type solutions, ("$\underline{B}^{(3)}$ Class" for short) implies that the topology induced on the underlying manifold by the functional forms of the potentials is not a connected topology. There does not exist a unique global smooth hypersurface connecting points on a "wave front". The one-form as Pfaffian equation is non-integrable in the sense of Frobenius. In such examples, when for certain other conditions $\underline{E} \cdot \underline{B} = 0$, the field can represent a characteristic system, and can therefore become a candidate for wave propagation and ~~that~~ the existence of discontinuities. The direction of the characteristic is in the direction of the torsion vector. In the earlier literature {2 - 10}, this direction is identified as longitudinal and represented by a single axial vector, the original $\underline{B}^{(3)}$ field. Such non-integrable systems are not Lorentz transformable in a global sense to the unconstrained vacuum, but they always exist over finite domains in the presence of matter.

It is of basic importance to note that the criterion of non-integrability leads to irreducibly three dimensional representations for a magnetic field in electrodynamics, and therefore indicates the existence of the $\underline{B}^{(3)}$ Class. Conversely, if a system is uniquely integrable it can be reduced to a "two dimensional" representation, such as that given by the U(1) group constraint of the standard model. The standard model thus disallows the $\underline{B}^{(3)}$ Class for this reason. In the non-integrable case the existence of the $\underline{B}^{(3)}$ Class is, however, without doubt. Examples can be constructed with a symbolic code such as MAPLE {31}, displayed and graphed. These can be constructed without any group constraint placed on the domain. Examples can be constructed using only the criterion that the domain of the vector field representing the potentials is irreducibly three dimensional (or more) in a topological (functional) sense. If the system of potentials is uniquely integrable in the sense of Frobenius then the irreducible magnetic representations need not have three components, and the $\underline{B}^{(3)}$ Class does not exist. This is characteristic of the standard model, which therefore puts a severe constraint on the topology underlying standard Maxwell Cartan electrodynamics. Thus, the "standard model" is not a standard model. The $\underline{E}$ and $\underline{B}$ fields computed by the MAPLE program, and therefore the $\underline{B}^{(3)}$ Class, are components of a second rank covariant tensor and are well behaved with respect to all diffeomorphisms, linear and non-linear. It is therefore trivial to show that the $\underline{B}^{(3)}$ Class is Lorentz covariant { 31 }.

The Frobenius integrability condition is not independent of the choice of gauge in gauge theories where a closed one-form is added to the one-form of action. Thus, the existence of the $\underline{B}^{(3)}$ Class in gauge theories is dependent on the choice of gauge. For example, $\underline{B}^{(3)}$ is not gauge invariant in a U(1) gauge theory, but is gauge invariant in an O(3) gauge theory {2-10}. Thus, gauge theories are intrinsically less satisfactory than those based on the Maxwell Cartan electrodynamics. The choice of U(1) versus O(3) is essentially subjective. The harmonic gauge contributions are those which carry much of the topological information in the four-vector potential. These contributions are closed in an exterior differential sense and do not contribute to the field intensities. However, they do contribute to the torsion current. Pure gradient additions to the four-potentials do not contribute to the topological prop~~p~~erties of the torsion current, nor to the field intensities. Thus, in gauge theories, if one defines a set of field intensities in terms of covariant derivatives instead of ordinary partial derivatives a topological constraint of a connection has been subsumed, and inevitably, classical Maxwell theory is modified. This is alse true for the U(1) group constraint because it also {1} leads to the use of a covariant derivative - the minimal prescription. The field intensities are not exact in the presence of such constraints because they depend not only on partial differential processes but also on connections (algebraic constraints). The classical Maxwell field axiom is violated. This procedure can lead to the Yang Mills class of gauge field theories, characterised by:

$$F = dA + A \hat{} A \qquad -(3)$$

The classical $F = dA$ defines a non-compact manifold, whereas the Yang Mills Class allows a compact manifold to be considered. The gauge field theories therefore reduce the generality of Maxwell Cartan theory, which is a generally applicable theory built on exterior forms: a theory which does not in general subsume a riemannian metric or affine connections. Therefore the question of vacuum structure, and the existence of the $\underline{B}^{(3)}$ Class in the vacuum, is left open by the Maxwell Cartan theory of electrodynamics. It is therefore logical to build a unified field theory on this structure, and not on a gauge field structure such as that used in the standard model {1}. In the latter, the connection is assumed to have a certain group property, which may at first be useful, but which is nevertheless subjective. The limitations of the U(1) group constraint are demonstrated clearly by the existence of the $\underline{B}^{(3)}$ Class; experimentally by the existence in nature of magneto-optical effects {2-10}.

In a torsion free space there exists a unique compat~~a~~ible set of connection coefficients, and such spaces are riemannian spaces. However, when the group used to define the connection is non-abelian, the non-unique connection used to define what is meant by a "covariant derivative" or a propagation that preserves local distance ds, has torsion of the affine type. With cert~~i~~an other constraints, the space is a non-riemannian Finsler space. The $\underline{B}^{(3)}$ Class in vacuo can exist only in a space with torsion. A Finsler space is an example.

Therefore the $\underline{B}^{(3)}$ Class in gauge theories needs for its existence a non-Abelian group constraint, such as that found in the O(3) group constraint {2- 10}. This type of constraint would lead to the concept of non-integrability and its attendant irreducibly three dimensional representations. However, the use of a covariant derivative of any kind eliminates local deformations, i.e. modifies the topology and loses the generality of Maxwell Cartan theory.

However successful these gauge models may appear to be {1}, the emergence of a concept such as $\underline{B}^{(3)}$ will sooner or later destroy their validity, and with it, that of the standard model. The $\underline{B}^{(3)}$ Class of solutions has therefore invalidated the standard model and has illustrated once more that the Maxwell field equations should not be constrained subjectively by metric, connection, or constitutive equation. The Maxwell equations are statements about topology on a domain. This is not to say that these equations cannot be generalized, but the use of covariant derivatives in such an attempt seems to result always in a loss of generality. This defeats the purpose of the attempt at generalization. Some other procedure may always be possible.

We conclude this section by asserting that for arbitrary constitutive equations, the $\underline{B}^{(3)}$ Class exists in the presence of matter and in gravitational domains with torsion; i.e. domains which are not uniquely integrable. The Godel solutions (rotational universes) are examples. In general, a sub-group constraint on a basis set is not a necessary condition for the existence of the $\underline{B}^{(3)}$ Class. In other words, a particular group structure or connection imposed on the Maxwell Cartan theory is not necessary for the existence of the $\underline{B}^{(3)}$ Class of solutions. Computations produced by the MAPLE program used in this work show the existence of the $\underline{B}^{(3)}$ Class within the Maxwell Cartan theory. This result counter-indicates several critical claims in the recent literature { 16 }.

Therefore the existence or non-existence of the $\underline{B}^{(3)}$ Class can be summarized by:

$$F = dA; \; dF = 0; \; A\hat{\ }dA <> 0 \text{ or } A\hat{\ }dA = 0 \; . \qquad \text{— (4)}$$

Thus the $\underline{B}^{(3)}$ Class arises from the Maxwell Cartan theory. There is no need to postulate the Yang Mills class of solutions. The three-form A^dA can be completely classical and is observable in the motion of fluids, in plasmas and in galactic structures as well as in classical magneto-optics. The topological torsion current exists on symplectic manifolds, and evolutionary motion in the direction of the torsion current can be shown to be irreversible in a thermodynamic sense, without invoking statistical mechanics. Within electrodynamics, these concepts have been self evident for about twenty years, and define the $\underline{B}^{(3)}$ Class in rigorous topology. Evidently, the critics of the original $\underline{B}^{(3)}$ concept { 16 } are unaware of this work and these ideas. The $\underline{B}^{(3)}$ Class of solutions can be observed not only in electrodynamics and unified field theory, but in hydrodynamics and cosmology.

3. TOPOLOGY VERSUS THE CRITICS OF $\underline{B}^{(3)}$.

There are no less than twenty six papers available in the recent literature { 16 } containing scientific exchange about the original concept of $\underline{B}^{(3)}$ {2-10}. The various critical claims are clearly incorrect at a basic level, because they assert that the $\underline{B}^{(3)}$ Class is incompatible with the Maxwell equations in all domains, including domains in which matter is present. Thus it is claimed that $A\hat{}dA <> 0$ is incompatible with $F = dA$, $dF = 0$. In fact, the three-form $A\hat{}dA <> 0$ coexists with F under well defined conditions. Thus Barron, Lakhtakia, Grimes, Comay, Buckingham et al. and Raja et al. are incorrect topologically. The net effect of these papers is to further cloud the issue and to cause subjective confusion. All these papers use the vector form of Maxwell's equations; a formalism due to Heaviside [32] and not the more elegant Cartan formalism used here. The papers assert subjectively that only transverse solutions are allowed of the Maxwell field equations and they therefore effectively assert that $A\hat{}dA =? 0$ under all conditions. These assertions are counter-indicated by the examples given in this paper from ABC flow, using the MAPLE software. Furthermore the critics are unaware of, or ignore, much recent work on relativistic helicity {11-14}, work which uses the concept A^dA < > 0. If so, there must exist at least three irreducible components of the magnetic field, as argued in Section 2.

The criticism by van Enk {16}, although obscure, appears to be an assertion that the existence of A^dA cannot be proven. In fact this is a three-form which emerges from the fundamental Maxwell-Cartan theory, one which has been self-evident in electrodynamics for about twenty years, and which causes observable effects in hydrodynamics. It is a well known property of classical physics.

Finally, the apparent failure of Raja et al. {16} and Rikken {16} to detect Faraday rotation due to A^dA is not due to the fact that A^dA =? 0, but to ~~very poor~~ inadequate experimental design, ~~an almost complete~~ a lack of theoretical understanding, and a subjective refusal to consult beforehand with the theorists.

The conclusion, inevitably, is that the critics do not have a sound knowledge of contemporary electrodynamics: their assertions should be read with considerable caution. The detailed answers to these critics [16] were based on the original concept of $\underline{B}^{(3)}$ as a vector, but can now be generalized with the use of topology, as developed in this paper. There is no further need to respond in detail to errors perpetrated by critics with an insufficient knowledge of the subject matter. One simply points out that A^dA < > 0 is consistent with F = dA.