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# THE PHOTOMAGNETON AND QUANTUM FIELD THEORY

## Vol. 1 of Quantum Chemistry

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**THE PHOTOMAGNETON AND QUANTUM FIELD THEORY**



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# PREFACE

It is well known that the time averaged energy (En) of one photon is $\hbar\omega$; that the magnitude ($|\boldsymbol{p}^{(3)}|$) of its time averaged linear momentum is $\hbar\omega/c$; and of its angular momentum, $\hbar$. Here $\hbar$ ($h/(2\pi)$) is the reduced Planck constant (or Dirac constant), $\omega$ the angular frequency (in radians $s^{-1}$) of electromagnetic radiation in a vacuum ("in vacuo"), and $c$ is the speed of light in vacuo. Following the arguments developed in the first book of this series, "The Photon's Magnetic Field", it is shown that electrodynamics can be developed in terms of a novel fundamental property of electromagnetic radiation: <u>*the Photomagneton*</u> $\hat{B}^{(3)}$, whose expectation value is the novel magnetic field in vacuo, $\boldsymbol{B}^{(3)}$. In terms of $\boldsymbol{B}^{(3)}$, the fundamental properties mentioned above become,

$$En = \hbar\omega = \frac{1}{\mu_0}\int \boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)}\,dV, \tag{1}$$

$$|\boldsymbol{p}^{(3)}| = \frac{En}{c}, \tag{2}$$

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}^{(3)}}{\hbar}, \tag{3}$$

where $\mu_0$ is the vacuum permeability, $V$ the volume occupied by one photon in vacuo; and $B^{(0)}$ is the scalar magnitude of $\boldsymbol{B}^{(3)}$ for one photon in vacuo. It is a magnetic flux density amplitude with the units of tesla.

In this part of the volume, the fundamental theory of $\hat{B}^{(3)}$ is developed in the form of eighteen scientific papers, each dealing with a different aspect of its properties in the quantum and classical theory of massless and massive electromagnetic fields. This part of the volume is therefore a radically new approach to electrodynamics which uses the photomagneton $\hat{B}^{(3)}$ as a fundamental conceptual entity. The papers provide a rigorous derivation of $\hat{B}^{(3)}$ from the existing ideas of electrodynamics, simultaneously simplifying and deepening them, making them more fully self-consistent and rigorous. Eqs. (1) to (3) show that $B^{(3)}$ is already well defined experimentally, but new experiments are suggested to look for effects specific to $\boldsymbol{B}^{(3)}$ which might not be expected to be found in the existing appreciation of electrodynamics, quantum or classical. Some of these are to be found in the inverse Faraday and optical Faraday effect, and the optical Bohm-Aharonov effect.

It is a pleasure to acknowledge the invaluable work of Dr. Laura J. Evans, who prepared the manuscript with skill and patience over many months of meticulous work. Many pleasant conversations by e mail are heartily acknowledged. Among those whose input has been invaluable are: Dr. Keith A. Earle of Cornell, Prof. Stuart K. Kurtz of Penn State, and Prof. Jean-Pierre Vigier of UPMC, Paris. Many other colleagues contributed with much appreciated criticism and ideas for development.

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*October, 1993*

# CONTENTS

# *Chapter 1*

## THE PHOTON HAS THREE POLARIZATIONS


M. W. Evans



Magnetization by light, an experimentally observed phenomenon, is interpreted to mean that the photon has three polarizations, right and left circular and longitudinal. The third is accompanied by a photomagneton $\hat{B}^{(3)}$ which has no energy or linear momentum and which is generated from photon spin. This is consistent with photon mass, however small in magnitude, and with Planck's radiation law. It implies that there is an anti-photon, whose kinematics are the same as those of the photon, but whose concomitant fields are changed in sign. The photon is therefore assigned a number $F$, akin to charge, baryon and lepton number and strangeness. Reversing the sign of $F$ produces the anti-photon.


It has been shown recently [1–3] that magnetization by light [4–11] can be interpreted through a *longitudinal* magnetic field of electromagnetism in free space, contrary to the standard theory [12–15], in which there are only two, circular, polarizations. This field, $\boldsymbol{B}^{(3)}$, has been named *the ghost field*, and is the expectation value of the novel longitudinal *photomagneton of light*, $\hat{B}^{(3)}$. The latter is an operator directly proportional to the angular momentum (spin) of the photon in free space, and its existence shows that *the photon has three polarizations, left and right circular, and longitudinal*. The latter has evaded detection to date because it is generated from the photon spin, which is independent of frequency ($\nu$). Therefore $\hat{B}^{(3)}$ is associated in the quantum theory with zero energy, $h\nu$, and zero linear momentum $h\nu/c$. Nevertheless $\hat{B}^{(3)}$ acts as a magnetic field, and causes magnetization when circularly polarized light interacts with liquids or solids. It cannot, however, be detected in Planck's radiation law [16, 17] because it has no Planck energy, $h\nu$.

The existence of $\hat{B}^{(3)}$ makes a fundamental difference to the theory of electromagnetism. For example, it supports the idea of finite photon rest mass, $m_0$, advocated by the School of de Broglie [18], and Vigier [19], because non-zero $m_0$, however small, means that the photon has three polarizations. This conclusion removes some of the discrepancies and obscurities of the electromagnetic part of field theory, which is based on a particle, the photon, with only two degrees of polarization, i.e. a dimensionality of two in three dimensional Euclidean space. In particular, the discovery of $\hat{B}^{(3)}$ removes the major problem that $A_\mu$, the potential four-vector that describes [20, 21] electromagnetism, corresponds to a field which conventionally has only two components. This is in conflict with the Bohm-Aharonov effect [22], which shows $A_\mu$ to be physically meaningful, with four components, as for any physical four-vector in special relativity. This had appeared to be irreconcilable with Planck's law, which was first derived in the quantum theory by Bose [23] on the ad hoc assumption of two photon polarizations only.

The missing longitudinal $\hat{B}^{(3)}$ is defined [1–3] by

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{1}$$

where $\hat{J}$ is the angular momentum of a photon beam, and where $B^{(0)}$ is the beam's magnetic field amplitude in tesla. For one photon, the classical equivalent of Eq. (1) becomes,

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}, \tag{2}$$

where $\boldsymbol{k}$ is a unit axial vector in the direction of travel of the light. *Therefore the field $\hat{B}^{(3)}$ is carried by photon spin in free space, a spin which becomes 0, $\pm\hbar$ in the photon with mass, or $\pm\hbar$ in the photon without mass.*

To date, clear experimental evidence for the effect of $\hat{B}^{(3)}$ at second order is available from the inverse Faraday effect [4–9], which is the name given to magnetization by light. There is also evidence from the optical Faraday effect [10] and light shifts in atomic spectra [11]. Symmetry [3] forbids the existence of a real longitudinal electric field, but there is, in classical electrodynamics, a longitudinal $i\boldsymbol{E}^{(3)}$ which is pure imaginary. The existence of $i\boldsymbol{E}^{(3)}$ means that the Poynting vector $\boldsymbol{N}^{(3)} = \boldsymbol{B}^{(3)} \times i\boldsymbol{E}^{(3)}/\mu_0$ vanishes, so that $\hat{B}^{(3)}$ does not contribute to classical electromagnetic energy flux density. This is equivalent to the fact that there is no Planck energy in the quantum theory.

Finally, it seems appropriate to associate with the photon an "$F$ number" which on reversal produces the anti-photon, whose kinematics are the same as those of the photon, but whose field is reversed. The charge conjugation operator $\hat{C}$ reverses $F$ in the same way as it reverses charge, baryon and lepton numbers, strangeness, and so on, to produce anti-particles. The existence of $\hat{B}^{(3)}$ means that the photon has all the attributes of a particle in three dimensions. In the Einstein-de Broglie theory of light, $\hat{B}^{(3)}$ becomes the pilot or guiding field of the photon spin in the direction of propagation of light as a needle wave (Nadelstrahlung). The discovery of $\hat{B}^{(3)}$ means that a fully consistent theory emerges naturally.

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# Chapter 2

## THE PHOTOMAGNETON $\hat{B}^{(3)}$ AND LONGITUDINAL GHOST FIELD $B^{(3)}$ OF ELECTROMAGNETISM

M. W. Evans

**Abstract**

The concepts are introduced of the longitudinal ghost field $\boldsymbol{B}^{(3)}$ and photomagneton $\hat{B}^{(3)}$ of electromagnetism: $\boldsymbol{B}^{(3)} = \langle\hat{B}^{(3)}\rangle = B^{(0)}\langle\hat{J}\rangle/\hbar$ where $B^{(0)}$ is the magnetic flux density amplitude and $\hat{J}$ the angular momentum operator of a photon beam. The major implication is that the individual photon has *three* degrees of polarization, the longitudinal one being accompanied by the ghost field $\boldsymbol{B}^{(3)}$ which has no energy or linear momentum, and is generated from the angular momentum of the photon.

### 1. Introduction

In order to derive without classical electrodynamics Planck's radiation law [1], S. N. Bose [2] used the notion that light has only two degrees of polarization. In this context Pais [3] clearly explains that Bose used this notion to derive the correct value of the premultiplier in Planck's law, i.e., of $8\pi\nu^2/c^2$. Here $\nu$ is the oscillator frequency in the old quantum theory of light, and $c$ is the speed of light in vacuo. At that time (1924) the idea that a particle (named the photon in 1926 by Lewis) can have only two polarizations in three dimensional space was unprecedented. In Euclidean and Minkowski spaces it is intuitively expected that a particle have three degrees of space-like freedom. Pais further recounts [3] that there exists no rest frame for photon spin *if the photon is massless*; and contemporary gauge invariance means that the separation between orbital and intrinsic (spin) angular momentum is ambiguous. The restriction to two polarizations also means that the little group of the Poincaré group of electromagnetism [4] is E(2), which is well known [4] to have no physical meaning. Most seriously, the customary approach is deeply flawed in that it leads to a loss of manifest covariance in special relativity, in that the four-potential $A_\mu$ must give a field with only two, transverse components. The Bohm-Aharonov effect [5] shows that $A_\mu$ is physically meaningful, and so must have four components in free space.

In this Letter, we demonstrate the fact that in the classical and quantum theories of electromagnetism, there exist the longitudinal *ghost field* (Gespensterstrahlung) $\boldsymbol{B}^{(3)}$, which is the expectation value of the longitudinal *photomagneton* $\hat{B}^{(3)}$:

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k} = \langle\hat{B}^{(3)}\rangle = B^{(0)}\frac{\langle\hat{J}\rangle}{\hbar}. \tag{1}$$



Thus $\boldsymbol{B}^{(3)}$ and $\langle\hat{B}^{(3)}\rangle$ are frequency independent magnetic flux densities in free space. Here $B^{(0)}$ is the scalar amplitude of the electromagnetic flux density, $\boldsymbol{k}$ an axial unit vector in the propagation axis, $\hat{J}$ the beam angular momentum and $\hbar$ the reduced Planck constant, $h/2\pi$. It is shown that $\hat{B}^{(3)}$ has no energy, no linear momentum, and does not affect the Planck law because its associated oscillator frequency $\nu$ is zero. It owes its existence purely to the intrinsic spin of the photon, whose scalar magnitude in the quantum theory is $\hbar$. The photomagneton and its classical equivalent, the ghost field, can nevertheless be detected experimentally through it ability to magnetize, and phenomena such as the inverse Faraday effect [6–13], and optical Faraday effect [14, 15] can be described in terms of $\hat{B}^{(3)}$ at first and higher orders. This provides unequivocal experimental support for the existence of $\hat{B}^{(3)}$. The latter is consistent with the idea that the photon may have a tiny mass [16–18]. However small, finite photon mass means immediately that the photon must have three well defined polarizations, and consequently, the existence of $\boldsymbol{B}^{(3)}$ in Maxwellian theory and $\hat{B}^{(3)}$ in the quantum theory means that finite photon mass is a consistent and natural idea.

### 2. Classical (Maxwellian) Electrodynamics

Within Maxwellian theory in free space, the ghost field $\boldsymbol{B}^{(3)}$ is related to the usual wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ by a cyclical Lie algebra [19]:

$$\left.\begin{aligned}
\boldsymbol{B}^{(1)}\times\boldsymbol{B}^{(2)} &= iB^{(0)}\boldsymbol{B}^{(3)*} = iB^{(0)}\boldsymbol{B}^{(3)},\\
\boldsymbol{B}^{(2)}\times\boldsymbol{B}^{(3)} &= iB^{(0)}\boldsymbol{B}^{(1)*} = iB^{(0)}\boldsymbol{B}^{(2)},\\
\boldsymbol{B}^{(3)}\times\boldsymbol{B}^{(1)} &= iB^{(0)}\boldsymbol{B}^{(2)*} = iB^{(0)}\boldsymbol{B}^{(1)}.
\end{aligned}\right\} \tag{2}$$

Here $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are complex conjugate wave fields (the usual magnetic components in circular polarization of the electromagnetic plane wave), and represent two transverse modes with orthogonal (circular) polarizations. In the standard Maxwellian theory of electrodynamics [20, 21] these are the only two polarizations considered, and describe left and right circularly polarized plane waves. However, Eqs. (2) show clearly that this picture is incomplete, *because if the ghost field $\boldsymbol{B}^{(3)}$ were zero, $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ would vanish, and all electromagnetism would be lost.*

*We assert therefore that in classical electrodynamics there are three magnetic components $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ of a travelling plane wave in vacuo. These are interrelated in the circular basis by Eq. (2). The third component, the ghost field*

$$B^{(3)} = \frac{B^{(1)} \times B^{(2)}}{(iB^{(0)})} = B^{(0)} \mathbf{k}, \tag{3}$$

*is real and independent of phase [19].*

By considerations of the Planck law of radiation, which is known [1, 3] to be valid experimentally, and which is based on only two polarizations (1) and (2), it follows that $B^{(3)}$ cannot contribute to free space electromagnetic energy density. It can be shown [19, 23] that this is true if $B^{(3)}$ is accompanied by the imaginary $iE^{(3)}$, so that,

$$U^{(3)} = \frac{1}{2}\left(\epsilon_0 iE^{(3)} \cdot iE^{(3)} + \frac{1}{\mu_0} B^{(3)} \cdot B^{(3)}\right) = 0. \tag{4}$$

Since $B^{(3)}$ is a real magnetic flux density it causes magnetization, as observed in the inverse and optical Faraday effects [6-15]. It can be shown [23] that the inverse Faraday effect *vanishes*, contrary to observation [6-13], if $B^{(3)} = 0$. In contrast, since $iE^{(3)}$ is imaginary, it is considered not to be a physical electric field strength, and to produce no spectral effects. Significantly, no spectral effects due to a longitudinal electric field of this type have been recorded, in contrast to the magnetic inverse and optical Faraday effects. The expression (4) is also consistent with the fact that $B^{(3)}$ and $iE^{(3)}$ do not contribute to the Poynting vector, and therefore do not contribute to the intensity of radiation, $I(\nu)$, the density of states, or the Planck law. This again is as observed experimentally.

## 3. The Fundamental Photomagneton $\hat{B}^{(3)}$

The transmutation of these classical ideas to the quantum theory takes place through the usual concepts of photon and photon spin. Since [24]

$$I(\nu) = c\rho(\nu), \tag{5}$$

where $\rho(\nu)$ is the density of radiation oscillator states, it is seen that $B^{(3)}$ and $iE^{(3)}$ do not affect the Planck law, because they add nothing to $I(\nu)$ or to $\rho(\nu)$. In Planck's law,

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\left(\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\right), \tag{6}$$



so that the Planck frequency, $\nu$, associated with $B^{(3)}$ and $iE^{(3)}$ is zero. This is consistent with the fact that these fields do not contribute to the classical equivalent of Eq. (6), the Rayleigh-Einstein-Jeans law. Nevertheless, in the quantum theory, it can be shown [19] that $B^{(3)}$ becomes the *photomagneton* operator

$$\hat{B}^{(3)} = B^{(0)} \frac{\hat{J}}{\hbar}, \tag{7}$$

where $\hat{J}$ is the angular momentum of the photon beam. For one photon, the magnitude of $\hat{J}$ is $\hbar$, so that the expectation value $\langle\hat{J}\rangle/\hbar = \mathbf{k}$, the axial unit vector that defines the classical ghost field $B^{(3)}$,

$$B^{(3)} = \langle\hat{B}^{(3)}\rangle = B^{(0)} \mathbf{k}. \tag{8}$$

*We are led to the conclusion that the photon has three degrees of polarization.*

In the quantum theory the photomagneton $\hat{B}^{(3)}$ is phase free, and as we have seen is associated with $\nu = 0$, i.e., with zero energy ($h\nu$) and zero *linear* momentum ($h\nu/c$). It is directly proportional, however, to the frequency independent photon *angular* momentum, which is $0, \pm\hbar$, not $\pm\hbar$ as in the original theory of S. N. Bose [2]. Because of the existence of $\hat{B}^{(3)}$, the photon becomes a boson with three degrees of polarization; and this does not affect the validity of the Planck law, Eq. (6).

## 4. The Existence of Photon Rest Mass, $m_0$

A boson with three degrees of polarization is a particle with mass. A boson with two degrees of polarization is a massless particle with only two helicities, the latter being components of particle spin along the direction of particle translation. In the standard Poincaré group [14] the helicity is the ratio of the Pauli Lubansky pseudo four-vector to the generator of space-time translation. In contemporary gauge theory, however, the existence of photon rest mass is usually not considered, because the gauge invariance condition,

$$m_0 A_\mu A_\mu = 0, \tag{9}$$

is solved by $m_0 = 0$. It has been shown recently, however [25], that the alternative solution,

$$m_0 \neq 0, \qquad A_\mu A_\mu = 0, \tag{10}$$

is consistent with the Dirac condition, and importantly, with the *experimental* fact that $A_\mu$ is physically meaningful through the Bohm-Aharonov effect [5]. The solution (10) allows $m_0$ to be non-zero, and this is consistent with our finding that the photon has three degrees of polarization (left and right circular and longitudinal).

The extensive and detailed work of de Broglie [16] and Vigier [18] and others on photon mass is therefore supported *strongly* by the existence of $\hat{B}^{(3)}$. We note that there is irrefutable experimental evidence for the magnetizing effects of $\hat{B}^{(3)}$ in the inverse Faraday effect [6-13] and optical Faraday effect [14, 15]. Recently the optical Cotton-Mouton effect has been reported experimentally [26] and interpreted [27] with $B^{(3)2}$. Note also that if $\hat{B}^{(3)}$ were zero, as required by the customary two polarization model, then the antisymmetric part of light intensity would vanish from Eq. (3), in conflict with experimental evidence from light scattering, for example. The Stokes $S_3$ parameter would vanish [19, 25], an incorrect result. In other words if $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$; $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} =? \boldsymbol{0}$, and it is known experimentally that $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is not zero, so $\boldsymbol{B}^{(3)}$ is not zero, and the photon has three polarizations.

Finally, it appears that the Einstein-de Broglie theory of light [28], in which waves and particles in light are both real, and co-exist, can accommodate the photomagneton $\hat{B}^{(3)}$ through the existence of photon spin, which, if the photon has mass, is now well defined in its rest frame as the angular momenta $0, \pm\hbar$. The field $\hat{B}^{(3)}$ becomes the Einstein-de Broglie guiding (or pilot) field of the frequency independent photon (particle) spin. This makes a profound difference to the basic theory of electromagnetism, for example $A_\mu$ becomes manifestly covariant, with four components, as needed, and the obscure E(2) little group [4] is replaced by a well defined and physically transparent rotation group in three dimensions. These are two of the consequences of the existence of the photomagneton $\hat{B}^{(3)}$ of light; and the ghost field $\boldsymbol{B}^{(3)}$, its expectation value. There is no algebra akin to Eq. (2) for $i\boldsymbol{E}^{(3)}$, and it appears that this field is unphysical. The photon spin cannot generate a longitudinal electric field by symmetry [19].

Further experimental work on the inverse and optical Faraday effects is urgently required to elucidate the intensity dependence of the influence of $\hat{B}^{(3)}$. The original experiment of van der Ziel *et al* [6] showed a dominating second order effect, but effects at first order in $\hat{B}^{(3)}$ are also expected.

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# Chapter 3

## THE RELATION BETWEEN TRANSVERSE AND LONGITUDINAL SOLUTIONS OF MAXWELL'S EQUATIONS

M. W. Evans

**Abstract**

It is shown that the antisymmetric part of the standard light intensity tensor is proportional in vacuo to a longitudinal magnetic field, denoted $\boldsymbol{B}^{(3)}$. This means that the cross product of electric or magnetic components of the usual transverse electromagnetic waves are linked to $\boldsymbol{B}^{(3)}$. The latter is a real, physically meaningful, uniform magnetic field. It is shown by considerations of energy conservation that its concomitant electric field is the imaginary $i\boldsymbol{E}^{(3)}$, directed in the same, propagation, axis of the transverse plane waves. Thus, $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are shown to be consistent with Poynting's theorem and with the inverse Faraday effect, the magnetization of material by circularly polarized light. The net contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to electromagnetic energy density, U, is zero,

$$U \equiv \frac{1}{2\mu_0}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)} + \frac{\epsilon_0}{2}i\boldsymbol{E}^{(3)}\cdot i\boldsymbol{E}^{(3)} = 0,$$

where $\epsilon_0$ and $\mu_0$ are respectively the permittivity and permeability in vacuo.

### 1. Introduction

It is well known that Maxwell's equations are linear partial differential equations in field vectors, charge density, and current density. The principle of superposition therefore asserts that the resulting field vector at a given point is the sum of vectors produced at that point by various sources [1-5]. Therefore Maxwell's equations also apply to sums of electric and magnetic fields and waves. They are the four fundamental equations of electromagnetism, and apply to electromagnetic waves and to uniform magnetic and electric fields in all media (including the vacuum) that are rest with respect to the coordinate system used. In a vacuum two of the equations reduce to the differential forms,

$$\nabla\cdot\boldsymbol{E} = 0, \tag{1}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{2}$$

where $\boldsymbol{E}$ is electric field strength in volts $m^{-1}$ and $\boldsymbol{B}$ is magnetic flux density in tesla. These conditions are satisfied by the usual transverse electromagnetic waves in vacuo [1–5].

However, they are also satisfied by uniform, longitudinal $\boldsymbol{E}$ and $\boldsymbol{B}$, for example, Eq. (2) is satisfied by the Z axis field,

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}, \tag{3}$$

where $B^{(0)}$ is a scalar amplitude independent of Z and where $\boldsymbol{k}$ is a unit axial vector in Z. Furthermore, a uniform string of magnetic flux such as $\boldsymbol{B}^{(3)}$ exists in vacuo, and is a "wave of infinitely low frequency". It is usual to develop the theory of electromagnetic waves without specific regard for uniform fields. The latter are removed customarily by setting them to zero, a procedure which is equivalent to assuming that there is no link between transverse electromagnetic waves and uniform longitudinal fields of type (3). For example, in a textbook such as that of Corson and Lorrain [1], on page 317, this habitual procedure is embodied in the statement:

> "Thus $E_Z$ cannot be a function of $Z$. We shall set $E_Z = 0$ since we are interested in waves and not in uniform fields".

Similarly, it is asserted that the longitudinal uniform magnetic field which is a solution of Maxwell's equations in vacuo is zero. These arbitrary assertions appear to be justified by experimental data, because light, unlike a uniform electric field, does not appear to cause polarization to first order in $\boldsymbol{E}$, or magnetization to first order in $\boldsymbol{B}$. It also appears possible to express the Poynting theorem in terms only of the transverse electromagnetic waves [1–5].

However, it is known experimentally that light can magnetize [6] through the inverse Faraday effect, cause shifts in atomic spectra [7], and in NMR lines [8]. Indeed, the theory of these effects has been developed [9–14] in terms of an effective, uniform, frequency dependent, magnetic field. In Sec. 2 of this paper the optical property usually used to describe the inverse Faraday effect [9–11], the antisymmetric part ($I_{ij}^A$) of the light intensity tensor, $I_{ij}$, is related directly to $\boldsymbol{B}^{(3)}$ of Eq. (3). This means that *transverse electromagnetic waves and uniform longitudinal fields are linked in free space*. If it is arbitrarily asserted that fields such as $\boldsymbol{B}^{(3)}$ are zero, then the transverse electromagnetic waves also disappear. Section 3 shows that the existence of a real $\boldsymbol{B}^{(3)}$ implies that of an imaginary $i\boldsymbol{E}^{(3)}$ in the same propagation axis (Z) of the plane wave. The net contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to electromagnetic energy density is therefore zero. The existence of these uniform, longitudinal fields does not affect the Poynting theorem. Since $i\boldsymbol{E}^{(3)}$ is imaginary, its real part vanishes and cannot cause electric polarization at first order. On the other hand $\boldsymbol{B}^{(3)}$ is real and can magnetize at first order. The experimental evidence for this finding is examined with particular reference to the inverse Faraday effect [6, 9–11]. Finally, in Sec. 4, it is shown that the existence of $I_{ij}^A$, the antisymmetric part of the light intensity tensor, implies the existence of a real $\boldsymbol{B}^{(3)}$ in vacuo. The latter is a new fundamental property of light.



## 2. Link Between Longitudinal and Transverse Magnetic Fields

The antisymmetric part of the well known light intensity tensor, $I_{ij}$, of nonlinear optics [15, 16] is defined in terms of the product of a transverse solution of Maxwell's equations (for example the oscillating electric field, $\boldsymbol{E}^{(1)}$ with its complex conjugate, denoted $\boldsymbol{E}^{(2)}$. Thus,

$$I_k \equiv \frac{1}{2}\epsilon_{ijk} I_{ij}^A \equiv \frac{1}{2}\epsilon_0 \boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}, \tag{4}$$

where $\epsilon_0$ is the permittivity in vacuo. The conjugate product is imaginary, but can form a real interaction Hamiltonian by multiplication with the imaginary part of electric polarizability [17]. In the inverse Faraday effect [9–11], a real magnetic dipole moment is induced from the product of $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ with the imaginary part of a molecular hyperpolarizability. The conjugate product is independent of frequency, and as first shown by Kielich [18], it is the quantity responsible for static magnetization by circularly polarized light.

It is a simple matter to show [19] that the conjugate product can be expressed in terms of the real longitudinal magnetic field, $\boldsymbol{B}^{(3)}$,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = iE_0^2\boldsymbol{k} = ic^2B^{(0)2}\boldsymbol{k} = ic^2B^{(0)}\boldsymbol{B}^{(3)} = c^2\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}. \tag{5}$$

Using the fundamental free space relation between electric and magnetic field amplitudes in vacuo,

$$E_0 = cB^{(0)}, \tag{6}$$

Eq. (5) leads to a cyclical relation between $\boldsymbol{B}^{(3)}$, the usual transverse magnetic field $\boldsymbol{B}^{(3)}$, and the latter's complex conjugate $\boldsymbol{B}^{(2)}$. The three equations are

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*} = iB^{(0)}\boldsymbol{B}^{(3)}, \tag{7a}$$

$$\boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = iB^{(0)}\boldsymbol{B}^{(1)*} = iB^{(0)}\boldsymbol{B}^{(2)}, \tag{7b}$$

$$\boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = iB^{(0)}\boldsymbol{B}^{(2)*} = iB^{(0)}\boldsymbol{B}^{(1)}, \tag{7c}$$

and show that $\boldsymbol{B}^{(3)}$ *is linked to* $\boldsymbol{B}^{(1)}$ *and* $\boldsymbol{B}^{(2)}$. This means that it is not possible, as is the customary practice, to discard longitudinal solutions of Maxwell's equations in free space, or in matter. If an attempt is made to set $\boldsymbol{B}^{(3)}$ to zero in Eq. (7), the transverse fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ also vanish. Conversely, if $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are not zero, then $\boldsymbol{B}^{(3)}$ cannot be zero. Note from Eqs. (7) that $\boldsymbol{B}^{(3)}$ has all the known properties of uniform magnetic flux density in tesla. The equations (7) are invariant to motion reversal ($\hat{T}$), parity inversion ($\hat{P}$), and charge conjugation ($\hat{C}$), and form a Lie algebra because $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are proportional to rotation generators [20] of the three dimensional group O(3). In space-time, these rotation generators form a subgroup of the Lorentz group through the same type of Lie algebra. The cyclical equations (7) are symmetrical and are geometrical in nature [20, 21]. The customary assertion that $\boldsymbol{B}^{(3)}$ is zero (see Introduction) while $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are non-zero is geometrically unsound [20, 21], both in three dimensional space, and four dimensional space-time. The reason is that $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are each proportional to rotation generators, and there are three non-zero rotation generators in space. The rotation generators form the Lie algebra of O(3) in three dimensional space. In space-time, the rotation generators are well known [20] to be four by four matrices, and the generators in this case obey a Lie algebra which describes a subgroup of the Lorentz group. It is geometrically incorrect, either in O(3) or in the Lorentz group, to assert that $\boldsymbol{B}^{(3)}$ is zero while at the same time asserting that $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are non-zero. Unfortunately, these assertions have become so familiar through habit that they are to be found in numerous textbooks on electrodynamics.

What is the experimental evidence for $\boldsymbol{B}^{(3)}$?

It is simple to see that experimental evidence for the existence of the well known light intensity tensor is also evidence for the product $ic^2B^{(0)}\boldsymbol{B}^{(3)}$. Is it then possible to argue that $\boldsymbol{B}^{(3)}$ is zero while at the same time accepting the existence of $I_{ij}^{A}$? The ultimate answer to this resides in experimental observation, and in this context it is known that circularly polarized light magnetizes [6, 9–11] liquids and solids through the inverse Faraday effect. The latter can be described theoretically [9–11] in terms of the conjugate product, and therefore in terms of $I_{ij}^{A}$, the antisymmetric intensity of light, an axial vector. It is well established therefore that there is experimental evidence for the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$. From Eq. (5) there is also evidence for the product $ic^2B^{(0)}\boldsymbol{B}^{(3)}$. Does this mean that there is evidence for $\boldsymbol{B}^{(3)}$? Let us consider the possibilities.

1. If $\boldsymbol{B}^{(3)} = 0$, then the antisymmetric intensity of light is always zero. This contradicts experimental data. Therefore $\boldsymbol{B}^{(3)}$ is not zero.

2. There is undisputed experimental evidence for the product $ic^2B^{(0)}\boldsymbol{B}^{(3)}$. This can be written out in three ways: (a) $i(B^{(0)2}\boldsymbol{k})$; (b) $iB^{(0)}(B^{(0)}\boldsymbol{k})$; and (c) $(iB^{(0)2})\boldsymbol{k}$. The first is simply the conjugate product itself. The second is the real magnetic field $\boldsymbol{B}^{(3)}$ multiplied by $iB^{(0)}$; and the third is equivalent to the first. The property appearing in cases i) and iii) can produce the inverse Faraday effect as in standard theory [9–11]. In case ii), the same property has been expressed in terms of a real, fundamental vector $\boldsymbol{B}^{(3)}$ which has all the known properties of real uniform magnetic flux density. In all three cases, the most fundamental element is $\boldsymbol{B}^{(3)}$, *which is therefore a fundamental property of light*. The way in which this property manifests itself experimentally can be determined only by further investigation. In the absence of contrary indications, it must be assumed that the real part of $\boldsymbol{B}^{(3)}$ is a magnetic field, and has all the associated physical properties. The consequent spectroscopic effects expected from $\boldsymbol{B}^{(3)}$ have been discussed fully elsewhere [22].



In the inverse Faraday effect, for example, it is expected that $\boldsymbol{B}^{(3)}$ can form a direct interaction Hamiltonian with a real electronic magnetic dipole moment, if non-zero, as in a paramagnetic. In this case there would be magnetization due to $ic^2B^{(0)}\boldsymbol{B}^{(3)}$, proportional to light intensity, *plus* magnetization due to $\boldsymbol{B}^{(3)}$, proportional to the square root of light intensity. The relative contributions would depend [23] on the relative magnitudes of the property tensors mediating the two mechanisms, as discussed elsewhere in detail. In the one available experiment on the inverse Faraday effect [6], (doped) $CaF_2$ glasses were investigated at liquid helium temperatures. It is not clear whether these glasses contained a net magnetic dipole moment. The majority of data in these experiments were obtained on diamagnetic liquids at room temperature, which were magnetically non-dipolar and in which an effect to first order in $\boldsymbol{B}^{(3)}$ is not expected because there is no net magnetic dipole moment with which $\boldsymbol{B}^{(3)}$ can interact. If the glasses used in this pioneering (and difficult) experiment did contain a net magnetic dipole moment, an effect is expected which is proportional to $\boldsymbol{B}^{(3)}$, because the latter has all the known properties of uniform magnetic flux density. The available data [6] can be interpreted in terms of the product $ic^2B^{(0)}\boldsymbol{B}^{(3)}$ whose magnitude is $c^2B^{(0)2}$, i.e. proportional to the *square* of the fundamental property $\boldsymbol{B}^{(3)}$. The data in no way contradict the existence of $\boldsymbol{B}^{(3)}$, and the available results are unequivocal evidence for the existence of $\boldsymbol{B}^{(3)}$ multiplied by $ic^2B^{(0)}$.

It is clear that the inverse Faraday effect is experimental evidence for $\boldsymbol{B}^{(3)}$. The next section shows that the existence of $\boldsymbol{B}^{(3)}$ is consistent with experimental data concerning electromagnetic energy density and the Poynting theorem. In other words there is no fundamental contradiction whatsoever with experimental data.

### 3. Considerations of Electromagnetic Energy Density

The existence of longitudinal solutions of Maxwell's equations such as $\boldsymbol{B}^{(3)}$ suggests a 50% increase in electromagnetic energy density (U). However, Planck's radiation law [24] is derived from the transverse electromagnetic waves and is known to account precisely for experimental measurements of light intensity. Therefore the magnetic field $\boldsymbol{B}^{(3)}$ cannot contribute to U in free space. This seems to contradict the fact that the total work which must be done to establish a magnetic field in terms of the magnetic flux density in free space is the integral,

$$W_B = \frac{1}{2\mu_0}\int \boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)}\,dV, \tag{8}$$

over the volume of interest, which can be taken to be the volume occupied by the light beam. Here $\mu_0$ is the magnetic permeability in vacuo. The energy density in free space associated with $\boldsymbol{B}^{(3)}$ is therefore

$$U_B = \frac{dW_B}{dV} = \frac{1}{2\mu_0}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)}, \tag{9}$$

at each point in space. This is not zero in general, and there appears to be a contradiction with experimental data on light intensity and with the Planck law.

It is shown in this Sec. that this contradiction can be resolved through the fact that associated with $\boldsymbol{B}^{(3)}$ in free space is a pure imaginary, uniform, longitudinal, electric field $i\boldsymbol{E}^{(3)}$. The total energy density due to $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ is zero,

$$U \equiv \frac{1}{2\mu_0}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)} + \frac{1}{2}\epsilon_0\, i\boldsymbol{E}^{(3)}\cdot i\boldsymbol{E}^{(3)}, \tag{10}$$

in agreement with experimental data. The existence of $i\boldsymbol{E}^{(3)}$ can be deduced as follows using a form of Lorentz's Lemma [1]. It is assumed that,

$$\boldsymbol{E}_G = \boldsymbol{E} + \boldsymbol{E}^{(3)}, \tag{11a}$$

$$\boldsymbol{B}_G = \boldsymbol{B} + \boldsymbol{B}^{(3)}, \tag{11b}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are transverse electromagnetic waves. The uniform $\boldsymbol{B}^{(3)}$ is real and is defined by Eq. (5), and in general $\boldsymbol{E}^{(3)}$ may be complex. Using the vector identities

$$\nabla\cdot(\boldsymbol{E}^G\times\boldsymbol{B}^G) = \nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}) + \nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}^{(3)}) + \nabla\cdot(\boldsymbol{E}^{(3)}\times\boldsymbol{B}) + \nabla\cdot(\boldsymbol{E}^{(3)}\times\boldsymbol{B}^{(3)}), \tag{12}$$



and

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}^{(3)}) = \boldsymbol{B}^{(3)}\cdot(\nabla\times\boldsymbol{E}) - \boldsymbol{E}\cdot(\nabla\times\boldsymbol{B}^{(3)}), \tag{13}$$

and the Maxwell equation

$$\nabla\times\boldsymbol{B}^{(3)} = \boldsymbol{0}, \qquad \nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \tag{14}$$

leads to the Lorentz Lemma [1]

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}^{(3)}) = \nabla\cdot(\boldsymbol{E}^{(3)}\times\boldsymbol{B}). \tag{15}$$

This can be re-expressed, using the Maxwell equations, as,

$$\boldsymbol{B}^{(3)}\cdot\frac{\partial\boldsymbol{B}}{\partial t} + \frac{1}{c^2}\boldsymbol{E}^{(3)}\cdot\frac{\partial\boldsymbol{E}}{\partial t} = 0, \tag{16}$$

from which it can be seen that the contributions of products $\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}$ and $\boldsymbol{E}^{(3)}\cdot\boldsymbol{E}$ to electromagnetic energy density are zero. This is of course consistent with the fact that $\boldsymbol{B}^{(3)}$ is orthogonal to $\boldsymbol{B}$, and $i\boldsymbol{E}^{(3)}$ is orthogonal to $\boldsymbol{E}$. Furthermore, integrating Eq. (15) over all space, and using the Divergence theorem,

$$\int(\boldsymbol{E}\times\boldsymbol{B}^{(3)} - \boldsymbol{E}^{(3)}\times\boldsymbol{B})\cdot d\boldsymbol{a} = 0. \tag{17}$$

This implies that the integrand is zero. Taking the $\boldsymbol{B}^{(1)}$ and $\boldsymbol{E}^{(1)}$ components, for example,

$$\boldsymbol{E}^{(1)}\times\boldsymbol{B}^{(3)} = \boldsymbol{E}^{(3)}\times\boldsymbol{B}^{(1)}, \tag{18}$$

and for the $\boldsymbol{B}^{(2)}$ and $\boldsymbol{E}^{(2)}$ components,

$$\boldsymbol{E}^{(2)}\times\boldsymbol{B}^{(3)} = \boldsymbol{E}^{(3)}\times\boldsymbol{B}^{(2)}, \tag{19}$$

Using the definitions in circular polarization,

$$\boldsymbol{E}^{(1)} = \frac{E_0}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi}, \qquad \boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(+i\boldsymbol{i} + \boldsymbol{j})e^{i\phi}, \tag{20}$$

it follows that the postulated longitudinal electric field $i\boldsymbol{E}^{(3)}$ is imaginary if $\boldsymbol{B}^{(3)}$

is real.

This result produces $U=0$ from Eq. (10), so the net contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to $U$ is zero, even though the individual contributions are not zero, and is consistent with Planck's law. In arriving at it we have assumed only that the longitudinal $\boldsymbol{B}^{(3)}$ is accompanied by the longitudinal $i\boldsymbol{E}^{(3)}$. The existence of a real $\boldsymbol{B}^{(3)}$ and imaginary $i\boldsymbol{E}^{(3)}$ is also consistent with the fact that $\boldsymbol{B}^{(3)}$ is observable through the inverse Faraday effect, while there is no evidence for electric polarization by light apart from optical rectification [25]. The latter occurs in chiral materials and is a second order process. The real part of $i\boldsymbol{E}^{(3)}$ is zero, and therefore there is no electric polarization. Furthermore, it is not possible to construct $\boldsymbol{E}^{(3)}$ from vector cross products of transverse fields, such as $\boldsymbol{E}^{(1)}\times\boldsymbol{E}^{(2)}$, $\boldsymbol{E}^{(1)}\times\boldsymbol{B}^{(2)}$ and so on, because the vector products are $\hat{T}$ negative while $\boldsymbol{E}^{(3)}$ must be $\hat{T}$ positive if it is to be a *real* electric field. There is no Lie algebra for a real $\boldsymbol{E}^{(3)}$ analogous with the Lie algebra (7) for $\boldsymbol{B}^{(3)}$.

The main result of this section therefore is that the Poynting theorem,

$$\nabla\cdot\boldsymbol{N}_G = -\frac{\partial U_G}{\partial t}, \qquad \boldsymbol{N}_G = \frac{1}{\mu_0}\boldsymbol{E}_G\times\boldsymbol{B}_G, \tag{21}$$

is unchanged by the existence of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$. These fields do not contradict the law of conservation of electromagnetic energy density in free space. From the Lie algebra (7) the source of $\boldsymbol{B}^{(3)}$ is the same as that of $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ and it is not possible to assert that $\boldsymbol{B}^{(3)}$ cannot exist because there is no source for it in free space. Clearly if $\boldsymbol{B}^{(3)}$ vanished for this reason so would $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. A real $\boldsymbol{B}^{(3)}$ and an imaginary $i\boldsymbol{E}^{(3)}$ is consistent with experimental data, which shows the existence of magnetizing effects due to circularly polarized light, but no polarizing effects. (Optical rectification occurs in linear polarization, but the inverse Faraday effect requires circular polarization. In linear polarization both $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are zero, showing that optical rectification is not due to a longitudinal electric field.)

## 4. Antisymmetric Light Intensity and $\boldsymbol{B}^{(3)}$

The antisymmetric part of the light intensity tensor

$$I_{ij} \equiv \epsilon_0 c E_i E_j^*, \tag{22}$$

is defined in the literature, for example by Knast and Kielich [26] as the pure imaginary axial vector



$$I_k = \frac{1}{2}\epsilon_{ijk}(I_{ij}-I_{ji}) = i\epsilon_0 c^2 E_0 \boldsymbol{B}^{(3)} \equiv i I_0 \boldsymbol{k}, \tag{23}$$

where $\boldsymbol{k}$ is a unit axial vector in the propagation direction. The $\boldsymbol{B}^{(3)}$ vector is therefore proportional to the square root of $I_k$.

$$\boldsymbol{B}^{(3)} = \left(\frac{I_0}{\epsilon_0 c^2 E_0}\right)\boldsymbol{k} = \left(\frac{I_0}{\epsilon_0 c^3}\right)^{\frac{1}{2}}\boldsymbol{k}, \tag{24}$$

The antisymmetric part of light intensity is also related to the third Stokes parameter $S_3$, and it is well known that the latter is non-zero only in circularly or elliptically polarized light. The field $\boldsymbol{B}^{(3)}$ is therefore zero in linearly polarized or incoherent light, and by Eq. (18) so is the imaginary $i\boldsymbol{E}^{(3)}$. Effects due to $\boldsymbol{B}^{(3)}$ are expected therefore only in circularly polarized light interacting with matter. For example, the inverse Faraday effect is magnetization due to circularly polarized light, and the sign of magnetization reverses experimentally with the sense of circular polarization [6, 9–11]. It can be said that the inverse Faraday effect is magnetization due to the angular momentum of antisymmetric light intensity, the latter can be expressed in terms of the fundamental $\boldsymbol{B}^{(3)}$. Unfortunately, the data available on the inverse Faraday effect do not give a sufficient idea of whether the field $\boldsymbol{B}^{(3)}$ can act directly, to produce magnetization proportional to the square root of intensity. There is in fact only one reported experiment [9–11], and most of the data from that study were obtained in molecular liquids at room temperature. In these diamagnetics there is no permanent net magnetic dipole moment, and the observed magnetization is proportional to $|\boldsymbol{B}^{(3)}|^2$ and therefore to light intensity. Some data were reported at low temperatures in doped $CaF_2$ glass, and appear at first glance to be proportional to intensity. However a least mean squares fit would reveal whether these data indicate the presence of a contribution from the square root of intensity. A recent reanalysis [23] of the inverse Faraday effect indicates the orders of magnitude of contributions to the total magnetization from $ic^2B^{(0)}\boldsymbol{B}^{(3)}$ and from $\boldsymbol{B}^{(3)}$, assuming that the latter acts as a magnetic field. This assumption is analyzed in more detail in the following discussion.

## 5. Discussion

The quantity $\boldsymbol{B}^{(3)}$ has all the known properties of magnetic flux density, i.e. it has the units of tesla, is an axial vector which is $\hat{C}$ negative, $\hat{P}$ positive, $\hat{T}$ negative, and is a solution of Maxwell's equations. It is however, a property of light, and is not a conventionally generated uniform magnetic field, in the sense that it is not generated by an electric current, as in a wound solenoid, or a magnetic material such as an ordinary magnet. The conjugate product can be expressed in terms of $\boldsymbol{B}^{(3)}$, and the latter is an elementary property of circularly polarized light, or electromagnetic radiation. The source of $\boldsymbol{B}^{(3)}$ must therefore be thought of in the same terms as the source of $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ in free space, i.e. in terms of the source of electromagnetic radiation in free space. The concept of $\boldsymbol{B}^{(3)}$ is therefore new to the theory of electromagnetism, and its properties must be investigated experimentally in order to determine whether it acts as a magnetic field or as part of the light intensity tensor. In either case the Lie algebra (7) shows that $\boldsymbol{B}^{(3)}$ is linked to $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, which are of course magnetic waves.

It is clear that $\boldsymbol{B}^{(3)}$ is not zero, and it seems basically self contradictory to assert that a quantity which has all the known properties of a magnetic field cannot act like one experimentally. If such a paradox is confirmed experimentally, it would indicate that electrodynamics is not complete, and not self consistent. Since $\boldsymbol{B}^{(3)}$ has all the properties of magnetic flux density an interaction Hamiltonian of the type,

$$\Delta H = -\boldsymbol{m}\cdot\boldsymbol{B}^{(3)}, \tag{25}$$

is expected with a magnetic dipole moment $\boldsymbol{m}$, and a variety of other effects specifically due to $\boldsymbol{B}^{(3)}$ regarded as a magnetic field are listed in Refs. [20–22]. An interesting example is the Bohm-Aharonov effect, in which a circularly polarized laser guided through a 2 $\mu m$ optical fibre is expected to produce a fringe shift in interfering electron beams [27]. This shift would indicate the existence of the vector potential due to the magnetic field $\boldsymbol{B}^{(3)}$ generated by the circularly polarized laser passing through the optical fibre. If no effect is observed then $\boldsymbol{B}^{(3)}$ does not act as a magnetic field despite having all the known properties of a magnetic field. This would be a fundamental paradox in electrodynamics. If an effect is observed, the existence of $\boldsymbol{B}^{(3)}$ would be verified. Therefore, either way, the experiment is of great interest [28].

The net contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to electromagnetic energy density is zero, as discussed in this paper. However, in Ref. [1], it is made clear that the assignment of an energy density from a magnetic or electric field to a point in space is an entirely arbitrary procedure, which is described in Ref. [1] as "meaningless" except as a means of computing the overall magnetic or electric energy. It is also made clear that it is meaningless to discuss energy as residing in a magnetic or electric field. Arguments against the existence of $\boldsymbol{B}^{(3)}$ on the grounds of electromagnetic energy density are therefore not valid. This conclusion is reinforced by the fact that it is the scalar part of light intensity that is normally associated with electromagnetic energy density, and not the antisymmetric part, which generates $\boldsymbol{B}^{(3)}$. The latter is not therefore expected to contribute to electromagnetic energy, and this result is reinforced by the calculations of this paper, which show that $\boldsymbol{B}^{(3)}$ is accompanied by an imaginary $i\boldsymbol{E}^{(3)}$. The dot product of the complex vector $\left(\boldsymbol{B}^{(3)}/\mu_0^{1/2} + i\epsilon_0^{1/2}\boldsymbol{E}^{(3)}\right)$ with itself is zero, and we define this dot product as the (zero) contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to electromagnetic energy density in vacuo.



It is clear that if $\boldsymbol{B}^{(3)}$ is a magnetic field, it is uniform and independent of frequency, and as such is a legitimate solution of Maxwell's equations. The imaginary $i\boldsymbol{E}^{(3)}$ is similarly a solution of Maxwell's equations, but having no real part, is not conventionally expected to produce physical effects such as electric polarization. Therefore it is not possible to assert that $\boldsymbol{B}^{(3)}$ is not a magnetic field because it is not a solution of Maxwell's equations. It has been shown recently [27] that $\boldsymbol{B}^{(3)}$ is also a zero frequency solution of Proca's equation for electromagnetism, first suggested in 1930, and extensively developed by de Broglie and his School [28]. The $\boldsymbol{B}^{(3)}$ from Proca's equation is for all practical purposes the same as the $\boldsymbol{B}^{(3)}$ from the d'Alembert equation, and therefore from Maxwell's equations. This reinforces the expectation that $\boldsymbol{B}^{(3)}$ is a physical magnetic field. Longitudinal solutions of the Proca equation were first proposed by de Broglie and Schrödinger [28], but are tiny at finite frequencies because they are multiplied by a factor $(m_0/\overline{\nu})^2$ where $m_0$ is the mass and where $\overline{\nu}$ is the frequency of the longitudinal photon. The Maxwellian $\boldsymbol{B}^{(3)}$ is recovered as the mass and frequency both go to zero, so that this factor approaches unity. It is clear therefore that $\boldsymbol{B}^{(3)}$ in this limit is not a minute, unobservable field.

Finally, there is no reason to assert that $\boldsymbol{B}^{(3)}$ cannot exist on the grounds of symmetry, as in a recent suggestion [29], because it is defined according to the Lie algebra (7), an algebra which conserves $\hat{C}$, $\hat{P}$, and $\hat{T}$ [30]. In quantum field theory $\boldsymbol{B}^{(3)}$ becomes the operator

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{26}$$

where $\hat{J}$ is photon angular momentum. This equation also conserves $\hat{C}$, $\hat{P}$, and $\hat{T}$, and in particular, $\hat{B}^{(3)}$ is generated from quantized photon angular momentum through the intermediacy of the scalar magnitude $B^{(0)}$ in tesla, a $\hat{C}$ negative quantity.

In conclusion there is no conventional reason to assert that $\boldsymbol{B}^{(3)}$ is not a magnetic field, and available data, for example from the inverse Faraday effect, are consistent with the existence of $\boldsymbol{B}^{(3)}$. The antisymmetric tensor $I_{ij}^{A}$ and the Stokes parameter $S_3$ can both be expressed in terms of $\boldsymbol{B}^{(3)}$, and this shows that $\boldsymbol{B}^{(3)}$ is non-zero. Unfortunately there are no data available to show unequivocally whether or not $\boldsymbol{B}^{(3)}$ can act as a magnetic field, for example in an optical Bohm-Aharonov effect. Such an experiment would be of central importance therefore in any further investigation of $\boldsymbol{B}^{(3)}$. This paper has shown that $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are consistent with electromagnetic energy density considerations.

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# Chapter 4

**THE ONE ELECTRON INVERSE FARADAY EFFECT: THE ROLE OF THE LONGITUDINAL MAGNETIC FIELD $B^{(3)}$**

M. W. Evans

**Abstract**

It is shown that the effect of a circularly polarized laser field on an electron can be described in terms of a latent or equivalent magnetic field $\boldsymbol{B}^{(3)}$, defined through the vector product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ of the rotating electric field $\boldsymbol{E}^{(1)}$ and its complex conjugate $\boldsymbol{E}^{(2)}$. The study of the intensity dependence of the magnetization in a one electron plasma reveals whether the field $\boldsymbol{B}^{(3)}$ has an intrinsic meaning in free space, or whether it becomes effective only when the laser beam interacts with matter. In the former case, there is a square root intensity dependence present in addition to the intensity dependence normally expected through the inverse Faraday effect. In both cases, the field $\boldsymbol{B}^{(3)}$ is the fundamental property responsible for magnetization by light, and is defined through the product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$.

## 1. Introduction

It is well known that the inverse Faraday effect is magnetization by circularly polarized light [1–3] and was first observed by van der Ziel *et al.* [2, 3] in molecular liquids and doped glasses. The effect was first observed in plasma by Deschamps *et al.* [4]. Its influence on ferromagnetism has been observed by Sanford *et al.* [5] and on conductivity in phthalocyanines by Barrett *et al.* [6]. It is well established theoretically [7–12]. Recently [12], it has been expressed in terms of the vector product of the rotating electric field $\boldsymbol{E}^{(1)}$ of the circularly polarized light beam with its complex conjugate $\boldsymbol{E}^{(2)}$. This is an axial vector which can be expressed as the product $ic^2B^{(0)}\boldsymbol{B}^{(3)}$, where $B^{(0)}$ is the scalar magnetic flux density amplitude of the light beam and where $\boldsymbol{B}^{(3)}$ is a quantity which has the units and symmetry of a phase independent magnetic field [13–17]. The interaction of the light beam with a single electron has been investigated theoretically by Talin *et al.* [18], who considered the angular momentum of an electron driven in a circular orbit by the rotating electric field of the light beam. From relativistic electrodynamics this angular momentum is proportional to the square of the amplitude $E^{(0)}$ of the electric field strength of the laser in volt $m^{-1}$.

In this paper it is shown that the results of Talin *et al.* [18] can be expressed in terms of the magnetic field $\boldsymbol{B}^{(3)}$. From first principles, it is clear that the effect of a rotating electric field, driving an electron around in a circular orbit, is entirely equivalent to that of a magnetic field. If the



sense of the electron's motion were not known, it would not be possible to know whether it was a rotating electric field or a magnetic field. This statement is equivalent to the relation,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = ic^2B^{(0)}\boldsymbol{B}^{(3)} \tag{1}$$

between the conjugate product and $\boldsymbol{B}^{(3)}$. The question addressed in this communication is that of the dependence of the magnetization on laser intensity. This addresses the fundamental question of whether the conjugate product generates the magnetic field $\boldsymbol{B}^{(3)}$ in free space, so that the photon may be thought of as carrying an intrinsic elementary unit of magnetic flux density. If so, it is shown from first principles in Sec. 2 that the magnetization in the one electron inverse Faraday effect (IFE) would be a sum of two terms, one proportional to the square root of the light intensity (watts per square meter) and one proportional to the light intensity, as described by Talin *et al.* [18]. Section 3 makes a careful distinction between the physical magnetic field generated in free space by $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$, and the effective magnetic field through which the IFE is usually described. It is shown that the results of Talin et al [18] can be expressed in terms of an effective magnetic field, through which the magnetization is generated via a susceptibility. The physical magnetic field (if it exists) is proportional to the square root of light intensity and the effective magnetic field to the light intensity itself.

## 2. Motion of An Electron in a Rotating Electric Field

From fundamental relativistic electrodynamics, the rotating electric field of a circularly polarized light beam drives an electron in a circle with radius [18],

$$r = \frac{ecB^{(0)}}{m_0\omega^2\gamma}, \tag{2}$$

where $e$ is the charge on the electron, $c$ the speed of light in vacuo, $B^{(0)}$ the magnetic flux density amplitude of the light beam, $m_0$ the electron rest mass, $\omega$ the angular frequency of the light, and $\gamma$ is a relativistic factor,

$$\gamma^2 = 1 + \left(\frac{eE^{(0)}}{m_0c\omega}\right)^2. \tag{3}$$

At visible frequencies and for an intensity of say 1.0 watt per meter squared the factor $\gamma$ is unity to an excellent approximation, and will henceforth be taken as such. S.I. (mks) units have been used in these equations. The transverse momentum of the electron is given from these first principles as

$$p_t = \frac{ecB^{(0)}}{\omega}, \tag{4}$$

so that the angular momentum of the electron is

$$J_z = rp_t = \frac{e^2c^2}{m_0\omega^3}B^{(0)2}, \tag{5}$$

and is proportional to $B^{(0)2}$. Finally the induced magnetic dipole moment is the gyromagnetic ratio multiplied by the angular momentum,

$$m_z = -\frac{e}{2m_0}J_z = -\frac{e^3c^2}{2m_0^2\omega^3}B^{(0)}|\mathbf{B}^{(3)}|. \tag{6}$$

The magnetization is proportional to the square of $B^{(0)}$ through a one electron hyperpolarizability and is therefore proportional to the laser intensity. This is the calculation of the IFE from first principles [18].

It is shown in this section that the action of the rotating electric field is entirely equivalent to that of a magnetic field, leading to a first order inverse Faraday effect. In the magnetic field $\mathbf{B_s}$, the radius of the electron's circular orbit is, from first principles [19],

$$r_0 = \frac{v_{ot}}{\Omega}, \tag{7}$$

where $v_{ot}$ is the initial transverse velocity defined by Landau and Lifshitz [19] and $\Omega$ is the frequency defined by the Lorentz equation,

$$\Omega = \frac{e}{m_0}|\mathbf{B_s}|. \tag{8}$$

This angular frequency is the ratio $e/m_0$ multiplied by the magnetic field $B_S$. The transverse momentum of the electron is [19]

$$p_t = er_0|\mathbf{B_s}| \tag{9}$$

and the angular momentum of the electron in $\mathbf{B_s}$ is

$$L_z = er_0^2|\mathbf{B_s}| \tag{10}$$



The angular momentum $L_z$ is therefore $er_0^2$ multiplied by the magnetic field $\mathbf{B_s}$. The magnetic dipole moment induced by $\mathbf{B_s}$ is therefore

$$m_z = -\left(\frac{e^2r_0^2}{2m_0}\right)|\mathbf{B_s}|. \tag{11}$$

where the quantity in brackets is the one electron susceptibility.

The result (6) for the one electron Faraday effect is the traditional theory [18], expressed in terms of the product $B^{(0)}\mathbf{B}^{(3)}$ through Eq. (1). It is second order in the magnetic flux density amplitude of the beam ($B^{(0)}$). However, because $\mathbf{B}^{(3)}$ is non-zero by Eq. (1), and has the units and symmetry of magnetic flux density there is an additional *first order inverse Faraday effect*, given by Eq. (11) with $\mathbf{B_s}$ identified as $\mathbf{B}^{(3)}$,

$$m_Z^{(1)} = -\left(\frac{e^2r_0^2}{2m_0}\right)|\mathbf{B}^{(3)}|. \tag{12}$$

The magnetic dipole moment induced by the beam is therefore the sum of first and second order effects,

$$\mathbf{m} = -\left(\frac{e^2r_0^2}{2m_0}\right)\mathbf{B}^{(3)} - \left(\frac{e^3c^2}{2m_0^2\omega^3}\right)B^{(0)}\mathbf{B}^{(3)}. \tag{13}$$

For $\omega$ about $10^{15}$ rad. $sec^{-1}$; and for a first order electron radius of about 10 Å ($10^{-9}$ m) this is, roughly,

$$\mathbf{m} \sim -10^{-26}\mathbf{B}^{(3)} - 10^{-25}B^{(0)}\mathbf{B}^{(3)}. \tag{14}$$

For a beam intensity of about $10^{14}$ watts $m^{-2}$; $B^{(0)}$ is about one tesla, and the second order effect is ten times bigger than the first order effect. It would appear in this case that there were only a second order effect present, but under different conditions, the first order effect should become visible superimposed on the second order induced dipole moment. The latter is proportional to the square root of light intensity. Therefore the complete effect is a sum of (6) and (12), which is the conclusion of this section.

## 3. The "Effective" and Physical Magnetic Fields

From Eq. (6) it is seen that the second order effect can expressed through an "effective" magnetic field $\mathbf{B_{eff}}$ defined by,

$$\boldsymbol{B}_{eff} \equiv \left( \frac{eB^{(0)}}{\omega m_0} \right) \boldsymbol{B}^{(3)} . \tag{15}$$

This *effective* magnetic field is what is usually referred to in the literature on the inverse Faraday effect, and was first mentioned in refs. (1) to (3). The *physical* magnetic field $\boldsymbol{B}^{(3)}$ is an elementary field carried by the electromagnetic wave. It is clear that the inverse Faraday effect for one electron depends on a non-zero $\boldsymbol{B}^{(3)}$, however *both at first and second orders*. In linear polarization, the net $\boldsymbol{B}^{(3)}$ is zero, because there is 50% of $\boldsymbol{B}^{(3)}$ and 50% of $-\boldsymbol{B}^{(3)}$ present in the beam. Therefore no inverse Faraday effect occurs in linear polarization. From Eq. (1) it is seen that $\boldsymbol{B}^{(3)}$ is the elementary unit of the conjugate product, and therefore of the antisymmetric part of light intensity [20] in free space. As always, radiation becomes manifest only when it interacts with matter, and similarly for $\boldsymbol{B}^{(3)}$. From Eq. (15), if $\boldsymbol{B}^{(3)}$ did not exist, then $\boldsymbol{B}_{eff}$ would also vanish, so the effective magnetic field depends on $\boldsymbol{B}^{(3)}$. In setting up the effect of Eq. (12), we have simply assumed that the physical magnetic field causes magnetization at first order through a one electron susceptibility. This appears to be reasonable, but since $\boldsymbol{B}^{(3)}$ is a property of light, it is clearly not an ordinary magnetostatic field. The hypothesis (12) does not contradict laws of conservation of energy and momentum because $\boldsymbol{B}^{(3)}$, being a magnetic field, does not contribute to the Poynting theorem through the usual term $\boldsymbol{E}\cdot\boldsymbol{J}$ [21]. We have simply assumed that there is a first order inverse Faraday effect corresponding to the usual second order effect described by Eq. (6). This hypothesis, must be tested experimentally, but it is clear without further data that the fundamental entity responsible for magnetization by light is the elementary magnetic field $\boldsymbol{B}^{(3)}$ of the photon. Without this, the effective field $\boldsymbol{B}_{eff}$ would be zero, and so would magnetization by circularly polarized light. It is physically transparent that magnetization by light is due to the magnetic field $\boldsymbol{B}^{(3)}$.

## 4. Discussion

Talin *et al.* [18] have discussed the conservation of energy in the inverse Faraday effect, showing that the circularly polarized laser gives up angular momentum to the electron, which acquires orbital angular momentum as we have seen. When the laser field is switched on, a current is induced because the electron is driven around in a circular orbit. If the electromagnetic field is switched on adiabatically (constant entropy) then the final orbital electronic angular momentum at the end of the switching procedure can be expressed in terms of the work done during the adiabatic switching, which is $\int \boldsymbol{E}\cdot\langle\boldsymbol{j}\rangle dt$, where $\langle\boldsymbol{j}\rangle$ is the induced current. The energy density stored in the medium is therefore $\int \boldsymbol{E}\cdot\langle\boldsymbol{j}\rangle dt$. The induced current $\langle\boldsymbol{j}\rangle$ can be expressed in terms of material



polarization and magnetization,

$$\langle\boldsymbol{j}\rangle = \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}, \tag{16}$$

and so the field $\boldsymbol{B}^{(3)}$ appears in the definition of $\langle\boldsymbol{j}\rangle$ because $\boldsymbol{M}$ is defined in terms of $\boldsymbol{B}^{(3)}$. We see that $\boldsymbol{B}^{(3)}$ does not add anything more to the free space energy density [22],

$$U_F = \frac{1}{2}\left(\epsilon_0 \boldsymbol{E}^{(1)}\cdot\boldsymbol{E}^{(1)} + \frac{1}{\mu_0}\boldsymbol{B}^{(1)}\cdot\boldsymbol{B}^{(1)}\right), \tag{17}$$

and for this reason does not upset the Planck radiation law and conservation of electromagnetic energy density in free space. It is also clear that $\boldsymbol{M}$ can be made up of terms to any power in $\boldsymbol{B}^{(3)}$ without affecting the law of conservation of energy in any way. $\boldsymbol{B}^{(3)}$ simply results in a redistribution of the energy density stored in the medium. When the latter is one electron, $\boldsymbol{B}^{(3)}$ provides the electron with orbital angular momentum and rotational kinetic energy, which is accounted for in Poynting's theorem by the term $\int \boldsymbol{E}\cdot\langle\boldsymbol{j}\rangle dt$. The latter is present only when there is interaction of an electromagnetic beam with matter.

In this sense, therefore, $\boldsymbol{B}^{(3)}$ is a stored magnetic field carried by the electromagnetic wave, a field which is switched on when the wave interacts with matter, in the simplest case one electron. Since $\boldsymbol{B}^{(3)}$ in free space is "latent" or unused then it does not contribute to free space electromagnetic energy density. The latter exists in the absence of matter, and defines the beam intensity, to which $\boldsymbol{B}^{(3)}$ does not contribute.

The additional effects expected in the inverse Faraday effect to first order in $\boldsymbol{B}^{(3)}$ have been treated in Ref. [23], and although the inverse Faraday effect has been observed on several occasions [2–6] there is still only one study available of its intensity dependence, the original series of experiments by van der Ziel *et al.* [2, 3].

### 4.1. Demonstration of Conservation of Energy in the First Order Interaction

It is well known [20] that light intensity is tensor quantity, with symmetric and antisymmetric components. If $\epsilon_0$ is the free space permittivity, the light intensity tensor is

$$I_{ij} = \epsilon_0 c E_i E_j = \frac{1}{2}\epsilon_0 c\left(E_i E_j^{(S)} + E_i E_j^{(A)}\right). \tag{18}$$

In vector notation, the antisymmetric component $E_i E_j^{(A)}$ is expressed through the vector product on the right hand side of Eq. (1), and is therefore directly proportional to $\boldsymbol{B}^{(3)}$. The total magnitude of the light intensity is made up of 50% symmetric and 50% antisymmetric contributions,

$$I = \frac{\epsilon_0 c}{2}(\boldsymbol{E}^{(1)} \cdot \boldsymbol{E}^{(2)} - i|\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}|) = \frac{\epsilon_0 c}{2}(|S_0| + |S_3|) = \epsilon_0 c E^{(0)2}, \tag{19}$$

where $S_0$ and $S_3$ are the zero'ith and third Stokes parameters, and it is this total magnitude that appears in the usual [21] expression for electromagnetic energy flux density in free space. The vector $\boldsymbol{B}^{(3)}$ is, in this sense, *already included* in $U_F$. Quite simply, therefore, $\boldsymbol{B}^{(3)}$ is the fundamental vector quantity that defines the antisymmetric part of light intensity in free space. Since $\boldsymbol{B}^{(3)}$ is already included in $U_F$ and in Poynting's theorem, it is incorrect to assert that $\boldsymbol{B}^{(3)}$ would add anything more to electromagnetic energy density. For the same reason, $\boldsymbol{B}^{(3)}$ is already included in Planck's radiation law. Also, if $\boldsymbol{B}^{(3)}$ were zero, as recently asserted [24], the antisymmetric part of light intensity would vanish, an incorrect conclusion.

For a circularly polarized laser interacting with one electron, the induced rotational kinetic energy can be expressed as

$$En^{(ind)} = \frac{1}{2} I^{(ind)} \omega^2 = -m_Z^{(ind)} B_Z^{(3)}, \tag{20}$$

where $I^{(ind)}$ is the beam induced moment of inertia of the electron,

$$I^{(ind)} = m_0 r_0^2. \tag{21}$$

This beam induced rotational electronic kinetic energy is proportional to $B_Z^{(3)2}$,

$$En^{(ind)} = -\left(\frac{e^2 r_0^2}{2m_0}\right) B_Z^{(3)2}. \tag{22}$$

The factor $B_Z^{(3)2}$ can be expressed as

$$B_Z^{(3)2} = |-i\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}| = |B^{(0)}\boldsymbol{B}^{(3)}|, \tag{23}$$

and contributes, as we have argued, to the antisymmetric part of light intensity. In a circularly polarized laser beam this is of course non-zero, and acts upon the electron through the fundamental field $\boldsymbol{B}^{(3)}$.

Therefore the antisymmetric contribution to the electromagnetic energy density in free space is transferred into electronic rotational kinetic energy through $\boldsymbol{B}^{(3)}$. There is conservation of energy.



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# Chapter 5

## LONGITUDINAL SOLUTIONS OF MAXWELL'S EQUATIONS IN THE LORENTZ GAUGE IN FREE SPACE

M. W. Evans and F. Farahi

**Abstract**

It is shown that there exist real magnetic and imaginary electric longitudinal components of electromagnetism in free space. These phase independent electric and magnetic fields are shown to be rigorously consistent in the Lorentz gauge with the concepts of relativistic quantum field theory, in which admixtures of the time-like and longitudinal space-like photon polarizations may be physically meaningful.

### 1. Introduction

In the contemporary theory [1] of massless gauge fields relativistic quantization of the electromagnetic field in the Lorentz gauge leads to the result that the photon has one time-like and three space-like polarizations, a total of four polarizations in free space. On the other hand, well known [1] considerations of the Poincaré group in the massless limit lead to the conclusion that the photon can have only two helicities, +1 and -1; conventionally identified with the two transverse space-like polarizations (1) and (2) and known as the left and right circularly polarized components of the electromagnetic field. What, therefore, is the physical significance of the time-like polarization (0) and the longitudinal space-like polarization (3) of the photon, and what do these mean in the classical theory of electromagnetic fields?

In this letter, it is shown that the photon polarizations (0) and (3) can be interpreted classically in terms of imaginary electric and real magnetic longitudinal components of the electromagnetic plane wave in free space, defined in the circular basis by

$$i\boldsymbol{E}^{(3)} = iE^{(0)}\boldsymbol{\hat{e}}^{(3)}, \quad \boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{\hat{e}}^{(3)}, \tag{1}$$

where $\boldsymbol{\hat{e}}^{(3)}$ is a unit vector in the circular basis in the propagation axis of the wave. Here $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are longitudinal magnetic and electric components respectively and $B^{(0)}$ and $E^{(0)}$ are time-like scalar amplitudes. In this interpretation, valid in the Lorentz gauge, manifest covariance [1] is retained, and all four polarizations (0), (1), (2), and (3) are physically meaningful. The two helicities of the photon are related to the polarization states (0) and (3) through the equation [2, 3],

$$\boldsymbol{B}_1^{(3)} = \langle\psi|\hat{\boldsymbol{B}}_1^{(3)}|\psi\rangle = \frac{B_1^{(0)}}{\hbar}\langle\psi|\hat{\mathcal{J}}_1|\psi\rangle, \tag{2}$$



Here $|\psi\rangle$ is an eigenstate of the photon field, and $\boldsymbol{B}_1^{(3)}$ has become the expectation value of the single photon operator $\hat{\boldsymbol{B}}_1^{(3)}$. The quantity $B_1^{(0)}$ is a scalar magnetic flux density amplitude for one photon, and $\hat{\mathcal{J}}_1$ is the angular momentum operator of one photon. The eigenvalues [4] of $\hat{\mathcal{J}}_1$ are +1 and -1, corresponding with the two photon helicities. Therefore it is possible to relate the two allowed photon helicities to the (0) and (3) polarizations equally as well as to the (1) and (2) polarizations. This removes obscurities [1] of conventional interpretation, in which the (0) and (3) polarizations are "non-physical". The conventional interpretation however *allows admixtures of the (0) and (3) polarizations to be physically meaningful*, but the (0) and (3) states taken separately have no physical significance. This is the result of the need to reconcile four polarizations to two helicities.

### 2. Background in Relativistic Quantum Field Theory

In special relativity there exists in the Lorentz gauge the well known potential four-vector [5],

$$A_\mu = (\boldsymbol{A}, i\phi), \tag{3}$$

whose vector part $\boldsymbol{A}$ has three space-like components and whose scalar part has one time-like component. In this gauge,

$$\phi = |\boldsymbol{A}|, \tag{4}$$

because the photon travels at the speed of light in free space. Equation (4) is a direct consequence of Eq. (3) for the photon in free space. In the Coulomb or radiation gauge $\phi = 0$ and $\boldsymbol{A} \neq \boldsymbol{0}$ in free space, so the Coulomb gauge in a rigorous interpretation is inconsistent with special relativity. It may be argued that $\phi$ and $\boldsymbol{A}$ are not observable quantities, but it is well known through the Bohm-Aharonov effect that in the quantum field these quantities too have a physical effect. If a four-vector $A_\mu$ is defined as such, then Eq. (4) is a logical consequence for the photon in free space.

In the Lorentz gauge, the Maxwell equations in free space reduce to the well known d'Alembert equation [1],

$$\Box A_\mu = 0, \tag{5}$$

which in the Lorentz gauge quantization of the quantum field theory, becomes the well known [1] Gupta-Bleuler condition,

$$\partial_\mu \hat{A}^{(+)\mu}|\psi\rangle = 0, \tag{6}$$

where $\hat{A}^{(+)}$ is an operator. The condition (6) leads directly to the result [1],

$$(\hat{a}^{(0)} - \hat{a}^{(3)})|\psi\rangle = 0, \tag{7}$$

where $\hat{a}^{(0)}$ and $\hat{a}^{(3)}$ are photon annihilation operators corresponding to polarizations (0) and (3). Equation (7) leads to [1],

$$\langle\psi|\hat{a}^{(0)+}\hat{a}^{(0)}|\psi\rangle = \langle\psi|\hat{a}^{(3)+}\hat{a}^{(3)}|\psi\rangle, \tag{8}$$

where $\hat{a}^{(0)+}$ and $\hat{a}^{(3)+}$ are photon creation operators. In the conventional interpretation [1] Eqs. (7) and (8) are physically meaningful admixtures in the manifestly covariant Lorentz gauge. Furthermore, the Hamiltonian operator in the quantized field is proportional to an integral [1] over the sum,

$$\sum_{\lambda=1}^{3} (\hat{a}^{(\lambda)+}\hat{a}^{(\lambda)} - \hat{a}^{(0)+}\hat{a}^{(0)}), \tag{9}$$

so that the contributions of the (3) and (0) photon states cancel, leaving only those from the (1) and (2) states. In other words, the (0) and (3) states do not contribute to the electromagnetic energy density.

## 3. Classical Interpretation

In this section it is shown that the results (7) to (9) imply the classical Eq. (1), which means that there exist physically meaningful longitudinal magnetic and electric field components of the electromagnetic wave in the classical theory. The starting point of the demonstration is the definition of the oscillating electric and magnetic field components associated with the usual transverse polarizations (1) and (2),

$$\boldsymbol{E}^{(1)} = \frac{E^{(0)}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi}}{\sqrt{2}}, \qquad \boldsymbol{E}^{(2)} = \frac{E^{(0)}(\boldsymbol{i} + i\boldsymbol{j})e^{-i\phi}}{\sqrt{2}}, \tag{10a}$$

$$\boldsymbol{B}^{(1)} = \frac{B^{(0)}(\boldsymbol{j} + i\boldsymbol{i})e^{i\phi}}{\sqrt{2}}, \qquad \boldsymbol{B}^{(2)} = \frac{B^{(0)}(\boldsymbol{j} - i\boldsymbol{i})e^{-i\phi}}{\sqrt{2}}, \tag{10b}$$

where $\phi = \omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}$ is the phase (not to be confused with the scalar potential). Here $E^{(0)}$ and $B^{(0)}$ are defined in Eq. (1); $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors mutually orthogonal to the propagation axis $Z$ of the electromagnetic plane wave in free space. The angular frequency of the plane wave is given by $\omega$ at the instant $t$ and its wave vector by $\boldsymbol{\kappa}$ at the point $\boldsymbol{r}$ in free space. Note that $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$



are complex conjugates, as are $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$.

Having defined the fields associated with the transverse polarizations (1) and (2) our task is to find the fields associated with the polarizations (0) and (3). The latter is a space-like polarization in the propagation axis. The transverse unit polarization vectors in the circular basis are defined as [7]

$$\hat{\boldsymbol{e}}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \qquad \hat{\boldsymbol{e}}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}), \tag{11}$$

and the longitudinal unit vector in the circular basis is found through the cross product

$$\hat{\boldsymbol{e}}^{(1)} \times \hat{\boldsymbol{e}}^{(2)} = i\hat{\boldsymbol{e}}^{(3)} \equiv i\boldsymbol{k}, \tag{12}$$

where $\hat{\boldsymbol{e}}^{(3)}$ is a unit vector in the propagation axis $Z$. It is clear that the longitudinal unit vector $\hat{\boldsymbol{e}}^{(3)}$ is the natural third component of the set $(\hat{\boldsymbol{e}}^{(1)}, \hat{\boldsymbol{e}}^{(2)}, \hat{\boldsymbol{e}}^{(3)})$ in the circular basis.

It follows that longitudinal polarizations can be generated algebraically through cross products such as,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = iE^{(0)2}\hat{\boldsymbol{e}}^{(3)}, \qquad \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)2}\hat{\boldsymbol{e}}^{(3)}, \tag{13}$$

from which

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = c^2(\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}). \tag{14}$$

It is clear that

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)}, \tag{15}$$

where

$$\boldsymbol{B}^{(3)} \equiv B^{(0)}\hat{\boldsymbol{e}}^{(3)}, \tag{16}$$

can be a real magnetic field because [3] the parity inversion ($\hat{P}$) and motion reversal ($\hat{T}$) symmetries of the right and left hand sides are the same ($\hat{P} = +$, $\hat{T} = -$).

However, if we attempt the definition,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} =?\ iE^{(0)}\boldsymbol{E}^{(3)}, \tag{17}$$

where

$$\boldsymbol{E}^{(3)} \equiv E^{(0)}\hat{\boldsymbol{e}}^{(3)}, \tag{18}$$

then the cross product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ cannot produce the real electric field $\boldsymbol{E}^{(3)}$ because such a field requires negative $\hat{P}$ and positive $\hat{T}$ symmetry [3]. None of the possible cross products between $\boldsymbol{E}^{(1)}$, $\boldsymbol{E}^{(2)}$, $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ produce the necessary fundamental symmetries of a longitudinal electric field of the type (18). However, we shall see later that an electric field of the type $i\boldsymbol{E}^{(3)}$ can be defined through considerations of the electromagnetic energy and momentum density of the classical field, considerations akin to Eq. (9) of the quantum field.

Equations (16) and (18) can be rewritten as

$$E^{(0)} - |\boldsymbol{E}^{(3)}| = 0, \qquad B^{(0)} - |\boldsymbol{B}^{(3)}| = 0, \tag{19}$$

which are the classical counterparts of Eq. (7) that we are seeking. To see this, recall that the classical and quantum fields are related through the field operators

$$\hat{E}^{(0)} = \left(\frac{2\hbar\omega}{\epsilon_0 V}\right)^{\frac{1}{2}}\hat{a}^{(0)}, \qquad \hat{E}^{(3)} = \left(\frac{2\hbar\omega}{\epsilon_0 V}\right)^{\frac{1}{2}}\hat{a}^{(0)},$$
$$\hat{B}^{(0)} = \left(\frac{2\mu_0\hbar\omega}{V}\right)^{\frac{1}{2}}\hat{a}^{(0)}, \qquad \hat{B}^{(3)} = \left(\frac{2\mu_0\hbar\omega}{V}\right)^{\frac{1}{2}}\hat{a}^{(3)}, \tag{20}$$

where $\mu_0\epsilon_0 = 1/c^2$ as usual; and where $V$ is the well known quantization volume [6, 7]. From Eqs. (20),

$$(\hat{E}^{(0)} - \hat{E}^{(3)})|\psi\rangle = (\hat{B}^{(0)} - \hat{B}^{(3)})|\psi\rangle = (\hat{a}^{(0)} - \hat{a}^{(3)})|\psi\rangle = 0. \tag{21}$$

It is therefore clear that the classical results corresponding to the admixture (7) of the quantum theory are the Eqs. (16) and (18). Here $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are related to the longitudinal space-like polarizations and $B^{(0)}$ and $E^{(0)}$ to the time-like component of the admixture in quantum theory. It is easily checked that $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ are solutions of Maxwell's equations in free space, because,

$$\nabla\cdot\boldsymbol{E}^{(3)} = \nabla\cdot\boldsymbol{B}^{(3)} = \frac{\partial\boldsymbol{E}^{(3)}}{\partial t} = \frac{\partial\boldsymbol{B}^{(3)}}{\partial t} = 0, \tag{22}$$

and



$$\nabla\times\boldsymbol{E}^{(3)} = -\frac{\partial\boldsymbol{B}^{(3)}}{\partial t} = 0, \qquad \nabla\times\boldsymbol{B}^{(3)} = \mu_0\epsilon_0\frac{\partial\boldsymbol{E}^{(3)}}{\partial t} = 0. \tag{23}$$

*The key to Eqs. (22) and (23) is that $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ are independent of the phase $\phi$.*

Another manifestation of the fact that $B^{(0)}$ and $\boldsymbol{B}^{(3)}$ are admixtures [1] is the existence [3] of four-vectors in vacuo,

$$B_\mu = (\boldsymbol{B}^{(3)}, iB^{(0)}), \tag{24a}$$

and

$$E_\mu = (\boldsymbol{E}^{(3)}, iE^{(0)}), \tag{24b}$$

in the Minkowski space-time of special relativity, meaning that $B^{(0)}$ and $\boldsymbol{B}^{(3)}$ are components of the same four-vector. In free space, electromagnetic radiation travels at the speed of light, and Eqs. (24) are consequences of Eqs. (19) in pseudo-Euclidean geometry. From Eqs. (19) and (24), it is obvious that

$$E^{(0)2} - \boldsymbol{E}^{(3)}\cdot\boldsymbol{E}^{(3)} = 0, \qquad B^{(0)2} - \boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)} = 0, \tag{25}$$

and this is the classical equivalent of the quantum condition (8), i.e.

$$\langle\psi|\hat{a}^{(0)+}\hat{a}^{(0)} - \hat{a}^{(3)+}\hat{a}^{(3)}|\psi\rangle = 0. \tag{26}$$

From the condition (9) of the quantum field, it becomes clear that the correct combination of $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ cannot contribute to the electromagnetic energy density. The same conclusion was reached by Farahi and Evans [8] from considerations of the continuity equation linking the electromagnetic energy density and momentum density. These considerations led to the results that both $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ are in general complex quantities, (but if $\boldsymbol{B}^{(3)}$ is real and physical, $i\boldsymbol{E}^{(3)}$ is imaginary and unphysical).

$$\boldsymbol{E}^{(3)} = \frac{E^{(0)}}{\sqrt{2}}(i-1)\hat{\boldsymbol{e}}^{(3)}, \qquad \boldsymbol{B}^{(3)} = \frac{B^{(0)}}{\sqrt{2}}(i+1)\hat{\boldsymbol{e}}^{(3)}, \qquad \boldsymbol{E}^{(3)}\times\boldsymbol{B}^{(2)} = \boldsymbol{B}^{(3)}\times\boldsymbol{E}^{(2)}, \tag{27}$$

under which neither $i\boldsymbol{E}^{(3)}$ nor $\boldsymbol{B}^{(3)}$ if correctly used in combination, contribute to the classical electromagnetic energy density. Equation (27) shows that if there is a non-zero $\boldsymbol{B}^{(3)}$ there must also be a non-zero $i\boldsymbol{E}^{(3)}$. This classical result [8] is consistent with the relativistic quantum field equation (9), because from Eqs. (20), annihilation and creation operators can be defined in terms both of magnetic and electric time-like and longitudinal field operators.

In the Coulomb gauge, the scalar part of $A_\mu$ is set to zero, so from the requirement (4) of special relativity, it follows in our rigorous interpretation that $\boldsymbol{A}$ must vanish in free space, meaning that there can be no electromagnetic wave or fields. Customarily, however, $\phi$ is set to zero in the Coulomb gauge, and $\boldsymbol{A}$ is set to non-zero. This loses manifest covariance, and is from our point of view especially unsatisfactory in that the significance of standard equations of the relativistic quantum field becomes clouded. In quantum mechanics, furthermore, the potentials $\boldsymbol{A}$ and $\phi$ have *physical and measurable* effects, recorded in the well known Bohm-Aharonov experiment. The relativistic *quantum* field is the most rigorous contemporary description. In our interpretation, the Coulomb gauge used in free space is inconsistent with special relativity and the covariance of physical laws. However, $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ are by any reasonable definition electric and magnetic fields which are rigorously defined in the quantum and classical theory of the electromagnetic field.

## 4. Discussion

It is possible to construct quantities $\boldsymbol{B}_\phi^{(3)}$ and $\boldsymbol{E}_\phi^{(3)}$ using other cross products, for example, of $\boldsymbol{E}_\phi^{(2)} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} + \boldsymbol{j})e^{i\phi}$ and $\boldsymbol{E}_\phi^{(1)} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi}$, to give,

$$\boldsymbol{E}_\phi^{(3)} = E^{(0)} e^{2i\phi} \hat{\boldsymbol{e}}^{(3)}. \tag{28}$$

However, $\boldsymbol{E}_\phi^{(2)}$ is no longer the complex conjugate of $\boldsymbol{E}_\phi^{(1)}$, and $\boldsymbol{E}_\phi^{(3)}$ does not obey the fundamental equations (7) to (9) of relativistic quantum field theory, nor does it obey the classical equivalents (19) and (21). Furthermore, quantities of the type (28) do not obey Gauss's theorem in free space, i.e.

$$\nabla \cdot \boldsymbol{E}_\phi^{(3)} \neq 0. \tag{29}$$

(The time-like creation or annihilation operator corresponding to the hypothetical electric field (28) cannot by definition depend on space coordinates, and cannot depend therefore on the phase. It follows that the quantized version of Eq. (28) cannot obey Eqs. (7) to (9) of the quantum field theory.)

Therefore quantities such as $\boldsymbol{E}_\phi^{(3)}$, in contradistinction to $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ are *not* reasonably definable as electric fields in free space. The removal of the phase $\phi$ in the cross product (15) and, more indirectly, in Eq. (27), is a critically important element in our analysis. This type of cross product is often referred to in the literature [9] as the "conjugate product" because it is the cross product of a vector with its own complex conjugate vector. Note that such a cross product is possible only if the vector is complex, i.e. if the electromagnetic wave is a travelling wave. In a standing wave, such cross products are zero. In a linearly polarized travelling wave the conjugate product is also zero, because,



$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = -\boldsymbol{E}^{(2)} \times \boldsymbol{E}^{(1)}, \tag{30}$$

by definition, and switching from right to left circular polarization means that $\boldsymbol{E}^{(1)} \rightarrow \boldsymbol{E}^{(2)}$ and $\boldsymbol{E}^{(2)} \rightarrow \boldsymbol{E}^{(1)}$. It follows from Eq. (30) that the sum of $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ and $\boldsymbol{E}^{(2)} \times \boldsymbol{E}^{(1)}$ vanishes, linear polarization being a sum of equal parts of right and left circular polarization. In other words $i\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ vanish by cancellation in linear polarization because conjugate products vanish in linear polarization.

The existence of a physically meaningful, real $\boldsymbol{B}^{(3)}$ in circular or elliptical polarization means that there should be physically meaningful effects, for example magnetization at first and higher orders in this field. Magnetization due to $\boldsymbol{B}^{(3)}$ at second order has already been observed in the well known inverse Faraday effect [10], whose original theory did not however, propose the existence of $\boldsymbol{B}^{(3)}$ explicitly. There should also be magnetization due to $\boldsymbol{B}^{(3)}$ at first order, and it would be interesting to repeat the inverse Faraday effect experiment with this in mind. Recent tentative evidence for magnetization by light has been described through small but real shifts in NMR frequencies induced by a circularly polarized laser, shifts which vanish in linear polarization [11]. Such effects are also present in theory in ESR [12] and in magnetic resonance imaging [13].

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# Chapter 6

## IRREDUCIBLE TENSORIAL REPRESENTATIONS IN R(3) OF THE LONGITUDINAL GHOST FIELDS OF FREE SPACE ELECTROMAGNETISM

M. W. Evans

**Abstract**

Experimental observation of the inverse Faraday effect (phase free magnetization by light) is shown to be proportional to the conjugate product of oscillating magnetic field components $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)}$, where $\boldsymbol{B}^{(3)}$ is a longitudinal magnetic field associated with free space electromagnetism and where $B^{(0)}$ is the scalar amplitude. It is demonstrated rigorously that the field $\boldsymbol{B}^{(3)}$ can be expanded in terms of irreducible compound tensors, the vector spherical harmonics, which are eigenvectors of the rotation operator for a vector field. Such an analysis implies that the neglect of $\boldsymbol{B}^{(3)}$ in standard electrodynamics is equivalent to a violation of fundamental group theory. The inclusion of $\boldsymbol{B}^{(3)}$ implies that in the quantum field, the photon becomes a rigorously well defined boson, with three eigenvalues, as for any boson of unit spin angular momentum. The field $\boldsymbol{B}^{(3)}$ is linked geometrically to the transverse wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ in free space, but is phase free and has no Planck energy. It is therefore a ghost field of electromagnetism which cannot be absorbed or re-emitted in field-matter interactions.

### 1. Introduction

It has been shown recently [1, 2] that the experimentally observed [3–10] inverse Faraday effect is directly proportional to the longitudinal ghost field, $\boldsymbol{B}^{(3)}$, of free space electromagnetism. At second order,

$$\boldsymbol{M}(0) = AB^{(0)}\boldsymbol{B}^{(3)}, \tag{1}$$

where $\boldsymbol{M}(0)$ is the real, phase free ("static" [1, 2]) magnetization due to a circularly polarized light beam, $A$ is the ensemble averaged magnitude of a molecular or atomic property tensor, and $B^{(0)}$ is the scalar density of magnetic flux in the light beam. The ghost field [11–14], $\boldsymbol{B}^{(3)}$, is the expectation value of the longitudinal photomagneton, $\hat{B}^{(3)}$, which is directly proportional to the photon angular momentum operator, $\hat{J}$, in free space,

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{2}$$

where $\hbar$ is the reduced Planck constant. From Eq. (2) in Eq. (1),

$$\boldsymbol{M}(0) = \langle \hat{M}(0) \rangle = AB^{(0)2} \frac{\langle \hat{J} \rangle}{\hbar}, \tag{3}$$

showing that the magnetization at second order in $B^{(0)}$ (first order in the light intensity) is directly proportional to photon angular momentum.

This novel analysis of a well known, experimentally verified, phenomenon [3–10], magnetization by light, demonstrates that there is a need to improve the standard approach to electromagnetism to account for the existence of the ghost field $\boldsymbol{B}^{(3)}$. In this paper, it is shown that $\boldsymbol{B}^{(3)}$ can be represented by a combination of irreducible spherical tensors of the full rotation group, R(3), i.e. irreducible compound tensorial representations, the vector spherical harmonics [15]. Therefore, any attempt to assert [16] that $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$ violates the symmetry of R(3), i.e. violates fundamental group theory. A rigorous demonstration of this result is necessary because the standard approach to free space electromagnetism [17–19] uses only the transverse, oscillating, magnetic, wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. These are complex conjugates in a circular basis defined by the unit vectors,

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \quad \boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}), \quad \boldsymbol{e}^{(3)} = \boldsymbol{k}, \tag{4}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are Cartesian unit vectors in Euclidean space. In this basis the transverse fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are linked geometrically to $\boldsymbol{B}^{(3)}$ through a cyclically symmetric Lie algebra [11–14],

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \tag{5}$$

and cyclic permutations of (1), (2), and (3). Here * denotes "complex conjugate", and,

$$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*} = iB^{(0)}\boldsymbol{e}^{(1)}e^{-i\phi}, \quad \boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)}, \tag{6}$$

where $\phi = \omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}$ is the phase of the plane wave in free space; $\omega$ being the angular frequency of the beam at an instant $t$, and $\boldsymbol{\kappa}$ its wave vector at position $\boldsymbol{r}$. From Eq. (5), with $\boldsymbol{B}^{(3)} = \boldsymbol{B}^{(3)*}$, it is seen that Eq. (1) can be written as [1, 2],

$$\boldsymbol{M}(0) = -iA\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}, \tag{7}$$

where $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is referred to as the "conjugate product" in the literature on nonlinear optics [20]. The experimental observation of $\boldsymbol{M}(0)$ [3–10] therefore



implies that of $\boldsymbol{B}^{(3)}$, the ghost field, so-called because it is phase free, so that $\hat{B}^{(3)}$, being directly proportional to $\hat{J}$, *has no Planck energy*. In consequence, $\hat{B}^{(3)}$ takes no part in energy transfer from field to matter, i.e. is not absorbed or re-emitted. Its interaction with matter is "purely elastic", *and involves the transfer only of photon angular momentum as described by Eq.* (3). The field $\boldsymbol{B}^{(3)}$ is therefore very much more difficult to detect experimentally than the waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, which carry Planck energy proportional to their angular frequency $\omega$. It can be shown [11–13] that $\boldsymbol{B}^{(3)}$ adds nothing to light intensity in vacuo, and therefore does not affect the Rayleigh-Jeans law, the classical limit of the Planck radiation law [20]. In the quantum theory, therefore, the illusion is created that the photon can be described by two transverse space polarizations only. This long-standing difficulty in the quantum theory of light is neatly removed by the realization that $\boldsymbol{B}^{(3)}$ is linked to $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ by Eq. (5), and is phase free. For this reason,

$$\boldsymbol{B}^{(3)} \neq \boldsymbol{0}, \quad \nabla\cdot\boldsymbol{B}^{(3)} = 0, \tag{8}$$

as required by one of the Maxwell equations in free space (and in matter). Considerations of fundamental symmetry [11–14] and special relativity show that $\boldsymbol{B}^{(3)}$ is accompanied by an imaginary, longitudinal, electric field, $i\boldsymbol{E}^{(3)}$, which produces no physical electric polarization in field-matter interaction. The contribution of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ to light intensity is zero, because their contribution [20] to the Poynting vector is zero ($\boldsymbol{B}^{(3)}$ being parallel to $i\boldsymbol{E}^{(3)}$),

$$\boldsymbol{N}^{(3)} = \frac{1}{\mu_0}\boldsymbol{B}^{(3)} \times i\boldsymbol{E}^{(3)} = \boldsymbol{0}. \tag{9}$$

The longitudinal flux density of electromagnetic energy, $\boldsymbol{N}^{(3)}$, in free space is zero, and the associated, zero light intensity is simply the time average of $\boldsymbol{N}^{(3)}$. Formally, we also obtain [21],

$$U^{(3)} = \frac{1}{\mu_0}\boldsymbol{B}^{(3)}\cdot\boldsymbol{B}^{(3)} + \epsilon_0 i\boldsymbol{E}^{(3)}\cdot i\boldsymbol{E}^{(3)} = 0, \tag{10}$$

where $U^{(3)}$ is the longitudinal free space electromagnetic energy density (watts $m^{-3}$).

Equations (8) and (9) reveal that in the quantum theory, Planck's law can be derived, as first shown by Bose [22], with the use of only two, transverse, polarizations. This corresponds in the classical theory to the usual assertion that plane *wave* solutions of Maxwell's equations are transverse. The recently discovered [11] Eqs. (5), however, show that the usual transverse *wave* solutions, $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, are linked to the *phase independent* ghost field $\boldsymbol{B}^{(3)}$. From Eq. (1), the assertion $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$ means that there is no inverse Faraday effect to any order in $B^{(0)}$, contradicting experience [3–10]. Furthermore, the existence of $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ is consistent with special relativity; the first paper on which by Einstein [23] shows that there is a four-tensor $F_{\mu\nu}$ in Minkowski space-time which defines the Maxwell equations. For example,

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu} = 0, \tag{11}$$

is a concise way of writing two of the Maxwell equations in free space-time. Here $x_\mu = (X, Y, Z, ict)$ as usual. In S.I. units, if $\epsilon_0$ is the permittivity in vacuo,

$$F_{\mu\nu} = \epsilon_0 \begin{bmatrix} 0 & cB_Z & -cB_Y & -iE_X \\ -cB_Z & 0 & cB_X & -iE_Y \\ cB_Y & -cB_X & 0 & -iE_Z \\ iE_X & iE_Y & iE_Z & 0 \end{bmatrix}, \tag{12}$$

in a Cartesian basis. Here, the longitudinal $\boldsymbol{B}_z = \boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}_z \equiv i\boldsymbol{E}^{(3)}$. From Eqs. (10) and (11), it is clear that the Maxwell equations in free space-time are satisfied in general with a *non-zero longitudinal* $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$. This was realized in 1905 by Einstein to be compatible with Planck's radiation law of 1900 [24], and classical counterparts, the Rayleigh-Jeans and Wien laws. It is also apparent that if $\boldsymbol{B}^{(3)}$ is pure real, then $i\boldsymbol{E}^{(3)}$ is pure imaginary, a result which is consistent with the fact that in special relativity [25], the square of the complex vector $c\boldsymbol{B}^{(3)} + i\boldsymbol{E}^{(3)}$ is a Lorentz invariant.

Remarkably, the existence of the Lie algebraic [11–13] Eq. (5) appears to have gone unrealized until 1992 [11], but it is clear that Eq. (5) is consistent with the 1905 equation (10). We arrive at the conclusion that the photon, like any particle, has *three* space-like dimensions, or polarizations, in Euclidean space, and that classically, transverse solutions of Maxwell's equations in vacuo imply the existence of longitudinal solutions which are phase free through Eq. (5) in free space (more properly free space-time). One of them, $\boldsymbol{B}^{(3)}$, is real and physical, the other, $i\boldsymbol{E}^{(3)}$, is imaginary and unphysical. (We follow the rule that pure real fields are physical, pure imaginary ones are unphysical.)

This paper deals, essentially, with the geometrical foundations of $\boldsymbol{B}^{(3)}$, i.e. with the self-evident tri-dimensionality of Euclidean space. From Eq. (6b), it is clear that $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$ implies $\boldsymbol{e}^{(3)} =? \boldsymbol{0}$, and unnaturally reduces space to a plane. It is necessary to mention this wholly obvious deduction because in electrodynamics, it has become habitual to consider [26] only the wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. However, the existence of the ghost field $\boldsymbol{B}^{(3)}$ means that it becomes necessary to investigate its symmetry representation in R(3), meaning that the irreducible representations of $\boldsymbol{e}^{(3)}$ itself must be developed specifically. The important aim of such a theory is to demonstrate *rigorously* that the



neglect of $\boldsymbol{B}^{(3)}$ (e.g. $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$) in electrodynamics violates the *fundamentals* of group theory itself.

In Sec. 2, the real, physical, $\hat{B}^{(3)}$ is expressed in terms of the rotation generator, $\hat{J}^{(3)}$, of three dimensional space, a pure imaginary operator [27]. This procedure clarifies that the existence of $\hat{B}^{(3)}$ is compatible with the fact that in the quantum theory, the eigenvalues of photon spin angular momentum (intrinsic spin) become $-\hbar$, 0, $\hbar$. *The photon becomes a rigorously well defined boson.* This differs from the conventional approach, in which there are only two eigenvalues, namely $-\hbar$ and $\hbar$. An important property of the field $\hat{B}^{(3)}$ is that it also exists, after quantization, for the zero rest mass photon. More generally, the zero rest mass photon is the $m_0 = 0$ limit of the theory of finite photon rest mass, $m_0$ [28], in which the d'Alembert equation is replaced by the Proca equation. The latter gives [11–14] the *longitudinal* magnetic field in the photon rest frame,

$$\boldsymbol{B}^{(3)} = B^{(0)} \left(\frac{\hbar\omega_0}{m_0 c^2}\right)^2 e^{-2m_0 cZ/\hbar}\boldsymbol{k} \underset{m_0 \to 0}{\to} B^{(0)}\boldsymbol{k}, \tag{13}$$

which reduces to $\boldsymbol{B}^{(3)}$ in the limit of identically zero rest ($m_0 = 0$), when the premultiplying factor reduces to unity by the de Broglie guiding theorem, $\hbar\omega_0 = m_0 c^2$, and the phase dependence disappears. The existence of $\boldsymbol{B}^{(3)}$ and three degrees of photon polarization in the zero mass limit is therefore fully compatible with the existence of finite photon rest mass, experimental evidence for which is reviewed in Ref. [14].

In Sec. 3, the fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are expressed in terms of irreducible representations of the rotation group R(3), using the vector spherical harmonics [15]. This development shows that $\boldsymbol{B}^{(3)}$ is directly proportional to sums of vector spherical harmonics, i.e. to sums of irreducible representations of the full rotation group. These irreducible representations are non-zero vector functions in Euclidean space, and can be expressed in terms of the well known Clebsch Gordan coupling coefficients or Wigner 3-j symbols. Therefore the conventional assertion $\boldsymbol{B}^{(3)} =? \boldsymbol{0}$ *violates the fundamentals of group theory*. Furthermore, Sec. 3 shows that $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are related linearly, as well as non-linearly through Eq. (5) in Euclidean space. These linear relations occur through the corresponding link between $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$, and $\boldsymbol{e}^{(3)}$,

$$\boldsymbol{e}^{(1)} = \frac{2}{\sqrt{2}a}\boldsymbol{e}^{(3)} - \boldsymbol{b} - \boldsymbol{e}^{(2)}, \quad \boldsymbol{e}^{(2)} = \frac{2}{\sqrt{2}a}\boldsymbol{e}^{(3)} - \boldsymbol{b} - \boldsymbol{e}^{(1)}, \quad \boldsymbol{e}^{(3)} = \frac{\sqrt{2}}{2}a(\boldsymbol{e}^{(1)} + \boldsymbol{e}^{(2)}) + \boldsymbol{b}, \tag{14}$$

where $a$ and $\boldsymbol{b}$ are defined in Sec. 3 in terms of combinations of vector and scalar spherical harmonics. These linear relations show that each of the fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ is defined through combinations of the other two. In particular, $\boldsymbol{B}^{(3)}$ can be expressed as a sum over $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, showing conclusively that a) it is non-zero; b) it is part of the fundamental symmetry group R(3).

Section 4 shows, finally, that the expansion of the electromagnetic field in terms of the vector spherical harmonics, described for example by Silver [15], can be augmented using the results of Sec. 3 to include the expansion of the longitudinal field $\boldsymbol{B}^{(3)}$.

## 2. The Eigenvalues of the Massless and Massive Photons

It is customary to assert [27] that the massless photon has two eigenvalues of intrinsic (spin) angular momentum, namely $-\hbar$ and $\hbar$. In units of $\hbar$ these are -1 and +1. The eigenvalue 0 is missing from the standard theory, despite the fact that the photon is a spin one boson. The eigenvalues $-\hbar$ and $\hbar$ correspond to the two senses of circular polarization in the classical theory, and originate in the theory of vector fields, described for example by Silver [15]. In Euclidean space, rather than Minkowski space-time, the eigenvalues -1, 0 and +1 are derived from eigenequations of the type,

$$\hat{J}^{(3)}\boldsymbol{e}^{(1)} = +1\boldsymbol{e}^{(1)}, \qquad \hat{J}^{(3)}\boldsymbol{e}^{(2)} = -1\boldsymbol{e}^{(2)}, \qquad \hat{J}^{(3)}\boldsymbol{e}^{(3)} = 0\boldsymbol{e}^{(3)}, \tag{15}$$

where $\hat{J}^{(3)}$ is the rotation operator,

$$\hat{J}^{(3)} = i\boldsymbol{e}^{(3)} \times = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{16}$$

There is no paradox in the use of $\boldsymbol{e}^{(3)}$ as an operator as well as a unit vector, in the same sense that there is no paradox in the use of the scalar spherical harmonics as operators. This is discussed by Silver following Eq. (5.5) of Ref. [15]. The rotation operators in Euclidean space are first rank $\hat{T}$ operators [15], which are irreducible tensor operators and under rotations transform into linear combinations of each other. The $\hat{T}$ operators are directly proportional to the scalar spherical harmonic operators. The rotation operators, $\hat{J}$ of the full rotation group are related to the $\hat{T}$ operators as follows,

$$\hat{T}^1_{-1} = i\hat{J}^{(1)}, \qquad \hat{T}^1_1 = i\hat{J}^{(2)}, \qquad \hat{T}^1_0 = i\hat{J}^{(3)}, \tag{17}$$

and to the scalar spherical harmonic operators by



$$\hat{Y}^1_{-1} = \frac{i}{r}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\hat{J}^{(1)}, \qquad \hat{Y}^1_1 = \frac{i}{r}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\hat{J}^{(2)}, \qquad \hat{Y}^1_0 = \frac{i}{r}\left(\frac{3}{2\pi}\right)^{\frac{1}{2}}\hat{J}^{(3)}. \tag{18}$$

This implies in turn that the fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are also operators of the full rotation group, and are therefore *irreducible representations of the full rotation group*. Specifically,

$$\left.\begin{aligned} \hat{B}^{(1)} &= -B^{(0)}\hat{J}^{(1)}e^{i\phi} = B^{(0)}r\left(\frac{4\pi}{3}\right)^{\frac{1}{2}}\hat{Y}^1_{-1}e^{i\phi}, \\ \hat{B}^{(2)} &= -B^{(0)}\hat{J}^{(2)}e^{-i\phi} = B^{(0)}r\left(\frac{4\pi}{3}\right)^{\frac{1}{2}}\hat{Y}^1_1 e^{-i\phi}, \\ \hat{B}^{(3)} &= iB^{(0)}\hat{J}^{(3)} = B^{(0)}r\left(\frac{2\pi}{3}\right)^{\frac{1}{2}}\hat{Y}^1_0, \end{aligned}\right\} \tag{19}$$

which shows that $\hat{B}^{(3)} =? \hat{0}$ [16] violates the fundamentals of group theory. Essentially, Eqs. (19) represent $\hat{B}^{(1)}$, $\hat{B}^{(2)}$ and $\hat{B}^{(3)}$ in spherical polar coordinates ($r$, $\theta$, $\phi$), where $\phi$ in this context should not be confused with the phase $\phi$ of the plane wave.

We arrive at the conclusion that $\hat{B}^{(1)}$, $\hat{B}^{(2)}$ and $\hat{B}^{(3)}$ in operator form are all non-zero components of the same rank one scalar spherical harmonic $Y^1_M$, $M = -1, 0, 1$.

Furthermore, since the operators $\hat{J}^{(1)}$, $\hat{J}^{(2)}$ and $\hat{J}^{(3)}$ are components in a circular basis of the spin, or intrinsic, angular momentum of the vector field representing the electromagnetic field, the fields $\hat{B}^{(1)}$, $\hat{B}^{(2)}$ and $\hat{B}^{(3)}$ themselves are components of spin angular momentum. It is also clear that $\hat{J}^{(1)}$ is a lowering (annihilation) operator,

$$\hat{J}^{(1)}\boldsymbol{e}^{(2)} = +1\boldsymbol{e}^{(3)}, \qquad \hat{J}^{(1)}\boldsymbol{e}^{(3)} = -1\boldsymbol{e}^{(1)}, \qquad \hat{J}^{(1)}\boldsymbol{e}^{(1)} = 0\boldsymbol{e}^{(2)}, \tag{20}$$

and that $\hat{J}^{(2)}$ is a raising (creation) operator,

$$\hat{J}^{(2)}\boldsymbol{e}^{(2)} = 0\boldsymbol{e}^{(1)}, \qquad \hat{J}^{(2)}\boldsymbol{e}^{(3)} = -1\boldsymbol{e}^{(2)}, \qquad \hat{J}^{(2)}\boldsymbol{e}^{(1)} = +1\boldsymbol{e}^{(3)}. \tag{21}$$

The total angular momentum $J^2$ is also an eigenoperator, for example,

$$J^2\boldsymbol{e}^{(3)} = l(l+1)\boldsymbol{e}^{(3)}, \qquad l = 1. \tag{22}$$

The rotation operators therefore operate on the unit vectors $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$ and $\boldsymbol{e}^{(3)}$. The operator $\hat{J}^{(3)}$ is therefore also an intrinsic spin and can be identified in the quantum theory as an intrinsic spin of the massive photon, with eigenvalues $-\hbar$, 0, and $\hbar$; or $-\hbar$ and $\hbar$, for the massless photon.

For a classical vector field, its intrinsic (spin) angular momentum is identifiable with its transformation properties under rotations, and within a factor $\hbar$ the rotation operators $\hat{J}$ are spin angular momentum operators of the spin one boson. Recognition of a non-zero $\hat{B}^{(3)}$ is therefore compatible with *three* eigenvalues $-\hbar$, 0, $\hbar$ of spin angular momentum for the massive photon, or two eigenvalues, $-\hbar$ and $\hbar$, for the massless photon.

The conventional assertion that there are only two quantized eigenvalues, $-\hbar$ and $\hbar$, of spin angular momentum for the massless photon, is therefore compatible with the classical $\boldsymbol{B}^{(3)}$. The experimentally observed phenomena [3-10] of magnetization by light shows the presence of $\boldsymbol{B}^{(3)}$.

## 3. Expansion of $\hat{B}^{(3)}$ in Terms of the Vector Spherical Harmonics

The vector spherical harmonics [15] are specific vector fields which are eigenvalues of $j^2$ and $\hat{j}_Z$, where $\hat{j}$ is the operator for vector fields of infinitesimal rotations about axis (3). They have definite total angular momentum and occur in sets of dimension $(2j+1)$ which span in standard form the $D^{(j)}$ representations of the full rotation group, and are therefore irreducible tensors of rank $j$. Defining the total angular momentum as the sum of "orbital" angular momentum $\hat{l}$ and intrinsic (spin) angular momentum $\hat{J}$, we have,

$$\hat{j} = \hat{l} + \hat{J}, \tag{23}$$

and the vector spherical harmonics are compound irreducible tensor operators [15],

$$\hat{Y}^L_{Ml1} \equiv [\hat{Y}^l \otimes \hat{e}]^L_M . \tag{24}$$

They are formed from scalar spherical harmonic operators $\hat{Y}^l_m$, which form a complete set for scalar functions, and the $\hat{e}^{(i)}$ operators, which form a complete set for any vector in three dimensional Euclidean space. Therefore the vector spherical harmonics form a complete set for the expansion of any arbitrary classical vector field,

$$\boldsymbol{A} = A_X\boldsymbol{i} + A_Y\boldsymbol{j} + A_Z\boldsymbol{k}, \tag{25}$$

(in a Cartesian basis). For this vector the $\hat{l}_Z$ operates on the $A_X$, $A_Y$ and $A_Z$. The operator $\hat{J}_Z$ on $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$. Therefore $\hat{l}_Z$ operates on the spatial part of the field, and $\hat{J}_Z (= \hat{J}^{(3)})$ on the vector part.



Therefore the operator [15] for infinitesimal rotations about the $Z$ axis contains two "angular momentum" operators, $\hat{l}$ and $\hat{J}$, analogous to orbital and spin angular momentum in the quantum theory of atoms and molecules. The infinitesimal rotation therefore formally [15] a coupling of a set of spatial fields transforming according to $D^{(l)}$ with a set of three vector fields, ($\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$, $\boldsymbol{e}^{(3)}$) transforming according to $D^{(1)}$.

Equation (24) is an expression of this coupling, or combining, of entities in two different spaces to give a total angular momentum. It follows from these considerations that the vector spherical harmonics are defined by:

$$Y^L_{Ml1} = \sum_{mn} \langle l1mn|l1LM\rangle\, Y^l_m \boldsymbol{e}_M , \tag{26}$$

where $\langle l1mn|l1LM\rangle$ are Clebsch Gordan, or coupling, coefficients [15]. For photons, regarded as bosons of unit spin, it is possible to multiply Eq. (26) by $\langle l10M|l1LM\rangle$ and to sum over $L$ [15]. Using the orthogonality condition [15]:

$$\sum_j \langle j_1m_1'j_2m-m_1'|j_1j_2jm\rangle \langle j_1j_2jm|j_1m_1j_2m-m_1\rangle = \delta_{m_1'm_1} , \tag{27}$$

it is found that

$$Y^1_0(\theta, \phi)\boldsymbol{e}_M = \sum_{L=|l-1|}^{l+1} \langle l10M|l1LM\rangle\, \boldsymbol{Y}^L_{Ml1} , \tag{28}$$

which is an expression for the unit vectors $\boldsymbol{e}_M$ in terms of sums over vector spherical harmonics, i.e. of irreducible compound tensors, representations of the full rotation group of Euclidean space.

It is usually asserted in the conventional approach that in the theory of free space electromagnetism, the transverse components of $\boldsymbol{e}_M$ are physical, but the longitudinal component $M=0$, is unphysical. This deduction corresponds to only two states of polarization, left and right circular, in the classical theory of light. However, we see from our previous considerations that this assertion amounts to $\boldsymbol{e}_0 \equiv \boldsymbol{e}^{(3)} =?\ 0$, meaning the incorrect disappearance of some vector spherical harmonics, and meaning in turn a violation of fundamental group theoretical principles, because some irreducible representations are incorrectly set to zero.

Since this point is fundamental to the $\boldsymbol{B}^{(3)}$ theory, it proves convenient to emphasize it in this section by expanding $\boldsymbol{e}^{(3)}$ in terms of Wigner 3-j coefficients, through which Clebsch Gordan coefficients are usually defined. This analysis shows clearly that the vector $\boldsymbol{B}^{(3)}$ (or the operator $\hat{B}^{(3)}$) is experimentally and theoretically not equal to zero, whatever its physical significance, and has *all* the known properties of uniform magnetic flux density. Since unit vectors in the circular basis $M = (0)$, $(1)$, $(2)$ are linear combinations of vector spherical harmonic functions and therefore of irreducible compound tensor operators in R(3), then all three fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are non-zero, and all are made up of combinations of irreducible representations of the R(3) symmetry group, weighted by Clebsch-Gordan coefficients, which are 3-j symbols. Any assertion to the contrary breaches the structure of R(3). Specifically, from Eq. (28), the $\boldsymbol{e}^{(3)}$ vector can be expanded as

$$\boldsymbol{e}^{(3)} \equiv \boldsymbol{e}_0 = \frac{(-1)^{l+1}}{Y_0^1}\left[(2l+3)^{\frac{1}{2}}\begin{pmatrix} l & 1 & l+1 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{0l1}^{l+1} + (2|l-1|+1)^{\frac{1}{2}}\begin{pmatrix} l & 1 & |l-1| \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{0l1}^{|l-1|}\right], \tag{29}$$

for integer $l = 0, 1, \ldots$. For example,

$$l = 0, \quad \boldsymbol{e}^{(3)} = \frac{2(-1)^1}{Y_0^1}\left[\sqrt{3}\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{001}^{1}\right], \tag{29a}$$

$$l = 1, \quad \boldsymbol{e}^{(3)} = \frac{(-1)^2}{Y_0^1}\left[\sqrt{5}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{011}^{2} + \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{011}^{0}\right], \tag{29b}$$

$$l = 2, \quad \boldsymbol{e}^{(3)} = \frac{(-1)^3}{Y_0^1}\left[\sqrt{7}\begin{pmatrix} 2 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{021}^{3} + \sqrt{3}\begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{Y}_{021}^{1}\right], \tag{29c}$$

in terms of various non-zero Wigner 3-j symbols. The latter arise from the coupling of two angular momenta, the eigenfunctions $|jm\rangle$ being basis functions for representations of the group R(3). The coupled states span the direct product representations of R(3) in standard form. Evaluating the 3-j symbols finally gives $\boldsymbol{e}^{(3)}$ and $\boldsymbol{B}^{(3)}$ directly in terms of the vector spherical harmonics,

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)} = 2B^{(0)}\frac{\boldsymbol{Y}_{001}^{1}}{Y_0^1} - \frac{B^{(0)}}{\sqrt{3}}\frac{\left(\sqrt{2}\,\boldsymbol{Y}_{001}^{2} - \boldsymbol{Y}_{011}^{0}\right)}{Y_0^1}, \quad etc. \tag{30}$$

In the simplest case it is seen that the field $\boldsymbol{B}^{(3)}$ is non-zero and proportional to the vector spherical harmonic $\boldsymbol{Y}_{001}^{1}$, which is of course, also non-zero. There is no way of asserting that $\boldsymbol{B}^{(3)}$ is zero without destroying the fundamentals of group theory.

Finally it is clear that since all the three vectors $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$ and $\boldsymbol{e}^{(3)}$ can be expressed in terms of vector spherical harmonics, they are linked linearly, as well as non-linearly, in Euclidean space. For example, the $\boldsymbol{B}^{(3)}$ field can be expressed in terms of the transverse fields by



$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)} = \frac{\sqrt{2}}{2}aB^{(0)}(\boldsymbol{e}^{(1)} + \boldsymbol{e}^{(2)}) + B^{(0)}\boldsymbol{b} = -\frac{\sqrt{2}}{2}cB^{(0)}(\boldsymbol{e}^{(1)} - \boldsymbol{e}^{(2)}) + B^{(0)}\boldsymbol{d}, \tag{31}$$

where the coefficients are defined by the following combinations of scalar and vector spherical harmonics,

$$a = \frac{2}{\sqrt{2}}\left(\frac{Y_0^1}{Y_1^1 - Y_{-1}^1}\right), \qquad c = -\frac{2}{\sqrt{2}}\left(\frac{Y_0^1}{Y_1^1 + Y_{-1}^1}\right) \tag{32a}$$

$$\boldsymbol{b} = \sqrt{2}\left(\frac{\boldsymbol{Y}_{111}^{1} + \boldsymbol{Y}_{-111}^{1}}{Y_1^1 - Y_{-1}^1}\right), \qquad \boldsymbol{d} = \sqrt{2}\left(\frac{\boldsymbol{Y}_{111}^{1} - \boldsymbol{Y}_{-111}^{1}}{Y_1^1 + Y_{-1}^1}\right), \tag{32b}$$

This result shows that $\boldsymbol{B}^{(3)}$ is not zero because $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are not zero.

## 4. Extension of the Standard Theory

The conventional theory is described by Silver (chapter 29 of Ref. [15]), and concerns itself with the multipole expansion of a plane light wave in terms of the vector spherical harmonics. In this approach, there are considered to be only two physically significant values of $M$ in Eq. (28), corresponding to $M = +1$ and $-1$, which translates into our notation as follows:

$$\boldsymbol{e}_1 = -\boldsymbol{e}^{(2)}, \quad \boldsymbol{e}_{-1} = \boldsymbol{e}^{(1)}, \quad \boldsymbol{e}_0 = \boldsymbol{e}^{(3)}. \tag{33}$$

In our new approach, which considers the experimentally proven existence of $\boldsymbol{B}^{(3)}$, there is a need to consider the case $M = 0$, which is longitudinal. Therefore there is a need to amend the statement by Silver [15] that "The only values of $M$ that are physically significant for a wave in the $Z$ direction are $M = \pm 1$, in which case $L$ cannot be zero, since the only allowable $M$ value for $L = 0$ is $M = 0$." The experimentally verified existence of $\boldsymbol{B}^{(3)}$ means that the values of $L$ and $M$ to be considered are: $L = 1$, $M = -1, 0, +1$. These are the "natural" values, i.e. $M$ runs from $-L$ to $L$ from fundamental theory. Thus, the expressions given by Silver for $\boldsymbol{e}^{(1)}$ and $\boldsymbol{e}^{(2)}$ in terms of the vector spherical harmonics are augmented by Eq. (29) of this paper. This leads to the deduction that the value of M in Silver's Eq. (29.5) must also run over $-1$, $0$ and $+1$, not $-1$ and $+1$ as in the standard (transverse) theory. In consequence, there is an additional pure real magnetic $2^L$-pole component of the electromagnetic plane wave in vacuo, corresponding to $\boldsymbol{B}^{(3)}$. The longitudinal $2^L$-pole electric component is pure imaginary from fundamental considerations [11-13, 21], but is also non-zero.

Furthermore, the statement by Silver [15] that "The vector spherical harmonics $\boldsymbol{Y}_{MLl}^{L}$, with $l = L$, are transverse fields...." must be modified in light of the fact that the vector $\mathbf{e}^{(3)}$ which is longitudinal, can also be expressed in terms of the $L=1$, $M=0$ vector spherical harmonics as in Eq. (29) of this paper. This result simply augments Silver's Eq. (28.15), where are displayed other, transverse, combinations of vector spherical harmonics. Equation (29) of this paper shows that the longitudinal $\mathbf{e}^{(3)}$ (and therefore the longitudinal $\mathbf{B}^{(3)}$) can be expanded for all integer $l$ of that equation in terms of vector spherical harmonics. Since $\mathbf{B}^{(3)}$ and $i\mathbf{E}^{(3)}$ satisfy the four Maxwell equations in vacuo, so do these combinations of irreducible representations. As for the transverse fields [15], each value of $l$ for $M=0$ in $Y^L_{0ll}$ defines a different *non-zero* irreducible component of $\mathbf{B}^{(3)}$ and $i\mathbf{E}^{(3)}$. Therefore, the $L=1$ components in the expansion of $\mathbf{B}^{(3)}$ are dipolar fields. It also follows that the discussion on the coherency matrix given by Silver [15], using irreducible tensorial sets, must also be augmented to include the experimentally verified existence of $\mathbf{B}^{(3)}$, which implies the existence of the unphysical but non-zero $i\mathbf{E}^{(3)}$. This is outside the scope of the present paper, but will be the subject of future work. In general, the whole of classical and quantum electromagnetic field theory should in principle be augmented to consider $\mathbf{B}^{(3)}$ and $i\mathbf{E}^{(3)}$. This process will in the end analysis produce a strengthened and more internally consistent appreciation of electrodynamics.

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