Physica B 190 (1993) 310–313
North-Holland

PHYSICA B

SDI: 0921-4526(93)E0052-I

Reply to comment: "Charge conjugation symmetry and the nonexistence of the photon's static magnetic field"

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Received 14 December 1992

It is shown that the arguments of charge conjugation symmetry $\hat{C}$ used recently by Barron in an attempt to disprove the existence of the photon's magnetic field are erroneous.

1. Introduction

Recently [1–5], it has been shown that the longitudinal component, $\boldsymbol{B}^{(3)}$, of the photon's magnetic field in free space is given classically by the relation

$$\boldsymbol{B}^{(3)} = \frac{\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}}{2E^{(0)}ci}, \tag{1}$$

where $\boldsymbol{E}^{(1)}$, and its complex conjugate, $\boldsymbol{E}^{(2)}$, are associated with the transverse electric field polarizations of the photon in free space, and $E^{(0)}$ is the amplitude of $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$. It has been argued that the right and left hand sides of eq. (1) have the same, positive, parity inversion ($\hat{P}$) symmetry, and negative motion reversal ($\hat{T}$) symmetry. It is well known that electric and magnetic fields are both negative to charge conjugation [6], $\hat{C}$, so that

$$A_\mu \xrightarrow{\hat{C}} -A_\mu, \tag{2}$$

where A_μ is the potential four vector [7]. The operation $\hat{C}$ leaves all spacetime quantities unchanged [6]. An electric field can be expressed as

$$\boldsymbol{E} = E^{(0)}\boldsymbol{k}, \tag{3}$$

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where $E^{(0)}$ is a scalar amplitude ($\hat{P}$ and $\hat{T}$ positive), with the units of V m^{-1}, and where $\boldsymbol{k}$ is a polar unit vector. The scalar amplitude $E^{(0)}$ is negative to $\hat{C}$, because by definition [6], $\boldsymbol{k}$ is a spacetime quantity invariant under $\hat{C}$. Therefore the fundamental definition, eq. (1), is invariant to $\hat{C}$, $\hat{T}$ and $\hat{P}$, i.e. both sides have the same $\hat{C}$, $\hat{P}$ and $\hat{T}$ symmetries. This is in itself enough to show that Barron's [8] criticism, based on simple diagrams, is incorrect. Explicitly,

$$\hat{C}(c) = +, \quad \hat{C}(\boldsymbol{B}^{(3)}) = -,$$
$$\hat{C}(\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}) = +, \quad \hat{C}(E^{(0)}) = -,$$
$$\hat{T}(c) = +, \quad \hat{T}(\boldsymbol{B}^{(3)}) = -,$$
$$\hat{T}(\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}) = -, \quad \hat{T}(E^{(0)}) = +,$$
$$\hat{P}(c) = +, \quad \hat{P}(\boldsymbol{B}^{(3)}) = +,$$
$$\hat{P}(\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}) = +, \quad \hat{P}(E^{(0)}) = +,$$
$$\hat{P}(i) = \hat{T}(i) = \hat{C}(i) = +.$$

2. Detailed comments on Barron's critique

2.1. Point 1

It is well known in contemporary quantum field theory [7] that by integrating the d'Alembert equation in a manifestly covariant gauge

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such as the Lorentz gauge, it can be shown that the photon has four polarizations, one time-like (0), and three space-like, of which two are transverse (1) and (2), and the third is longitudinal (3). From the Gupta Bleuler condition, admixtures [7] of the (0) and (3) polarizations are physically meaningful photon states. The polarizations (1) and (2) are associated with the well known transverse, oscillating, electric and magnetic fields of the plane wave in vacuo. It is also well known that the photon is negative to $\hat{C}$ [6]; symbolically,

$$\hat{C}(\gamma) = -\gamma \,, \tag{4}$$

so that the $\hat{C}$ operation produces the antiphoton. This means that the electric and magnetic fields in polarizations (0)–(3) all change sign under $\hat{C}$. Without further development, this argument shows that the transverse (1) and (2) components (the everyday transverse fields) of an electromagnetic plane wave change sign under $\hat{C}$, whereas the propagation vector $\boldsymbol{\kappa}$, being a spacetime quantity, does not. According to Barron's pictorial argument [8], this would mean that all electromagnetic fields vanish in vacuo, a reductio ad absurdum. Therefore his argument is flawed fundamentally, and his critique is incorrect.

Barron does not consider the behaviour of $B^{(0)}, \boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ under $\hat{C}$ symmetry, and does not refer to Evans' fundamental equation [1], reproduced as eq. (1) of this reply. The $\hat{C}$ operator, by definition [6], however, takes the photon to the antiphoton, and also operates on the vacuum (the Dirac sea [7]), i.e. presumably changes the signs of all the negative energy particles making up the Dirac sea according to the Pauli Exclusion Principle. In this sense, therefore, the 'antivacuum' is also produced by $\hat{C}$ from the vacuum. The electrodynamical equation (1) is invariant to $\hat{C}$, $\hat{P}$ and $\hat{T}$, and the $\hat{C}$, $\hat{P}$ and $\hat{T}$ symmetries of $\boldsymbol{B}^{(3)}$ on the left hand side are those of magnetic flux density in tesla.

For example, the following classical relations emerge [2,5] from the fundamentals of quantum field theory [7]:

$$|\boldsymbol{B}^{(3)}| - B^{(0)} = 0 \,, \tag{5a}$$

$$\boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)} - B^{(0)2} = 0 \,. \tag{5b}$$

These are physically meaningful admixtures [7] of the longitudinal space-like (3) and time-like (0) polarizations of the photon. Since $B^{(0)}$ is the amplitude of the transverse as well as the longitudinal components, eqs. (5) imply that $\boldsymbol{B}^{(3)}$ is non-zero in free space if $B^{(0)}$ is non-zero, and the latter is non-zero as we have seen.

Lastly, since the $\hat{P}$, $\hat{T}$, and $\hat{C}$ symmetries of the two sides of eq. (1) are the same, and since $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$ are non-zero, then $\boldsymbol{B}^{(3)}$ is also non-zero. It is zero if and only if $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$ are both identically zero, since one is the conjugate of the other.

2.2. Point 2

Barron has misread the meaning of the vector $\boldsymbol{k}$ in the equation which he cites (Evans' eq. (14) of ref. [1]):

$$\boldsymbol{B}^{(3)}(\equiv \boldsymbol{B}_{II}) = -\left(\frac{\varepsilon_0}{8I_0 c}\right)^{\frac{1}{2}} |S_3| \boldsymbol{k} = -B^{(0)}\boldsymbol{k} \,. \tag{6}$$

Here ε_0 is the permittivity in vacuo, I_0 is the intensity of the electromagnetic radiation, c the speed of light, and $|S_3|$ the absolute (scalar) value of the Stokes parameter S_3. From fundamental electrodynamics [1–5],

$$|S_3| = 2E^{(0)2} \,, \quad I_0 = \tfrac{1}{2}\varepsilon_0 c E^{(0)2} \,, \quad E^{(0)} = cB^{(0)} \tag{7}$$

The vector $\boldsymbol{k}$ in eq. (6), as stated in the original paper [1] is a unit axial vector, with $\hat{\boldsymbol{P}}(\boldsymbol{k}) = +$, $\hat{T}(\boldsymbol{k}) = -$. In eq. (6), therefore, $\boldsymbol{k}$ is a spacetime quantity which is positive to $\hat{C}$ [6], and so $B^{(0)}$ is negative to $\hat{C}$. The combination of terms to which $B^{(0)}$ is equated in (6) is therefore also negative to $\hat{C}$.

The $\hat{P}$ and $\hat{T}$ symmetries of $|S_3|$ in eq. (6), as stated in the original paper [1] are both positive, because $|S_3|$ in eq. (6) denotes the absolute positive value of S_3. Evans' original eq. (14), is therefore consistent; $\boldsymbol{k}$ has the same $\hat{P}$ and $\hat{T}$ symmetries as $\boldsymbol{B}^{(3)}$, and $B^{(0)}$ has the same negative $\hat{C}$ symmetry as $\boldsymbol{B}^{(3)}$. Barron has mistaken $\boldsymbol{k}$ for $\boldsymbol{\kappa}$, which he calls a propagation vector, to which he then ascribes negative $\hat{P}$ symmetry. This obliges him to ascribe to $|S_3|$ a pseudoscalar symmetry with negative $\hat{P}$, despite the statement in Evans' paper [1] that $|S_3|$ in the absolute magnitude, a scalar positive to $\hat{P}$. Another critical part of Barron's argument therefore fails, because he has mistakenly ascribed to Evans' $\boldsymbol{k}$ the wrong $\hat{P}$ symmetry. In other words, Barron has erroneously identified the axial vector $\boldsymbol{k}$ with the polar vector $\boldsymbol{\kappa}$.

3. Discussion

Barron appears to have introduced charge conjugation symmetry [8] with the specific intent of disproving eq. (1), which, however, he never quotes, and which is invariant to $\hat{C}, \hat{P}$ and $\hat{T}$. The example of the inverse Faraday effect cited by Barron is in one respect irrelevant to the arguments in Evans' paper. The inverse Faraday effect as cited by Barron was treated by Woźniak et al. [9] in terms of the induction of a magnetic dipole moment by the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ via a hyperpolarisability $\boldsymbol{\beta}$. It is simple to note that $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ has positive $\hat{C}$ symmetry, $\boldsymbol{\beta}$ has negative $\hat{C}$ symmetry and the induced magnetic dipole moment has negative $\hat{C}$ symmetry. In another respect, however, the (observed) inverse Faraday effect exposes difficulties in the application of $\hat{C}$, because the latter changes all electrons to positrons and all protons to antiprotons, i.e. produces antimatter, including molecular antimatter. Thus $\hat{C}$ changes A_μ, but the electric and magnetic fields must now interact with molecular antimatter. The situation obtained after application of $\hat{C}$ is therefore never observed, since molecules are made up of matter, not antimatter, in the observable world. This exposes another flaw in Barron's argument, since $\hat{C}$ also changes the vacuum (the Dirac sea [7]) to the distinct state (the 'antivacuum') in which all the charges of the Dirac sea are reversed in sign, so that the situation obtained after $\hat{C}$ application is one of reversed A_μ, B_μ and E_μ (vide infra) propagating in the anti-vacuum, not the reversed A_μ, B_μ and E_μ propagating in the vacuum. To this author, it appears that the notion of applying $\hat{C}$ to electrodynamics, while possible, is fraught with subtlety, and errors can result, especially if consideration is restricted to simple pictorial arguments.

For example, Barron has restricted his pictorial consideration to the $\hat{C}$ symmetries of just $\boldsymbol{B}^{(3)}$ and $\boldsymbol{\kappa}$, while it is known [1–5,7] that there also exist $B^{(0)}, \boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. (Similar considerations apply to the electric field.) Both $\boldsymbol{B}^{(3)}$, and its electric counterpart $\boldsymbol{E}^{(3)}$, correspond to the longitudinal space-like components of the manifestly covariant four vectors $B_\mu \equiv (\boldsymbol{B}, \mathrm{i}B^{(0)})$ and $E_\mu \equiv (\boldsymbol{E}, \mathrm{i}E^{(0)})$ in vacuo (free spacetime). We have

$$\boldsymbol{E}_\mu \xrightarrow{\hat{C}} -\boldsymbol{E}_\mu\,, \qquad \boldsymbol{B}_\mu \xrightarrow{\hat{C}} -\boldsymbol{B}_\mu\,, \tag{8}$$

showing that all four components change sign with $\hat{C}$, in the same way that all four components of A_μ change sign (eq. (2)). Thus, if $\boldsymbol{B}^{(3)}$ is zero on the grounds of $\hat{C}$ symmetry, as asserted by Barron, then so must the other three components of $\boldsymbol{B}_\mu$, a fallacious conclusion,. It seems to this author that it is arbitrary to choose a 'system' in terms of $\boldsymbol{B}^{(3)}$ and $\boldsymbol{\kappa}$, leaving out other considerations, and that the elements making up Barron's pictorial representations must be chosen with great care. This is a largely subjective process, i.e. one does not know without calculation which elements to choose for a given picture or situation. Presumably, all possible elements must be chosen, including the vacuum itself, because the vacuum in contemporary thought is the Dirac sea, i.e. a collection of negative energy fermions arranged according to the Pauli Exclusion Principle. The Dirac sea of elementary particles is charged and charmed, and $\hat{C}$ reversed the sign of both the charge and the charm [6,7]. The vacuum, taken to be the Dirac sea therefore, is presumably not invariant to $\hat{C}$. Another possible conceptual error in Barron's analysis is the assumption that the photon is not distinct from the antiphoton. While it is stated [6] that the photon is its own antiparticle, it does not follow that this antiparticle is not distinct from the original particle. If this were the case, the $\hat{C}$ operation would be positive. However, eq. (4) shows that the photon is negative [6] to $\hat{C}$, and presumably this means that the entity obtained from the operation of $\hat{C}$ on the photon is a distinct entity. The situation obtained by Barron after operating on $\boldsymbol{B}^{(3)}$ by $\hat{C}$ is therefore a distinct situation, and his argument fails. In other words $\hat{C}$ operates on the photon's longitudinal magnetic field to produce the oppositely directed longitudinal field of the antiphoton.

It appears to this author that the correct diagramatic analysis is the one in which the $\hat{C}$ operator is applied to the two sides of the simple equation:

$$\boldsymbol{B}^{(3)} = \mp B^{(0)}\boldsymbol{k} , \tag{9}$$

where $\boldsymbol{k}$ is a unit axial vector, and $\boldsymbol{B}^{(0)}$ the scalar amplitude of longitudinal magnetic flux density. In eq. (9) $\hat{C}(\boldsymbol{B}^{(3)}) = -$, $\hat{C}(B^{(0)}) = -$ and $\hat{C}(\boldsymbol{k}) = +$, and the equation is invariant to $\hat{C}, \hat{\boldsymbol{P}}$ and $\hat{T}$, as required by fundamental symmetry. The definition (9) does not involve the propagation vector at all. Barron has mistaken $\boldsymbol{k}$ for a propagation vector, and therefore his digrammatic analysis is meaningless.

More generally, the difficulties with Barron's pictorial approach to electrodynamics in vacuo are exposed by applying $\hat{P}$ and $\hat{C}$ to an ordinary transverse electric field component of a plane wave:

$$\boldsymbol{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} (\boldsymbol{i} - \mathrm{i}\boldsymbol{j}) \exp(\mathrm{i}(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r})) ,$$

where, as usual, $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit polar vectors in X and Y, mutually orthogonal to the propagation axis Z, $E^{(0)}$ is the scalar field amplitude in volts per metre, ω its angular frequency at instant t, and $\boldsymbol{\kappa}$ its propagation vector at a point $\boldsymbol{r}$. We have

$$\hat{P}(\boldsymbol{E}^{(1)}) = -\boldsymbol{E}^{(1)} , \qquad \hat{P}(\boldsymbol{\kappa}) = -\boldsymbol{\kappa} , \tag{10a}$$

$$\hat{C}(\boldsymbol{E}^{(1)}) = -\boldsymbol{E}^{(1)} , \qquad \hat{C}(\boldsymbol{\kappa}) = \boldsymbol{\kappa} . \tag{10b}$$

$\hat{P}$ and $\hat{C}$ have the same negative effect on $\boldsymbol{E}^{(1)}$, but the opposite effect on $\boldsymbol{\kappa}$. Using Barron's approach, it is impossible to reconcile these results with a nonvanishing $\boldsymbol{E}^{(1)}$. In Barron's approach [8], $\hat{P}$ would have produced a 'distinct situation', but $\hat{C}$ would not have produced a 'distinct situation', and so $\boldsymbol{E}^{(1)}$ would for this reason have to be zero. However, $\boldsymbol{E}^{(1)}$ is an everyday observable, and Barron's argument is again shown to be fallacious, even when applied to a transverse wave in which $\boldsymbol{\kappa}$ appears explicitly. For $\boldsymbol{B}^{(3)}, \boldsymbol{\kappa}$ does not even appear in the definition, as we have seen, since it is removed by the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ in eq. (1).

Acknowledgements

Professor Laurence Barron is thanked for a preprint of his critique [8].

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