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THE PHOTON'S MAGNETOSTATIC FLUX QUANTUM: SYMMETRY AND WAVE PARTICLE DUALITY—FUNDAMENTAL CONSEQUENCES IN PHYSICAL OPTICS

I. INTRODUCTION

It has recently been demonstrated theoretically that there exists an operator $\hat{B}_{\Pi}$ of the quantized electromagnetic field that describes the photon's magnetostatic flux density:

$$\hat{B}_{\Pi} = B_0 \frac{\hat{J}}{\hbar} \tag{1}$$

Here B_0 has been interpreted[2–5] as a scalar magnetic flux density amplitude of a beam of circularly polarized light consisting of one photon, and $\hat{J}$ is the boson operator[6,7] describing that photon's quantized angular momentum. The classical equivalent of $\hat{B}_{\Pi}$ is a novel axial vector $\mathbf{B}_{\Pi}$, which is directed in the propagation axis of the beam. In this paper it is demonstrated using elementary tensor algebra, and from inspection of the Maxwell equations of the classical field, that there is another possible interpretation of the scalar amplitude B_0, designated henceforth by $(B_0)_+$,

where the + subscript is to be interpreted as "positive to parity inversion." It turns out that B_0 can be interpreted both as a scalar and as a pseudoscalar quantity, designated $(B_0)_-$, where the minus subscript means "negative to parity inversion." This is designated "symmetry duality," and is shown in this work to imply that $\hat{B}_{\Pi}$ can be defined simultaneously in terms of the photon's angular momentum operator $\hat{J}$ and linear momentum operator $\hat{p}$, a result that is a generalization of a keystone of wave mechanics, the de Broglie wave particle duality.[6] The latter is linked through $\hat{B}_{\Pi}$ to a symmetry duality in Maxwell's classical equations.

It has already been shown theoretically[2–5] and experimentally[8,9] that circularly polarized light can magnetize, leading, for example, to the inverse Faraday effect[10–13] and novel, potentially very useful, light-induced shifts in NMR spectroscopy[8,9] in one and more dimensions. The existence of the operator $\hat{B}_{\Pi}$ and its classical equivalent $\mathbf{B}_{\Pi}$ makes it much easier to interpret these magnetization effects by treating circularly polarized light as a "magnet" generating this novel flux quantum per photon. The $\hat{B}_{\Pi}$ concept also makes it relatively straightforward to forecast the existence of novel spectral phenomena, such as optical Zeeman, anomalous Zeeman, and Paschen-Back effects,[3] an optical Faraday effect and optically induced magnetic circular dichroism,[4] and optical Stern-Gerlach effect, using a focused laser beam to produce a light-induced magnetic field gradient, optical ESR effects, optically induced effects in interacting beams, such as a beam of circularly polarized photons reflected[5] from a beam of polarized electrons, and so on. All these effects can be thought of as arising from the replacement (or augmentation) of an ordinary magnet by or with a circularly polarized laser. These theories allow scope for the development of several novel analytically useful methods.

In this paper it is shown that $\mathbf{B}_{\Pi}$ is related directly to the ubiquitous,[14] pseudoscalar, third Stokes parameter S_3 of the classical electromagnetic plane wave, which becomes in quantum-field theory the third Stokes operator of Tanaś and Kielich.[1] Therefore, it follows immediately that several well-known phenomena of physical optics can be reinterpreted fundamentally in terms of the operator $\hat{B}_{\Pi}$, or its classical equivalent $\mathbf{B}_{\Pi}$. Examples include ellipticity in the plane wave, ellipticity developed in the measuring beam of the electrical Kerr effect, and circular dichroism, which are shown in this work to be magneto-optic phenomena. Therefore, not only does $\hat{B}_{\Pi}$ allow this reinterpretation, in both classical and quantum field theory, it also allows a link to be made between de Broglie wave particle duality and symmetry duality in the classical Maxwell equations. It appears, therefore, to go to the root of physical optics and field theory.

In Section II we develop the mathematical basis of symmetry duality with elementary vector and tensor algebra, before embarking in Section

III on a discussion of symmetry duality in the link between $\mathbf{B}_{\Pi}$ and S_3. In Section IV we develop the link between wave particle duality and the symmetry duality in Maxwell's equations demonstrated in Section III, and discuss qualitatively the implications for elementary particle theory. In Section V we develop the link between $\mathbf{B}_{\Pi}$ and S_3 into a novel explanation for ellipticity and circular dichroism in physical optics.

II. SYMMETRY DUALITY IN THE VECTOR PRODUCT OF TWO POLAR VECTORS

It is well known that the components of a vector that can be written as the cross product of two polar vectors do not change sign under parity inversion ($\hat{P}$) and that the vector so formed is an axial vector,[15] or pseudovector. The conjugate product of the classical electromagnetic field[2–5]

$$\mathbf{\Pi}^{(A)} = \mathbf{E} \times \mathbf{E}^* = 2(E_0^2)_+ \,\mathrm{i}\mathbf{e}_+ \tag{2}$$

where $\mathbf{E}^*$ is the polar complex conjugate of the polar electric field strength vector $\mathbf{E}$, is an axial vector, therefore. Here, $\mathbf{e}_+$ is an axial unit vector, positive to $\hat{P}$, and the quantity $(E_0^2)_+$ is a scalar, also positive to $\hat{P}$. The overall motion reversal ($\hat{T}$) symmetry of $\mathbf{\Pi}^{(A)}$ is negative, and it is natural to define $\mathbf{e}_+$ as a $\hat{T}$-negative unit vector, so that $(E_0^2)_+$ is a $\hat{T}$ positive scalar.

It appears at first sight that these definitions are both necessary and sufficient for the complete definition of the axial vector $\mathbf{\Pi}^{(A)}$; but mathematically, there is an alternative, which is revealed through writing any arbitrary axial vector as

$$\mathbf{C} = C_+\mathbf{e}_+ = C_-\mathbf{e}_- \tag{3}$$

where C_+ and $\mathbf{e}_+$ are respectively $\hat{P}$-positive scalar and unit axial vector quantities, and where C_- and $\mathbf{e}_-$ are respectively $\hat{P}$-negative pseudoscalar and $\hat{P}$-negative polar unit vector quantities. The overall $\hat{P}$ symmetry of the complete axial vector $\mathbf{C}$ is positive in both cases.

This seemingly mundane observation in elementary vector analysis has far-reaching consequences in the theory of the classical and quantized electromagnetic fields. In tensor algebra, the general vector cross product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ is written with the third rank antisymmetric (or alternating) unit tensor, $\varepsilon_{\alpha\beta\gamma}$, known as the Levi-Civita symbol[7,15]:

$$C_\alpha = \tfrac{1}{2}\varepsilon_{\alpha\beta\gamma}(A_\beta B_\gamma - A_\gamma B_\beta) \equiv \tfrac{1}{2}\varepsilon_{\alpha\beta\gamma}C_{\beta\gamma} \tag{4}$$

where the $\hat{P}$ symmetry of $\varepsilon_{\alpha\beta\gamma}$ is negative, so that $C_{\beta\gamma}$ is a $\hat{P}$-negative antisymmetric polar tensor of rank two. Evidently, C_α must be $\hat{P}$-positive, and is the rank one axial tensor (i.e., an axial vector). However, $C_{\beta\gamma}$ can also be written[15] as

$$C_{\beta\gamma} = -\mathrm{i}\varepsilon_{\beta\gamma}C_- \tag{5}$$

where $\varepsilon_{\beta\gamma}$ is the $\hat{P}$-positive, axial, unit antisymmetric tensor of rank two, and C_- is the pseudoscalar of Eq. (3). Equation (5) shows that the polar antisymmetric tensor of rank two can be reduced, quite generally, to a pseudoscalar, a particular result of a generalization in the relativistic theory of the classical electromagnetic field.[15] Note that $C_{\beta\gamma}$ is purely imaginary from the Hermitian properties of the general second-rank tensor, which can always be written as a sum of real symmetric and imaginary antisymmetric parts.[15]

Therefore,

$$C_\alpha = -\frac{\mathrm{i}}{2}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\beta\gamma}C_- \tag{6}$$

or

$$C_\alpha \equiv -\mathrm{i}C_+\varepsilon_{\alpha+} = -\frac{\mathrm{i}}{2}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\beta\gamma}C_- \tag{7}$$

where $\varepsilon_{\alpha+}$ is the rank one axial unit tensor, positive to $\hat{P}$, and C_+ is a $\hat{P}$-positive scalar. Recall that C_- is a $\hat{P}$-negative pseudoscalar. Equations (3) and (7), using vector and tensor notation, respectively, are expressions of symmetry duality, a purely mathematical result that shows that a scalar and pseudoscalar may both be used to define an axial vector. Clearly, if we take the magnitude ($|\mathbf{C}|$) of the axial vector $\mathbf{C}$ in Eq. (3), we obtain

$$\begin{aligned} C^2 &\equiv \mathbf{C}\cdot\mathbf{C} = C_+^2 = C_-^2 \\ |\mathbf{C}| &= \left|(C^2)^{1/2}\right| = |C_+| = |C_-| \end{aligned} \tag{8}$$

so that the positive parts of the scalar C_+ and pseudoscalar C_- are equal in absolute magnitude. This same result can be obtained from the tensor

Equation (7) by taking a particular Z component:

$$C_Z = -\mathrm{i}C_+\varepsilon_{Z+} = -\frac{\mathrm{i}}{2}C_-(\varepsilon_{ZXY}\varepsilon_{XY} + \varepsilon_{ZYX}\varepsilon_{YX}) \tag{9}$$

where the Einstein convention of summation over repeated indices has been used on the right side. With the component definitions $\varepsilon_{ZXY} = 1$, $\varepsilon_{XY} = 1$, $\varepsilon_{ZYX} = -1$, and $\varepsilon_{YX} = -1$, we obtain

$$C_+\varepsilon_{Z+} = C_-\varepsilon_{Z-} \tag{10}$$

where

$$\varepsilon_{Z-} \equiv \varepsilon_{ZXY}\varepsilon_{XY} + \varepsilon_{ZYX}\varepsilon_{YX} \tag{11}$$

is the Z component of the $\hat{P}$-negative polar unit tensor of rank one, $\varepsilon_{\alpha-}$. Note that Eqs. (3) and (10) are identical in symmetry character for the considered Z components of C. Equation (10), which is a direct and fundamental consequence of elementary tensor algebra, again shows the symmetry duality between scalar and pseudoscalar in the definition of the axial, or pseudo, vector. It is now possible to apply the purely mathematical principle of symmetry duality to the classical, nonrelativistic (or relativistic) field to obtain novel information of fundamental importance in physical optics, particularly in respect of a $\hat{P}$-positive, $\hat{T}$-negative axial vector, a novel magnetostatic field, $\mathbf{B}_\Pi$,[2-5] associated with the electromagnetic plane wave or in the quantized field, the magnetostatic flux density operator $\hat{B}_\Pi$ of the photon.

III. AN EXAMPLE OF SYMMETRY DUALITY: THE RELATION BETWEEN $\mathbf{B}_\Pi$ AND THE STOKES PARAMETER S_3

Consider the classical electromagnetic wave in free space, so that the real scalar refractive index is unity. It follows from Maxwell's equations for a plane wave that

$$E_0 = cB_0 \tag{12}$$

where E_0 and B_0 are $\hat{P}$- and $\hat{T}$-positive scalars, amplitudes, respectively, of the electric field strength and magnetic flux density. The intensity of the wave is defined in free space by

$$I_0 = \varepsilon_0 c E_0^2 \tag{13}$$

where ε_0 is the free space permittivity[6] in S.I. units, and c is the speed of light in vacuo. With Eqs. (12) and (13), Eq. (2) can be rewritten as

$$\mathbf{\Pi}^{(A)} = 2(E_0)_+ c\mathrm{i}\mathbf{B}_\Pi \tag{14}$$

where we have defined the magnetostatic flux density vector $\mathbf{B}_\Pi^{2-5}$ of the classical electromagnetic plane wave in free space:

$$\mathbf{B}_\Pi \equiv (B_0)_+ \mathbf{e}_+ \tag{15}$$

where $\mathbf{e}_+$ is a $\hat{P}$-positive unit axial vector. The overall $\hat{T}$ symmetry of $\mathbf{B}_\Pi$ is negative, and the overall $\hat{P}$ symmetry is positive. In the introduction we have given an account of the role of $\mathbf{B}_\Pi$ in the reinterpretation of well-known effects, such as circular dichroism and ellipticity, and its mediating role in new effects such as optical NMR and ESR,[9] optical Faraday[4] and Zeeman[3] effects, optical Stern-Gerlach effects, optical Compton scattering,[5] and so on.[2-5] Its quantized equivalent is the magnetostatic flux density operator of Eq. (1), in which $(B_0)_+$ is defined as the $\hat{P}$- and $\hat{T}$-positive scalar magnetic flux density amplitude of one photon.

Again, as in Section II, it would appear at first sight as if the definition of the seemingly mundane quantity B_0 as a $\hat{P}$- and $\hat{T}$-positive scalar is sufficient. Remarkably, however, this is not the case, there is an alternative definition possible of the novel classical vector $\mathbf{B}_\Pi$ which uses B_0 as a pseudoscalar. Not only does this emerge naturally from the Maxwell equations for the plane wave, it also provides a natural link between $\mathbf{B}_\Pi$ and the third Stokes parameter S_3.[1,7,14,15]

These conclusions emerge straightforwardly from the equations linking the $\mathbf{E}$ and $\mathbf{B}$ vectors of the classical electromagnetic plane wave in a medium of refractive index n, defined through the classical wave vector $\boldsymbol{\kappa}$, a $\hat{T}$- and $\hat{P}$-negative polar vector directed in the propagation axis Z of the plane wave[7]:

$$\boldsymbol{\kappa} = \frac{\omega}{c}\mathbf{n} \qquad n = \frac{c}{v} \tag{16}$$

Here ω is the angular frequency in radians per second of the plane wave, as usual. Maxwell's equations give[7]

$$\mathbf{B} = \frac{1}{c}\mathbf{n} \times \mathbf{E} \qquad \mathbf{E} = -\frac{c}{n^2}\mathbf{n} \times \mathbf{B} \tag{17}$$

In free space, the positive absolute magnitude of the $\hat{P}$- and $\hat{T}$-positive scalar n is unity. Using Equation (17) yields the conjugate product

$$\mathbf{E} \times \mathbf{E}^* = -\frac{c}{n^2}\mathbf{E} \times (\mathbf{n} \times \mathbf{B}^*) = -\frac{\mathbf{n}}{n}\frac{c(\mathbf{E} \cdot \mathbf{B}^*)}{n} \tag{18}$$

We note that the vector **n** is a $\hat{P}$-negative, $\hat{T}$-negative polar vector, defined as usual[7] as a propagation vector whose scalar magnitude is equal to the $\hat{T}$-positive scalar refractive index n; the dot product $\mathbf{E} \cdot \mathbf{B}^*$ is a $\hat{T}$-positive pseudoscalar. Equation (18) reduces to

$$\underbrace{\frac{E_0 B_0 c}{n}}_{\text{Scalar}}{}_{+}\ \underbrace{\mathbf{e}_+}_{\text{Axial unit vector}} = \underbrace{\frac{E_0 B_0 c}{n}}_{\text{Pseudoscalar}}{}_{-}\ \underbrace{\frac{\mathbf{n}}{|\mathbf{n}|}}_{\text{Polar unit vector}} \tag{19}$$

in which we have designated the various symmetries. It follows algebraically that

$$(B_0)_+ \mathbf{e}_+ = (B_0)_- \frac{\mathbf{n}}{n} \tag{20}$$

which can be rewritten in the notation of Section II as an example of symmetry duality in the Maxwell equations:

$$(B_0)_+ \mathbf{e}_+ = (B_0)_- \mathbf{e}_- \qquad \mathbf{e}_- \equiv \frac{\mathbf{n}}{n} \tag{21}$$

This shows that classical vector $\mathbf{B}_\Pi$ can be defined simultaneously in terms of the unit axial vector $\mathbf{e}_+$ and the unit polar vector $\mathbf{e}_-$, which is related to the propagation vector $\boldsymbol{\kappa}$, the photon linear momentum. In free space, with $n = 1$,

$$\mathbf{B}_\Pi = (B_0)_+ \mathbf{e}_+ = (B_0)_- \mathbf{e}_- = (B_0)_- \frac{c}{\omega}\boldsymbol{\kappa} \tag{22}$$

demonstrating a duality between the classical angular and linear momentum of the plane wave. We shall see that this is none other than the classical equivalent of the de Broglie wave particle duality for the photon in the quantized field. Before making the transition to the quantized field, however, another fundamentally new result emerges when we consider the

definition[15] of the Stokes parameter S_3:

$$E_\alpha E_\beta^* - E_\beta E_\alpha^* = -\mathrm{i}\varepsilon_{\alpha\beta}(S_3)_- \tag{23}$$

so that

$$(S_3)_- \equiv (E_0^2)_- \tag{24}$$

is a pseudoscalar quantity, implying inter alia the symmetry duality

$$\boldsymbol{\Pi}^{(A)} = 2(E_0^2)_+ \mathrm{i}\mathbf{e}_+ = 2(S_3)_- \mathrm{i}\mathbf{e}_- \tag{25}$$

It follows directly that the magnetostatic vector $\mathbf{B}_\Pi$ can be defined in free space ($n = 1$) in terms of $(S_3)_-$ as follows:

$$\mathbf{B}_\Pi = \frac{(S_3)_-}{2E_0 c}\mathbf{e}_- \equiv (B_0)_- \mathbf{e}_- \tag{26}$$

and we find that the role of B_0 as pseudoscalar is none other than the Stokes parameter S_3 scaled by an appropriate $\hat{P}$- and $\hat{T}$-positive scalar quantity. Thus, $\mathbf{B}_\Pi$ can be defined in free space through the symmetry duality

$$\mathbf{B}_\Pi = (B_0)_+ \mathbf{e}_+ = \frac{(S_3)_-}{2E_0 c}\mathbf{n} = \frac{(S_3)_-}{2E_0 c}\frac{c}{\omega}\boldsymbol{\kappa} \tag{27}$$

where the unit polar vector **n** can be identified with the unit vector $\mathbf{e}_-$ of this section. We thus forge a novel and fundamental link between the pseudoscalar magnitude of $\mathbf{B}_\Pi$ and the pseudoscalar S_3.

VI. SYMMETRY DUALITY AND WAVE PARTICLE DUALITY FOR THE PHOTON

Equation (1) shows that the photon's novel magnetic field operator $\hat{B}_\Pi$ is directly proportional to its well-defined[6] angular momentum boson operator $\hat{J}$ through B_0 in its scalar representation $(B_0)_+$, interpreted as the magnetic flux density amplitude of a single photon. The latter is a massless lepton that propagates at the speed of light and is not localized in space[16] unlike a massive lepton such as the electron or proton. These well-known properties are contained in Eq. (1), in that B_0 varies with intensity I_0 for a beam of circularly polarized light containing one photon, and therefore B_0 for one photon depends on the beam cross section, a finite area. The eigenvalues of the operator $\hat{J}$ are known to be $M_J\hbar$, $M_J = \pm 1$; there is no $M_J = 0$ component from relativistic considerations.[6,7] Therefore, the eigenvalues of $\hat{B}_{\Pi}$ are $\pm(B_0)_+$, where $(B_0)_+$ is a scalar, the positive eigenvalue corresponds to one particular circular polarization, and vice versa,[7] as in the convention for the operator $\hat{J}$.

We now use the result[6,7] that the eigenvalue of the linear momentum operator $\hat{p}$ of the photon is

$$\mathbf{p} \equiv \langle \hat{p} \rangle = \hbar \boldsymbol{\kappa} \tag{28}$$

where $\boldsymbol{\kappa}$ is the wave vector as defined classically in the preceding section. It follows straightforwardly from Eqs. (22) and (28) that in free space $(n = 1)$

$$\hat{B}_{\Pi} = (B_0)_+ \frac{\hat{J}}{\hbar} = \frac{(B_0)_-}{n} \frac{c}{\omega} \frac{\hat{p}}{\hbar} \tag{29}$$

which expresses the duality of Eq. (22) in terms of quantum field theory, and shows that the $\hat{B}_{\Pi}$ operator of the photon is simultaneously proportional to both its angular and linear momentum operators. Equation (29) summarizes a duality in symmetry, linear/angular momentum, and wave–particle character with the results

$$\begin{aligned} \hat{T}[(B_0)_+] &= +\hat{T}[(B_0)_-] \\ \hat{P}[(B_0)_+] &= -\hat{P}[(B_0)_-] \end{aligned} \tag{30}$$

Equation (29) implies the free space relation

$$\hat{p} = n\frac{\omega}{c}\hat{J} \qquad n = 1 \tag{31}$$

The expectation value of $\hat{p}$ is therefore given by the expectation value of $\hat{J}$, which is $\pm\hbar$. Taking without loss of generality the positive eigenvalue $\hbar$, we have, with $n = 1$,

$$p = \frac{\omega}{c}\hbar \tag{32}$$

which is the de Broglie wave particle duality for the photon.

We have therefore succeeded in relating directly the de Broglie wave-particle duality of quantum mechanics to the novel symmetry duality (22) of classical electromagnetic field theory. It has also been shown that the novel flux quantum $\hat{B}_{\Pi}$ is definable simultaneously in terms of $\hat{J}$ and $\hat{p}$, one operator being directly proportional to the other, implying that both must be quantized in the same way. In a sense, therefore, $\hat{B}_{\Pi}$ is the keystone of de Broglie's concept of duality for the photon.

Furthermore, contemporary elementary particle theory argues that the photon is a chiral entity, a massless lepton that travels in any frame of reference at c, and whose chirality, in consequence,[17] is well defined in terms of the eigenvalues of Dirac's $\hat{\gamma}_5$ operator. The chirality of a lepton with mass (i.e., a massive lepton) such as the electron is not well defined, leading to the idea[17] that mass itself is ill-defined chirality. Well-defined chirality in the photon can be thought of as a consequence of superimposed linear and angular momentum, and Eq. (29) shows that there is a duality between these two fundamental quantities. It appears therefore that the novel $\hat{B}_{\Pi}$ operator of the photon is a true chiral influence as defined by Barron,[17] and is therefore fundamentally different in nature from a magnetostatic flux density, such as a magnetic field generated in an electromagnet. The latter is now known to be an example of a false chiral influence,[17] and cannot, for example, be a cause of enantioselective synthesis. This is in contrast to the circularly polarized electromagnetic field, which le Bel[18] in 1874 conjectured to be a truly chiral influence, and which is now known to influence enantioselectivity in chemical reactions. The definition of the $\hat{B}_{\Pi}$ operator in Eq. (29) also allows insight to the symmetry of natural optical activity, i.e., circular dichroism and optical rotatory dispersion, as developed in the next section.

It may be conjectured that a magnetostatic flux quantum $\hat{B}_{\Pi}$ is always carried by a massless lepton whose chirality can be precisely defined as the eigenvalues of the Dirac $\hat{\gamma}_5$ operator; and, conversely, that the massive lepton does not support $\hat{B}_{\Pi}$ and does not have precisely defined eigenvalues of $\hat{\gamma}_5$. This conjecture would imply that fundamentally, $\hat{B}_{\Pi}$ is always a consequence of the absence of mass. It would therefore follow that the neutrino (and antineutrino) carries a $\hat{B}_{\Pi}$ field, but that the electron, neutron, and proton do not. However, it is not clear whether the neutrino has a classical counterpart such as the classical electromagnetic plane wave, the counterpart of the photon. If the parallel between photon and neutrino can be carried further, it would appear that the neutrino must also be thought of as unlocalized in space. This would imply inter alia that localization in space implies the presence of mass and the absence of well-defined chirality (or well-defined eigenvalues of $\hat{\gamma}_5$), and that the absence of mass implies the absence of space localization. Carrying the argument further, wave particle duality in a massive lepton such as the electron has been observed, because an electron beam can be diffracted, for example, but since the electron is localized and does not have well-fined chirality, its wave nature must be fundamentally different from at of the photon, and in consequence, no $\hat{B}_{\Pi}$ can be constructed or fined for the electron. Wave particle duality in the electron is therefore ndamentally different in nature from duality in the photon. The electron s a magnetic dipole moment that is proportional to the electron's spin gular momentum operator through the gyromagnetic ratio. We there- e conjecture that a massless lepton cannot support a magnetic dipole oment, because its effective gyromagnetic ratio would be infinite, but can pport a magnetostatic flux quantum. The opposite is true for a massive oton. With these assumptions, the $\hat{B}_{\Pi}$ operator of a massless lepton ould always be able to form an interaction Hamiltonian operator to first der with the magnetic dipole moment operator of a massive lepton, an ample being a photon beam interacting with an electron beam,[5] or a utrino beam with a neutron beam and so on, giving rise to measurable fects in principle. The inference overall, therefore, is that a beam of assless leptons, for example, photons or neutrinos, can magnetize but nnot be magnetized, whereas a beam of massive leptons cannot magne- e but can be magnetized.

The charge conjugation symmetry operator can be defined as $\hat{C}$ (which erates to reverse the sign of charge), and with this definition we recall e fundamental Luders-Pauli-Villiers theorem[17]:

$$\hat{C}\hat{P}\hat{T} = \hat{1} \tag{33}$$

ne violation of $\hat{P}$ has been observed[17] in a number of different ways, hereas the violation of $\hat{T}$ has been observed in only one critical experi- ent.[17] The violation of $\hat{P}$ leads to the result that the space-inverted iantiomers of a truly chiral entity such as the photon or neutrino are not egenerate, or exactly the same in energy, because of the existence of the violating electroweak force.[17] In contrast, the space-inverted "enanti- mers" of a falsely or pseudo chiral entity, such as an ordinary magnetic eld, are precisely the same in energy.[17] Thus, it is important to note that e true enantiomer of the photon, or neutrino, is not generated by space version or by application of the $\hat{P}$ operator, i.e., by reversing the linear omentum and keeping the angular momentum the same. Assuming that e photon is uncharged, so that $\hat{C}$ has no effect, its true or exact nantiomer must be generated by simultaneous $\hat{P}$ and $\hat{T}$ violation in order conserve the validity of the Luders-Pauli-Villiers theorem. (33). The ue enantiomer of the left-handed photon is presumably, therefore, an oject that must be designated the right-handed "antiphoton," and there a very small, but nonzero, energy difference between the left-handed photon and the right-handed photon. If the right-handed photon is to be regarded as having a different energy from the left-handed photon, then either $\hat{P}$ has been violated and $\hat{T}$ and $\hat{C}$ have been conserved, or $\hat{T}$ has been violated and $\hat{P}$ and $\hat{C}$ have been conserved. Assuming that $\hat{C}$ has no effect on the photon, because it is uncharged, the combined operation $\hat{P}\hat{T}$ must be used to generate the right antiphoton from its true enantiomer, the left photon, and vice versa. The photon is an object whose chirality is generated only as a result of its simultaneous translational and rotational motion, and the novel $\hat{B}_{\Pi}$ operator is a fundamental manifestation of this chirality. The latter is conserved, furthermore, in the photon and antiphoton, because both travel at the speed of light and both are massless. It is also known[17] that neutrinos conserve chirality, in that only left-handed neutrinos and right-handed antineutrinos exist. The $\hat{P}$ violating weak force is known to play a critical part in the interaction of left-handed neutrinos with left, but not with right, spin-polarized relativistic electrons, and of right-handed antineutrinos with right, but not left, polarized relativistic electrons.[17]

These arguments lead to the interesting possibility that a beam of, say, left photons, each carrying a flux quantum $\hat{B}_{\Pi}$, may interact differently with a beam of left and right polarized relativistic electrons, each carrying the magnetic dipole moment $\hat{m}$, through the interaction Hamiltonian operator

$$\Delta\hat{H} = -\hat{m}\cdot\hat{B}_{\Pi}$$

This difference may be picked up by observation of the Zeeman splitting caused by $\Delta\hat{H}$ in, for example, a circularly polarized visible laser beam reflected from a polarized, relativistic, electron beam. Such an experiment has been proposed recently to evaluate the effect of $\hat{B}_{\Pi}$ (Ref. 5).

V. THE ROLE OF $\mathbf{B}_{\Pi}$ IN ELLIPTICITY AND ASSOCIATED EFFECTS IN PHYSICAL OPTICS, FOR EXAMPLE, CIRCULAR DICHROISM

The link between $|\mathbf{B}_{\Pi}|$ and the third Stokes parameter $(S_3)_-$ can be expressed through the intensity I_0 as

$$|\mathbf{B}_{\Pi}| = (B_0)_- = \left(\frac{\varepsilon_0}{4I_0c}\right)^{1/2}(S_3)_- \tag{34}$$

so that it follows that whenever $(S_3)_-$ occurs in physical optics, it can be replaced by the pseudoscalar quantity B_0, multiplied by the scalar $(4I_0c/\varepsilon_0)^{1/2}$. This is a key link between the photon's magnetostatic field operator $\hat{B}_\Pi$, in its classical limit, and the ubiquitous $(S_3)_-$, revealing immediately the root cause of several well-known phenomena in physical optics.

As an example, the ellipticity η of an elliptically polarized beam of light is related to $(S_3)_-$ by

$$(S_3)_- = 2E_0^2 \sin 2\eta \tag{35}$$

with

$$\eta = \tan^{-1}\frac{b}{a} \tag{36}$$

where a and b are respectively the major and minor axes of the polarization ellipse.[7] This shows that there is a direct link between ellipticity and the vector $\mathbf{B}_\Pi$, which is the classical equivalent of the operator $\hat{B}_\Pi$. In the theory of the electrically induced Kerr effect,[7] for example, ellipticity is developed in an initially circularly polarized measuring beam after it has passed through a material to which a static, uniform, electric field has been applied perpendicular to the propagation direction of the beam and at 45° to the aximuth of an incident linearly polarized beam. For the emerging beam in the electric Kerr effect it can be shown that

$$|\mathbf{B}_\Pi| = (B_0)_- = \left(\frac{I_0}{\varepsilon_0 c^3}\right)^{1/2} \sin(2\eta) \tag{37}$$

showing that the root cause of ellipticity in the Kerr effect is the pseudoscalar magnitude $(B_0)_-$ of $\mathbf{B}_\Pi$. Note that for the incident, linearly polarized beam, $\mathbf{B}_\Pi$ is zero, but that in the transmitted, elliptically polarized beam it is nonzero.

Another example of the fundamental role of the pseudoscalar $(B_0)_-$ is the phenomenon of circular dichroism, which is a manifestation of optical activity, whereby the intensity of initially linearly polarized electromagnetic radiation transmitted by a structurally chiral material contains an excess of left over right circularly polarized components, or vice versa. In this context[7]

$$\frac{(S_3)_-}{(S_0)_+} = \frac{I_L - I_R}{I_L + I_R} \tag{38}$$

where $(S_0)_+$ is the zeroth Stokes parameter, a scalar quantity defined by

$$(S_0)_+ = 2E_0^2 \tag{39}$$

Therefore, the root cause of circular dichroism is the pseudoscalar magnitude $(B_0)_-$ of the vector $\mathbf{B}_\Pi$:

$$|\mathbf{B}_\Pi| = (B_0)_- = \left(\frac{1}{\varepsilon_0 c^3 I_0}\right)^{1/2} (I_R - I_L) \tag{40}$$

an equation that is valid at all electromagnetic frequencies.

The origin of circular dichroism therefore resides in the photon's magnetostatic flux quantum $\hat{B}_\Pi$. In other words, circular dichroism is magneto-optic in origin, and the observable $(I_R - I_L)$ is a spectral consequence of the interaction of $\hat{B}_\Pi$ with structurally chiral material. From Eq. (40), $I_R - I_L$ is proportional to the real pseudoscalar quantity $(B_0)_-$ after each photon of the beam emerges from the chiral material through which the beam has passed, i.e., after interaction has occurred between the incident flux quantum $\hat{B}_\Pi$ per photon and the appropriate molecular property tensor of the material.[7] This leads to a new way of describing the fundamental mechanism of natural optical activity by considering the mechanism of interaction of $\hat{B}_\Pi$ with a structurally chiral molecule, or center of optical activity. A quantum $\hat{B}_\Pi$ per photon is evidently absorbed and reemitted with different characteristics imparted by the chiral structure.

For a beam consisting of one photon, the observable $I_R - I_L$ provides an experimental measure of the transmitted elementary $\hat{B}_\Pi$ at each frequency of that beam. Although $\mathbf{B}_\Pi$ is itself independent of the phase of the beam, the interacting molecular property tensor depends on the beam frequency through semiclassical perturbation theory,[7] which gives

$$|\mathbf{B}_\Pi| = (B_0)_- = \left(\frac{I_0}{\varepsilon_0 c^3}\right)^{1/2} \tanh[\omega\mu_0 c l N \zeta''_{XYZ}(g)] \tag{41}$$

where μ_0 is the permeability in vacuo, ω the angular frequency of the beam, l the length of sample through which the beam has passed, and ζ''_{XYZ} an appropriately averaged molecular property tensor component, a pseudoscalar.[7] Equation (41) shows that all circularly dichroic spectra are signatures of the reemitted $\hat{B}_\Pi$ property of the photon.

More generally, any property in physical optics that involves $(S_3)_-$, in classical or quantized[1] form, necessarily involves $\mathbf{B}_\Pi$ or the quantized $\hat{B}_\Pi$

per photon. There are several of these phenomena, each of whose origin can be traced to the novel elementary flux quantum $\hat{B}_{\Pi}$ of the photon. Rayleigh refringent scattering theory, for example,[7] shows that $(S_3)_-$ is associated with a change $d\eta/dz$ in ellipticity in a beam passing through a sample of thickness z. It is immediately possible to say, therefore, that $d\eta/dz$ measures changes in the flux quantum $\hat{B}_{\Pi}$ per photon as it passes through the sample, i.e., as $\hat{B}_{\Pi}$ is absorbed and reemitted, a process from Eq. (29) that must involve changes in the incident photon's angular and linear momentum. Ellipticity is therefore magneto-optic in fundamental origin.

VI. DISCUSSION

One of the interesting consequences of the development in the preceding sections is that the speed of light c must be regarded as a $\hat{T}$-positive scalar quantity. This is because c is a universal constant that is relativistically the same in any frame of reference, and cannot be reversed by the motion reversal operator $\hat{T}$, because c is independent of motion. However, a velocity v that is less than c is $\hat{T}$-negative, because it is reversed by motion reversal in a given reference frame. In consequence, the scalar refractive index n, defined by c/v in a material, must be a $\hat{T}$-positive quantity. The value of n in vacuo is numerically unity and is the mathematical limit of c/v as $v \to c$. It is proper to regard n as being $\hat{T}$-positive in this limit, and this is the point of view utilized in this paper.

It follows that the unit polar vector $\mathbf{n}/n$ must be $\hat{T}$-negative, because it is the quotient of $\hat{T}$-negative/$\hat{T}$-positive quantities. In Eq. (20), for example, $\hat{T}[(B_0)_-] = +$, $\hat{T}(\mathbf{e}_+) = -$, $\hat{T}[(B_0)_+] = +$, $\hat{T}(\mathbf{n}/n) = -$, so that there is a balance of net $\hat{T}$ symmetries on either side of the equation.

To interpret rigorously the generalization, Eq. (29), of the de Broglie equation (32) in its "textbook" form, it must be borne in mind that the de Broglie duality rigorously implies the symmetry duality summarized in Eq. (30) for $\hat{P}$. Equation (31), therefore, is more rigorously expressed as

$$\hat{P} = n\frac{(B_0)_+}{(B_0)_-}\frac{\omega}{c}\hat{J} \qquad n = 1 \tag{42}$$

and Eq. (32) as

$$p = n\frac{(B_0)_+}{(B_0)_-}\frac{\omega}{c}\hbar \qquad n = 1 \tag{43}$$

The quotient $(B_0)_+/(B_0)_-$, and the $\hat{T}$-positive free space refractive index ($n = 1$) are missing or implied in the usual textbook definition of the de Broglie wave particle duality.

In conclusion, it has been demonstrated that there is an inherent symmetry duality in the definition of the magnetostatic flux quantum $\hat{B}_{\Pi}$, which is the root of the de Broglie wave particle duality for the photon. The operator $\hat{B}_{\Pi}$ can be defined simultaneously in terms of the angular and linear momentum operators of the photon. This type of symmetry duality occurs throughout physical optics, and is inherent in the fact that $\hat{B}_{\Pi}$, or its classical equivalent $\mathbf{B}_{\Pi}$, is at the root of several well-known effects, such as circular dichroism and ellipsometry of various kinds. The operator $\hat{B}_{\Pi}$ can also be used straightforwardly to predict and describe novel and useful spectroscopic effects that depend on magnetization by circularly polarized light.

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