

(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $f(i, j) := \sum_{\sigma=0}^3 R_{i,j,\sigma,0} gContr_{i,\sigma} gContr_{j,0,\sigma,0,3} +$

$\sum_{\sigma=1}^3 R_{i,j,\sigma,1} gContr_{i,\sigma} gContr_{j,1,\sigma,0,3} +$

$\sum_{\sigma=2}^3 R_{i,j,\sigma,2} gContr_{i,\sigma} gContr_{j,2,\sigma,0,3} +$

$\sum_{\sigma=3}^3 R_{i,j,\sigma,3} gContr_{i,\sigma} gContr_{j,3,\sigma,0,3}$

(%i2) /* define coordinate vector */

```
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

(%o2) x

(%o3) [t , r , θ , ϕ]

(%i4) C(r) := ((abs(r-r0))^n + alpha^n)^(2/n);

(%o4) $C(r) := \left(|r - r0|^n + \alpha^n \right)^{2/n}$

(%i5) /* g1 is symm. metric with indices 1...4 */

```
g1: matrix(
  [-A*C(r)^(1/2),0,0,0],
  [0,B*C(r)^(1/2),0,0],
  [0,0,C(r),0],
  [0,0,0,C(r)*sin(theta)^2]
);
```

(%o5)

$$\begin{bmatrix} -\sqrt{\left(|r0 - r|^n + \alpha^n\right)^{2/n}} A & 0 & 0 & 0 \\ 0 & \sqrt{\left(|r0 - r|^n + \alpha^n\right)^{2/n}} B & 0 & 0 \\ 0 & 0 & \left(|r0 - r|^n + \alpha^n\right)^{2/n} & 0 \\ 0 & 0 & 0 & \left(|r0 - r|^n + \alpha^n\right)^{2/n} \sin(\theta)^2 \end{bmatrix}$$

(%i6) /* contravariant g is inverse of g */

```
gContr1: ratsimp(invert(g1));
```

$$\begin{array}{cccc}
 \frac{\sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}}{(|r_0 - r|^n + \alpha^n)^{2/n} A} & 0 & 0 & 0 \\
 0 & \frac{\sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}}{(|r_0 - r|^n + \alpha^n)^{2/n} B} & 0 & 0 \\
 0 & 0 & \frac{1}{(|r_0 - r|^n + \alpha^n)^{2/n}} & 0 \\
 0 & 0 & 0 & \frac{1}{(|r_0 - r|^n + \alpha^n)^{2/n} \sin(\theta)^2}
 \end{array}$$

(%o6)

```

/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g      [mu,nu]: g1      [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$

```

```

(%i8) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu]) +
            diff(g[rho,mu],x[nu]) -
            diff(g[mu,nu],x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$

```

```

(%i9) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}$

```

$$\Gamma_{0,0,1} = - \frac{|r_0 - r|^n}{2(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{0,1,0} = - \frac{|r_0 - r|^n}{2(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{1,0,0} = - \frac{|r_0 - r|^n A}{2(r_0 - r)(|r_0 - r|^n + \alpha^n) B}$$

$$\Gamma_{1,1,1} = - \frac{|r_0 - r|^n}{2(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{1,2,2} = \frac{|r_0 - r|^n \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}}{(r_0 - r)(|r_0 - r|^n + \alpha^n) B}$$

$$\Gamma_{1,3,3} = \frac{|r_0 - r|^n \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} \sin(\theta)^2}{(r_0 - r)(|r_0 - r|^n + \alpha^n) B}$$

$$\Gamma_{2,1,2} = - \frac{|r_0 - r|^n}{(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{2,2,1} = - \frac{|r_0 - r|^n}{(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{2,3,3} = - \cos(\theta) \sin(\theta)$$

$$\Gamma_{3,1,3} = - \frac{|r_0 - r|^n}{(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{3,2,3} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{3,3,1} = - \frac{|r_0 - r|^n}{(r_0 - r)(|r_0 - r|^n + \alpha^n)}$$

$$\Gamma_{3,3,2} = \frac{\cos(\theta)}{\sin(\theta)}$$

```
(%i10) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
  R[rho,sigma,mu,nu] :
  diff(Gamma[rho,nu,sigma],x[mu]) -
  diff(Gamma[rho,mu,sigma],x[nu]) +
  /* lambda sums by function call: */
  sum(
    Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
    Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
    lambda, 0, 3)
}}}}$
```

```
(%i11) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$R_{0,1,0,1} = \frac{|r_0 - r|^n (|r_0 - r|^n - \alpha^n n + \alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{0,1,1,0} = -\frac{|r_0 - r|^n (|r_0 - r|^n - \alpha^n n + \alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{0,2,0,2} = -\frac{|r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{0,2,2,0} = \frac{|r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{0,3,0,3} = -\frac{|r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} \sin(\theta)^2}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{0,3,3,0} = \frac{|r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} \sin(\theta)^2}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{1,0,0,1} = \frac{|r_0 - r|^n (|r_0 - r|^n - \alpha^n n + \alpha^n) A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{1,0,1,0} = -\frac{|r_0 - r|^n (|r_0 - r|^n - \alpha^n n + \alpha^n) A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{1,2,1,2} = \frac{|r_0 - r|^n (|r_0 - r|^n + \alpha^n)^{2/n-2} (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} B}$$

$$R_{1,2,2,1} = -\frac{|r_0 - r|^n (|r_0 - r|^n + \alpha^n)^{2/n-2} (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} B}$$

$$R_{1,3,1,3} = \frac{|r_0 - r|^n (|r_0 - r|^n + \alpha^n)^{2/n-2} (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n) \sin(\theta)^2}{2(r_0 - r)^2 \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} B}$$

$$R_{1,3,3,1} = - \frac{|r_0 - r|^n (|r_0 - r|^n + \alpha^n)^{2/n-2} (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n) \sin(\theta)^2}{2(r_0 - r)^2 \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}} B}$$

$$R_{2,0,0,2} = - \frac{|r_0 - r|^{2n} A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{2,0,2,0} = \frac{|r_0 - r|^{2n} A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{2,1,1,2} = - \frac{|r_0 - r|^n (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{2,1,2,1} = \frac{|r_0 - r|^n (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{2,3,2,3} = (\sin(\theta)^2 (r_0^2 |r_0 - r|^{2n} B - 2 r r_0 |r_0 - r|^{2n} B + r^2 |r_0 - r|^{2n} B + 2 \alpha^n r_0^2 |r_0 - r|^n B - 4 \alpha^n r r_0 |r_0 - r|^n B + 2 \alpha^n r^2 |r_0 - r|^n B + \alpha^{2n} r_0^2 B - 2 \alpha^{2n} r r_0 B + \alpha^{2n} r^2 B - |r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}})) / ((r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B)$$

$$R_{2,3,3,2} = - (\sin(\theta)^2 (r_0^2 |r_0 - r|^{2n} B - 2 r r_0 |r_0 - r|^{2n} B + r^2 |r_0 - r|^{2n} B + 2 \alpha^n r_0^2 |r_0 - r|^n B - 4 \alpha^n r r_0 |r_0 - r|^n B + 2 \alpha^n r^2 |r_0 - r|^n B + \alpha^{2n} r_0^2 B - 2 \alpha^{2n} r r_0 B + \alpha^{2n} r^2 B - |r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}})) / ((r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B)$$

$$R_{3,0,0,3} = - \frac{|r_0 - r|^{2n} A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{3,0,3,0} = \frac{|r_0 - r|^{2n} A}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B}$$

$$R_{3,1,1,3} = - \frac{|r_0 - r|^n (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{3,1,3,1} = \frac{|r_0 - r|^n (|r_0 - r|^n - 2\alpha^n n + 2\alpha^n)}{2(r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2}$$

$$R_{3,2,2,3} = - (r_0^2 |r_0 - r|^{2n} B - 2 r r_0 |r_0 - r|^{2n} B + r^2 |r_0 - r|^{2n} B + 2 \alpha^n r_0^2 |r_0 - r|^n B - 4 \alpha^n r r_0 |r_0 - r|^n B + 2 \alpha^n r^2 |r_0 - r|^n B + \alpha^{2n} r_0^2 B - 2 \alpha^{2n} r r_0 B + \alpha^{2n} r^2 B - |r_0 - r|^{2n} \sqrt{(|r_0 - r|^n + \alpha^n)^{2/n}}) / ((r_0 - r)^2 (|r_0 - r|^n + \alpha^n)^2 B)$$

$$R_{3,2,3,2} = (r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}) / ((r0 - r)^2 (|r0 - r|^n + \alpha^n)^2 B)$$

```
(%i12) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i13) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n n - \alpha^n) A}{2 (r0 - r)^2 (|r0 - r|^n + \alpha^n)^2 B}$$

$$Ric_{1,1} = \frac{|r0 - r|^n (3 |r0 - r|^n - 5 \alpha^n n + 5 \alpha^n)}{2 (r0 - r)^2 (|r0 - r|^n + \alpha^n)^2}$$

$$Ric_{2,2} = (r0^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 r r0 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + r^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r0^2 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 4 \alpha^n r r0 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r^2 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r0^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 \alpha^{2n} r r0 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - |r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{2/n} - \alpha^n n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n} + \alpha^n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n}) / ((r0 - r)^2 (|r0 - r|^n + \alpha^n)^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B)$$

$$Ric_{3,3} = (\sin(\theta)^2 (r0^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 r r0 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + r^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r0^2 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 4 \alpha^n r r0 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r^2 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r0^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 \alpha^{2n} r r0 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - |r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{2/n} -$$

$$\alpha^n n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n} + \alpha^n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n}) / ((r0 - r)^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B)$$

(%i14) /* Ricci Scalar */

```
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
      + sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
      + sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
      + sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

$$\begin{aligned} (\%o14) \quad & \left(2 (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (r0^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 r r0 \right. \\ & |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + r^2 |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r0^2 \\ & |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 4 \alpha^n r r0 |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + 2 \alpha^n r^2 \\ & |r0 - r|^n \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r0^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - 2 \alpha^{2n} r r0 \\ & \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B + \alpha^{2n} r^2 \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B - |r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{2/n} - \\ & \left. \alpha^n n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n} + \alpha^n |r0 - r|^n (|r0 - r|^n + \alpha^n)^{2/n}) / ((r0 - r)^2 \right. \\ & \left. \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} B) + \right. \\ & \left. \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (3 |r0 - r|^n - 5 \alpha^n n + 5 \alpha^n)}{2 (r0 - r)^2 B} - \right. \\ & \left. \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (|r0 - r|^n + \alpha^n n - \alpha^n)}{2 (r0 - r)^2 B} \right) \end{aligned}$$

(%i15) ratsimp(RicSc);

$$\begin{aligned} (\%o15) \quad & \left(\sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} ((2 r0^2 - 4 r r0 + 2 r^2) |r0 - r|^{2n} + \right. \\ & (4 \alpha^n r0^2 - 8 \alpha^n r r0 + 4 \alpha^n r^2) |r0 - r|^n + 2 \alpha^{2n} r0^2 - 4 \alpha^{2n} r r0 + 2 \alpha^{2n} r^2) B + \\ & \left. (|r0 - r|^n + \alpha^n)^{2/n} ((5 \alpha^n - 5 \alpha^n n) |r0 - r|^n - |r0 - r|^{2n}) / ((|r0 - r|^n + \alpha^n)^{2/n} \right. \\ & \left. \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} ((r0^2 - 2 r r0 + r^2) |r0 - r|^{2n} + (2 \alpha^n r0^2 - 4 \alpha^n r r0 + 2 \alpha^n r^2) \right. \\ & \left. |r0 - r|^n + \alpha^{2n} r0^2 - 2 \alpha^{2n} r r0 + \alpha^{2n} r^2) B) \right) \end{aligned}$$

(%i16)

```

/* Test for R^q */
for mu: 0 thru 3 do (
for sigma:0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
  R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
  if R_q # 0 then (
    display("====Einstein equation R^q=0 not fulfilled! "),
    display(mu,sigma,nu,rho),
    display(R_q)
  )
)))));
(%o16) done

```

```

(%i17) /* Raising of indices,
        contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);

```

$$(\%o17) \quad - \frac{\left| r0 - r \right|^n \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2} \left(\left| r0 - r \right|^n - \alpha^n n + \alpha^n \right)}{2 (r0 - r)^2 A B}$$

$$(\%o18) \quad \frac{\left| r0 - r \right|^{2n} \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2}}{2 (r0 - r)^2 A B}$$

$$(\%o19) \quad \frac{\left| r0 - r \right|^{2n} \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2}}{2 (r0 - r)^2 A B}$$

```

(%i20) R0101: factor(R0101);
R0202: factor(R0202);
R0303: factor(R0303);

```

$$(\%o20) \quad - \frac{\left| r0 - r \right|^n \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2} \left(\left| r0 - r \right|^n - \alpha^n n + \alpha^n \right)}{2 (r0 - r)^2 A B}$$

$$(\%o21) \quad \frac{\left| r0 - r \right|^{2n} \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2}}{2 (r0 - r)^2 A B}$$

$$(\%o22) \quad \frac{\left| r0 - r \right|^{2n} \left(\left| r0 - r \right|^n + \alpha^n \right)^{-\frac{2}{n} - 2}}{2 (r0 - r)^2 A B}$$


```
(%i23) R1010: f(1,0);
        R1212: f(1,2);
        R1313: f(1,3);
```

$$(\%o23) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - \alpha^n n + \alpha^n)}{2 (r0 - r)^2 B^2}$$

$$(\%o24) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B^2}$$

$$(\%o25) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B^2}$$

```
(%i26) R1010: factor(R1010);
        R1212: factor(R1212);
        R1313: factor(R1313);
```

$$(\%o26) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - \alpha^n n + \alpha^n)}{2 (r0 - r)^2 B^2}$$

$$(\%o27) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B^2}$$

$$(\%o28) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B^2}$$

```
(%i29) R2020: f(2,0);
        R2121: f(2,1);
        R2323: f(2,3);
```

$$(\%o29) - \frac{|r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}}{2 (r0 - r)^2 B}$$

$$(\%o30) \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B}$$

$$(\%o31) (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} (r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2$$

$$\frac{\begin{aligned} &|r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \\ &\alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} \end{aligned}}{(r0 - r)^2 B}$$

```
(%i32) R2020: factor(R2020);
      R2121: factor(R2121);
      R2323: factor(R2323);
```

$$(\%o32) \quad - \frac{|r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}}{2 (r0 - r)^2 B}$$

$$(\%o33) \quad \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 B}$$

$$\begin{aligned} (\%o34) \quad & ((|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} (r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 \\ & |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \\ & \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}})) / (\\ & (r0 - r)^2 B) \end{aligned}$$

```
(%i35) R3030: f(3,0);
      R3131: f(3,1);
      R3232: f(3,2);
```

$$(\%o35) \quad - \frac{|r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}}{2 (r0 - r)^2 \sin(\theta)^2 B}$$

$$(\%o36) \quad \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 \sin(\theta)^2 B}$$

$$\begin{aligned} (\%o37) \quad & ((|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} (r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 \\ & |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \\ & \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}})) / (\\ & (r0 - r)^2 \sin(\theta)^2 B) \end{aligned}$$

```
(%i38) R3030: factor(R3030);
      R3131: factor(R3131);
      R3232: factor(R3232);
```

$$(\%038) \quad - \frac{|r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}}{2 (r0 - r)^2 \sin(\theta)^2 B}$$

$$(\%039) \quad \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{2 (r0 - r)^2 \sin(\theta)^2 B}$$

$$(\%040) \quad ((|r0 - r|^n + \alpha^n)^{-\frac{4}{n} - 2} (r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}})) / ((r0 - r)^2 \sin(\theta)^2 B)$$

(%i41) /* Coulomb law */
DivE : R0101 + R0202 + R0303;

$$(\%041) \quad \frac{|r0 - r|^{2n} (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2}}{(r0 - r)^2 A B} - \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - \alpha^n n + \alpha^n)}{2 (r0 - r)^2 A B}$$

(%i42) ev(ratsimp(DivE),diff);

$$(\%042) \quad (|r0 - r|^{2n} + (\alpha^n n - \alpha^n) |r0 - r|^n) / ((|r0 - r|^n + \alpha^n)^{2/n} ((2 r0^2 - 4 r r0 + 2 r^2) |r0 - r|^{2n} + (4 \alpha^n r0^2 - 8 \alpha^n r r0 + 4 \alpha^n r^2) |r0 - r|^n + 2 \alpha^{2n} r0^2 - 4 \alpha^{2n} r r0 + 2 \alpha^{2n} r^2) A B)$$

(%i43) /* J[r] */
Jr : -(R1010 + R1212 + R1313);

$$(\%043) \quad - \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - \alpha^n n + \alpha^n)}{2 (r0 - r)^2 B^2} - \frac{|r0 - r|^n (|r0 - r|^n + \alpha^n)^{-\frac{2}{n} - 2} (|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n)}{(r0 - r)^2 B^2}$$

(%i44) ratsimp(Jr);

$$(\%044) \quad - (3 |r0 - r|^{2n} + (5 \alpha^n - 5 \alpha^n n) |r0 - r|^n) / ((|r0 - r|^n + \alpha^n)^{2/n} ((2 r0^2 - 4 r r0 + 2 r^2) |r0 - r|^{2n} + (4 \alpha^n r0^2 - 8 \alpha^n r r0 + 4 \alpha^n r^2) |r0 - r|^n + 2 \alpha^{2n} r0^2 - 4 \alpha^{2n} r r0 + 2 \alpha^{2n} r^2) B^2)$$

```
(%i45) /* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

$$\begin{aligned}
& - \left(\left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \left(r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 \right. \right. \\
& \left. |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \right. \\
& \left. \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}} \right) / \left(\right. \\
& \left. (r0 - r)^2 B \right) + \frac{|r0 - r|^{2n} \left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}}}{2 (r0 - r)^2 B} - \\
& \frac{|r0 - r|^n \left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}} \left(|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n \right)}{2 (r0 - r)^2 B}
\end{aligned}$$

```
(%i46) ratsimp(Jtheta);
```

$$\begin{aligned}
& \left(\left((-r0^2 + 2 r r0 - r^2) |r0 - r|^{2n} + (-2 \alpha^n r0^2 + 4 \alpha^n r r0 - 2 \alpha^n r^2) |r0 - r|^n \right. \right. \\
& \left. - \alpha^{2n} r0^2 + 2 \alpha^{2n} r r0 - \alpha^{2n} r^2 \right) B + \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}} \\
& \left(|r0 - r|^{2n} + (\alpha^n n - \alpha^n) |r0 - r|^n \right) / \left(\left(|r0 - r|^n + \alpha^n \right)^{4/n} \left((r0^2 - 2 r r0 + r^2) \right. \right. \\
& \left. |r0 - r|^{2n} + (2 \alpha^n r0^2 - 4 \alpha^n r r0 + 2 \alpha^n r^2) |r0 - r|^n + \alpha^{2n} r0^2 - 2 \alpha^{2n} r r0 + \alpha^{2n} r^2 \right. \\
& \left. \left. \right) B \right)
\end{aligned}$$

```
(%i47) /* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

$$\begin{aligned}
& - \left(\left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \left(r0^2 |r0 - r|^{2n} B - 2 r r0 |r0 - r|^{2n} B + r^2 \right. \right. \\
& \left. |r0 - r|^{2n} B + 2 \alpha^n r0^2 |r0 - r|^n B - 4 \alpha^n r r0 |r0 - r|^n B + 2 \alpha^n r^2 |r0 - r|^n B + \right. \\
& \left. \alpha^{2n} r0^2 B - 2 \alpha^{2n} r r0 B + \alpha^{2n} r^2 B - |r0 - r|^{2n} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}} \right) / \left(\right. \\
& \left. (r0 - r)^2 \sin(\theta)^2 B \right) + \frac{|r0 - r|^{2n} \left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}}}{2 (r0 - r)^2 \sin(\theta)^2 B} - \\
& \frac{|r0 - r|^n \left(|r0 - r|^n + \alpha^n \right)^{-\frac{4}{n} - 2} \sqrt{\left(|r0 - r|^n + \alpha^n \right)^{2/n}} \left(|r0 - r|^n - 2 \alpha^n n + 2 \alpha^n \right)}{2 (r0 - r)^2 \sin(\theta)^2 B}
\end{aligned}$$

```
(%i48) ratsimp(Jphi);
```

```
(%o48) ( ( (- r0^2 + 2 r r0 - r^2) |r0 - r|^{2 n} + (- 2 \alpha^n r0^2 + 4 \alpha^n r r0 - 2 \alpha^n r^2) |r0 - r|^n
- \alpha^{2 n} r0^2 + 2 \alpha^{2 n} r r0 - \alpha^{2 n} r^2 ) B + \sqrt{(|r0 - r|^n + \alpha^n)^{2/n}}
(|r0 - r|^{2 n} + (\alpha^n n - \alpha^n) |r0 - r|^n) ) / ( (|r0 - r|^n + \alpha^n)^{4/n} ( (r0^2 - 2 r r0 + r^2)
|r0 - r|^{2 n} + (2 \alpha^n r0^2 - 4 \alpha^n r r0 + 2 \alpha^n r^2) |r0 - r|^n + \alpha^{2 n} r0^2 - 2 \alpha^{2 n} r r0 + \alpha^{2 n} r^2
) \sin(\theta)^2 B )
```

```
(%i49) DivE_p: ratsimp(ev(DivE, [r0=0, alpha=1, n=3, A=1, B=1]));
```

```
(%o49) - \frac{(-2|r|^3 - r^6)(|r|^3 + 1)^{1/3}}{2r^2|r|^9 + 6r^2|r|^3 + 6r^8 + 2r^2}
```

```
(%i50) Jr_p: ratsimp(ev(Jr, [r0=0, alpha=1, n=3, A=1, B=1]));
```

```
(%o50) - \frac{(3r^6 - 10|r|^3)(|r|^3 + 1)^{1/3}}{2r^2|r|^9 + 6r^2|r|^3 + 6r^8 + 2r^2}
```

```
(%i51) Jtheta_p: ratsimp(ev(Jtheta, [r0=0, alpha=1, n=3, A=1, B=1]));
```

```
(%o51) - \frac{(-4|r|^3 - 2r^6)(|r|^3 + 1)^{1/3} + 4r^2|r|^3 + 2r^8 + 2r^2}{(|r|^3 + 1)^{1/3} (2r^2|r|^9 + 6r^2|r|^3 + 6r^8 + 2r^2)}
```

```
(%i52) Jphi_p: ratsimp(ev(Jphi, [r0=0, alpha=1, n=3, A=1, B=1]));
```

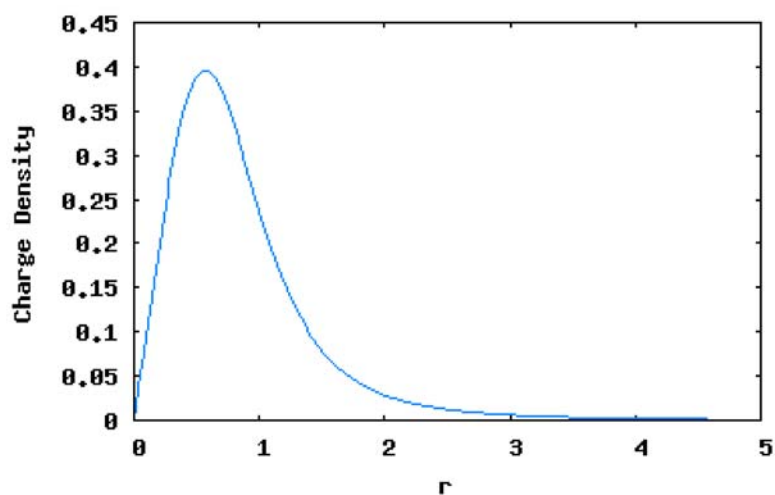
```
(%o52) - \frac{(-4|r|^3 - 2r^6)(|r|^3 + 1)^{1/3} + 4r^2|r|^3 + 2r^8 + 2r^2}{(|r|^3 + 1)^{1/3} (2r^2|r|^9 + 6r^2|r|^3 + 6r^8 + 2r^2) \sin(\theta)^2}
```

```
(%i53)
```

```
wxplot2d([DivE_p], [r,0,5], [y,0,.45], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t53)

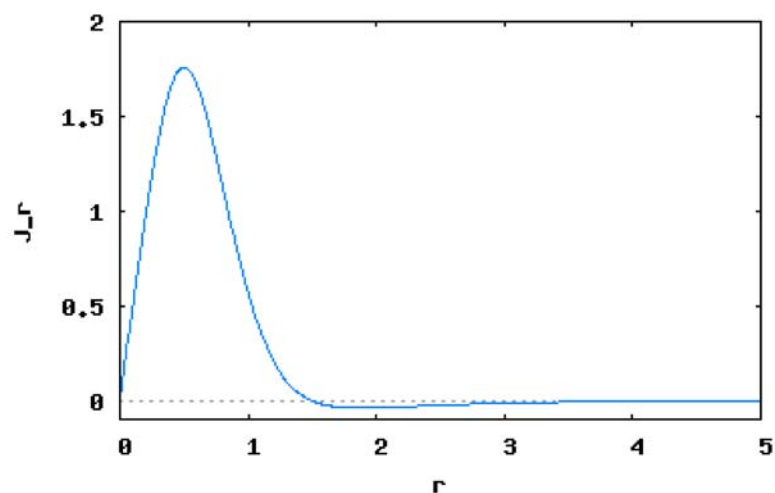


(%i54)

```
wxplot2d([Jr_p], [r,0,5],[y,-.1,2], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t54)

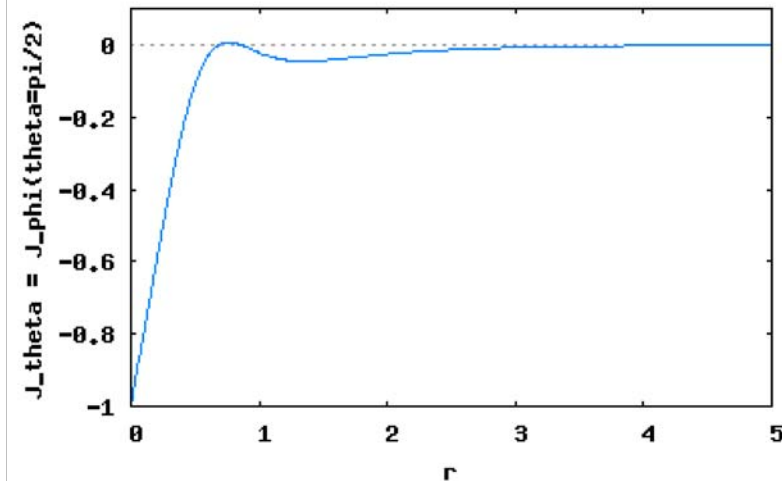


(%i55)

```
wxplot2d([Jtheta_p], [r,0,5],[y,-1,.1], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t55)



(%i56) DivE_p: ratsimp(ev(DivE, [r0=1,alpha=1,n=1, A=1, B=1]));

$$(\%o56) \quad \frac{1}{8|r-1|^3 + 8|r-1| + 2(r-1)^4 + 12(r-1)^2 + 2}$$

(%i57) Jr_p: ratsimp(ev(Jr, [r0=1,alpha=1,n=1, A=1, B=1]));

$$(\%o57) \quad -\frac{3}{8|r-1|^3 + 8|r-1| + 2(r-1)^4 + 12(r-1)^2 + 2}$$

(%i58) Jtheta_p: ratsimp(ev(Jtheta, [r0=1,alpha=1,n=1, A=1, B=1]));

$$(\%o58) \quad -\frac{|r-1| + r^2 - 2r + 1}{6|r-1|^5 + 20|r-1|^3 + 6|r-1| + (r-1)^6 + 15(r-1)^4 + 15(r-1)^2 + 1}$$

(%i59) Jphi_p: ratsimp(ev(Jphi, [r0=1,alpha=1,n=1, A=1, B=1]));

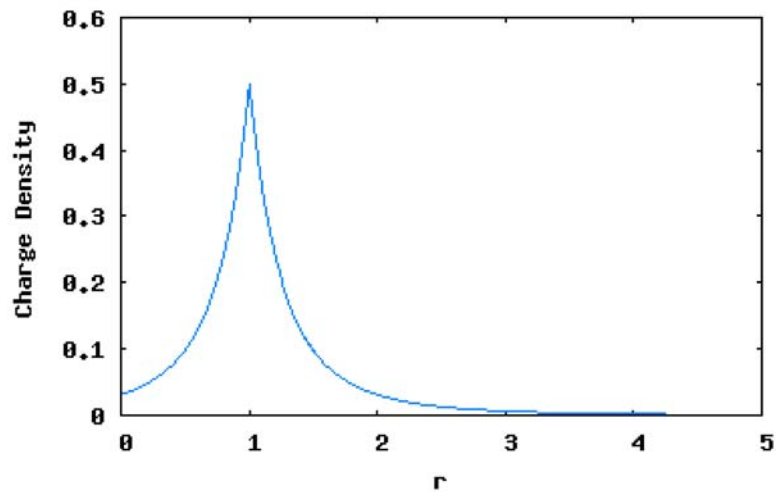
$$(\%o59) \quad -\frac{|r-1| + r^2 - 2r + 1}{(6|r-1|^5 + 20|r-1|^3 + 6|r-1| + (r-1)^6 + 15(r-1)^4 + 15(r-1)^2 + 1)\sin(\theta)^2}$$

(%i60)

```
wxplot2d([DivE_p], [r,0,5],[y,0,.6], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t60)

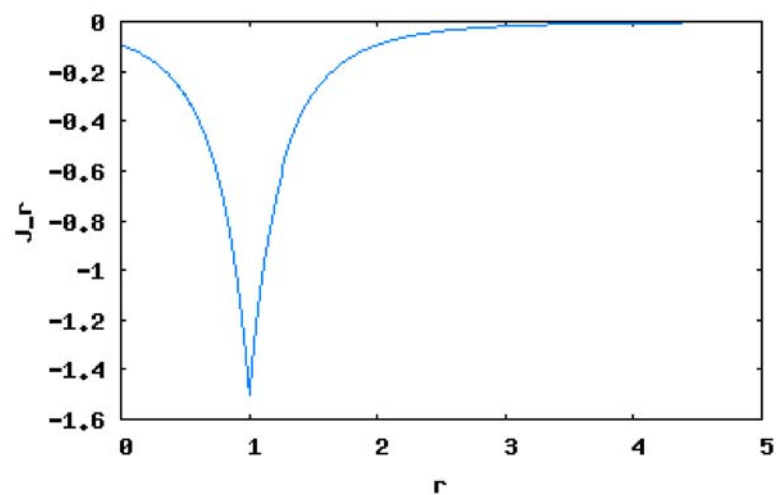


(%i61)

```
wxplot2d([Jr_p], [r,0,5],[y,-1.6,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t61)

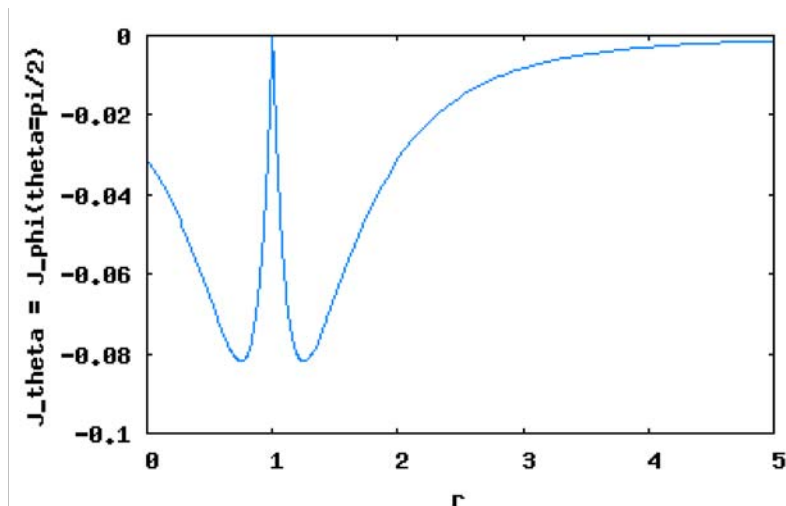


(%i62)

```
wxplot2d([Jtheta_p], [r,0,5],[y,-.1,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t62)



(%i63) DivE_p: ratsimp(ev(DivE,[r0=0,alpha=0,n=1, A=1, B=1]));

(%o63)
$$\frac{1}{2 r^4}$$

(%i64) Jr_p: ratsimp(ev(Jr,[r0=0,alpha=0,n=1, A=1, B=1]));

(%o64)
$$-\frac{3}{2 r^4}$$

(%i65) Jtheta_p: ratsimp(ev(Jtheta,[r0=0,alpha=0,n=1, A=1, B=1]));

(%o65)
$$\frac{|r| - r^2}{r^6}$$

(%i66)

Jphi_p: ratsimp(ev(Jphi,[r0=0,alpha=0,n=1, A=1, B=1, theta=%pi/2]));

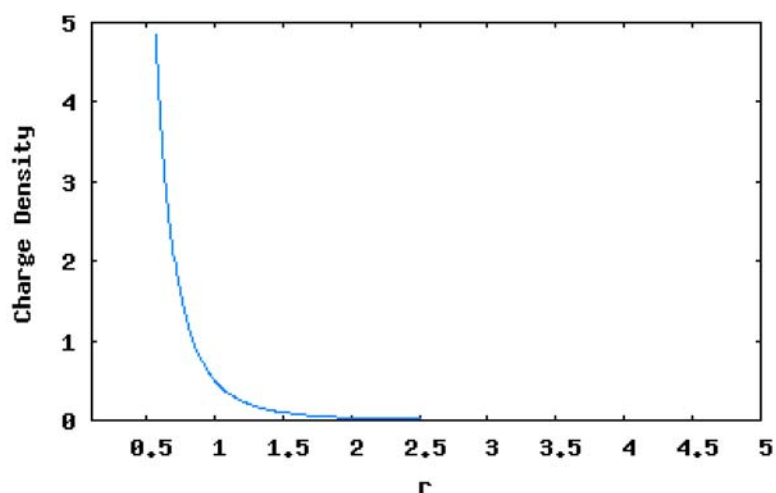
(%o66)
$$\frac{|r| - r^2}{r^6}$$

(%i67)

```
wxplot2d([DivE_p], [r,.1,5],[y,0,5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"]);
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t67)

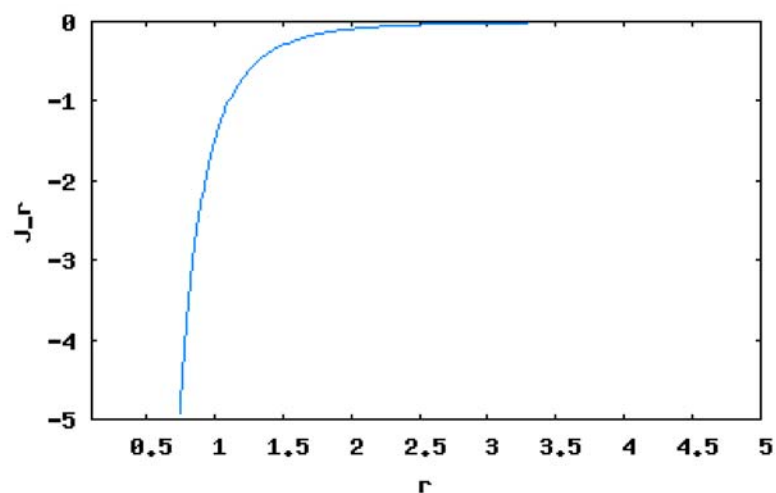


(%i68)

```
wxplot2d([Jr_p], [r,.1,5],[y,-5,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t68)

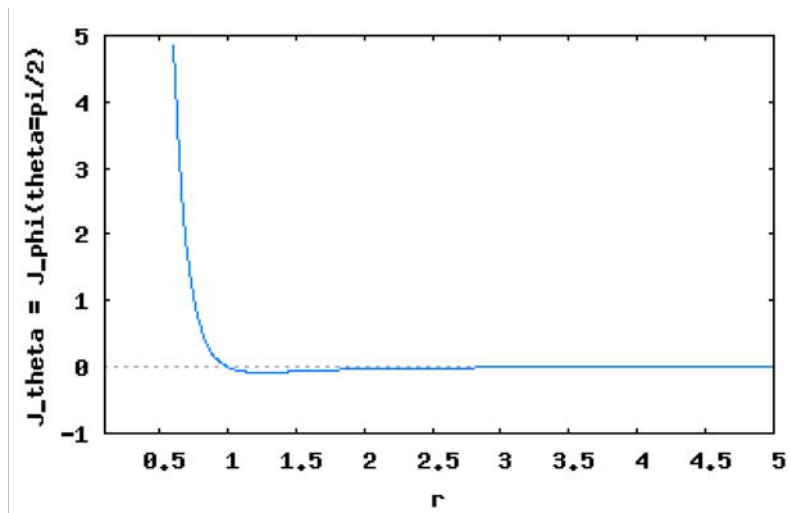


(%i69)

```
wxplot2d([Jtheta_p], [r,.1,5],[y,-1,5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t69)



(%i70)