(%i1)
```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $$f(i,j) := sum(R_{i,j,\sigma,0}\,gContr_{i,\sigma}\,gContr_{j,0},\sigma,0,3) + sum(R_{i,j,\sigma,1}\,gContr_{i,\sigma}\,gContr_{j,1},\sigma,0,3) + sum(R_{i,j,\sigma,2}\,gContr_{i,\sigma}\,gContr_{j,2},\sigma,0,3) + sum(R_{i,j,\sigma,3}\,gContr_{i,\sigma}\,gContr_{j,3},\sigma,0,3)$$

(%i2)
```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, x1, x2, x3];
```

(%o2) x

(%o3) $[\,t\,,\,x1\,,\,x2\,,\,x3\,]$

(%i4)
```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
  [-1/(2*omega^2),0,0,2*exp(x1)],
  [0,1/(2*omega^2),0,0],
  [0,0,1/(2*omega^2),0],
  [2*exp(x1),0,0,-1/(2*omega^2)*1/2*exp(2*x1)]
);
```

(%o4)
$$\begin{bmatrix} -\frac{1}{2\,\omega^2} & 0 & 0 & 2\,\%e^{x1} \\ 0 & \frac{1}{2\,\omega^2} & 0 & 0 \\ 0 & 0 & \frac{1}{2\,\omega^2} & 0 \\ 2\,\%e^{x1} & 0 & 0 & -\frac{\%e^{2\,x1}}{4\,\omega^2} \end{bmatrix}$$

(%i5)
```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o5)
$$\begin{bmatrix} \frac{2\,\omega^2}{32\,\omega^4-1} & 0 & 0 & \frac{16\,\omega^4\,\%e^{-x1}}{32\,\omega^4-1} \\ 0 & 2\,\omega^2 & 0 & 0 \\ 0 & 0 & 2\,\omega^2 & 0 \\ \frac{16\,\omega^4\,\%e^{-x1}}{32\,\omega^4-1} & 0 & 0 & \frac{4\,\omega^2\,\%e^{-2\,x1}}{32\,\omega^4-1} \end{bmatrix}$$

(%i6)
```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g     [mu,nu]: g1     [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

(%i7)
```
/* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
        diff(g[nu,rho],x[mu] ) +
        diff(g[rho,mu],x[nu] ) -
        diff(g[mu,nu] ,x[rho])),
    rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

(%i8)
```
/* dislplay Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
}}}}$
```

$$\Gamma_{0,0,1} = \frac{16\,\omega^4}{32\,\omega^4 - 1}$$

$$\Gamma_{0,1,0} = \frac{16\,\omega^4}{32\,\omega^4 - 1}$$

$$\Gamma_{0,1,3} = -\frac{2\,\omega^2\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$\Gamma_{0,3,1} = -\frac{2\,\omega^2\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$\Gamma_{1,0,3} = -2\,\omega^2\,\%e^{x1}$$

$$\Gamma_{1,3,0} = -2\,\omega^2\,\%e^{x1}$$

$$\Gamma_{1,3,3} = \frac{\%e^{2\,x1}}{2}$$

$$\Gamma_{3,0,1} = \frac{4\,\omega^2\,\%e^{-x1}}{32\,\omega^4 - 1}$$

$$\Gamma_{3,1,0} = \frac{4\,\omega^2\,\%e^{-x1}}{32\,\omega^4 - 1}$$

$$\Gamma_{3,1,3} = \frac{16\,\omega^4}{32\,\omega^4 - 1} - \frac{1}{32\,\omega^4 - 1}$$

$$\Gamma_{3,3,1} = \frac{16\,\omega^4}{32\,\omega^4 - 1} - \frac{1}{32\,\omega^4 - 1}$$

```
(%i9)  /* compute Riemann tensor elements */
       for rho:0 thru 3 do {
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           R[rho,sigma,mu,nu] :
           diff(Gamma[rho,nu,sigma],x[mu]) -
           diff(Gamma[rho,mu,sigma],x[nu]) +
           /* lambda sums by function call: */
           sum(
               Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
               Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
               lambda, 0, 3)
       }}}}$
```

```
(%i10)  /* dislplay R's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
        for k:0 thru 3 do {
        for l:0 thru 3 do {
            R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
            if R[i,j,k,l] # 0 then display(R[i,j,k,l])
            }}}}$
```

$$R_{0,0,0,3} = -\frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$R_{0,0,3,0} = \frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$R_{0,1,0,1} = -\frac{8\,\omega^4}{32\,\omega^4 - 1}$$

$$R_{0,1,1,0} = \frac{8\,\omega^4}{32\,\omega^4 - 1}$$

$$R_{0,1,1,3} = -\frac{4\,\omega^2\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$R_{0,1,3,1} = \frac{4\,\omega^2\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$R_{0,3,0,3} = \frac{4\,\omega^4\,\%e^{2\,x1}}{32\,\omega^4 - 1}$$

$$R_{0,3,3,0} = -\frac{4\,\omega^4\,\%e^{2\,x1}}{32\,\omega^4-1}$$

$$R_{1,0,0,1} = -\frac{8\,\omega^4}{32\,\omega^4-1}$$

$$R_{1,0,1,0} = \frac{8\,\omega^4}{32\,\omega^4-1}$$

$$R_{1,0,1,3} = -\frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

$$R_{1,0,3,1} = \frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

$$R_{1,3,0,1} = \frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

$$R_{1,3,1,0} = -\frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

$$R_{1,3,1,3} = \frac{(40\,\omega^4-1)\,\%e^{2\,x1}}{2\,(32\,\omega^4-1)}$$

$$R_{1,3,3,1} = -\frac{(40\,\omega^4-1)\,\%e^{2\,x1}}{2\,(32\,\omega^4-1)}$$

$$R_{3,0,0,3} = -\frac{8\,\omega^4}{32\,\omega^4-1}$$

$$R_{3,0,3,0} = \frac{8\,\omega^4}{32\,\omega^4-1}$$

$$R_{3,1,1,3} = \frac{8\,\omega^4-1}{32\,\omega^4-1}$$

$$R_{3,1,3,1} = -\frac{8\,\omega^4-1}{32\,\omega^4-1}$$

$$R_{3,3,0,3} = \frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

$$R_{3,3,3,0} = -\frac{32\,\omega^6\,\%e^{x1}}{32\,\omega^4-1}$$

```
(%i11)  /* Ricci tensor Ric[mu,nu] */
        for mu:0 thru 3 do {
        for nu:0 thru 3 do {
            Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
        }}$
```

```
(%i12)  /* dislplay Ric's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
            Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
            if Ric[i,j] # 0 then display(Ric[i,j])
            }}$
```

$$Ric_{0,0} = \frac{16\,\omega^4}{32\,\omega^4 - 1}$$

$$Ric_{0,3} = -\frac{64\,\omega^6\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$Ric_{1,1} = -\frac{(2\,\omega - 1)(2\,\omega + 1)(4\,\omega^2 + 1)}{32\,\omega^4 - 1}$$

$$Ric_{3,0} = -\frac{64\,\omega^6\,\%e^{x1}}{32\,\omega^4 - 1}$$

$$Ric_{3,3} = \frac{(48\,\omega^4 - 1)\,\%e^{2\,x1}}{2\,(32\,\omega^4 - 1)}$$

```
(%i13)  /* Ricci Scalar */
        RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
            +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
            +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
            +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
        ;
```

$$(\%o13)\quad \frac{2\,\omega^2(48\,\omega^4 - 1)}{(32\,\omega^4 - 1)^2} - \frac{2\,\omega^2(2\,\omega - 1)(2\,\omega + 1)(4\,\omega^2 + 1)}{32\,\omega^4 - 1} - \frac{2048\,\omega^{10}}{(32\,\omega^4 - 1)^2} + \frac{32\,\omega^6}{(32\,\omega^4 - 1)^2}$$

```
(%i14)  ratsimp(RicSc);
```

$$(\%o14)\quad -\frac{96\,\omega^6 - 4\,\omega^2}{32\,\omega^4 - 1}$$

```
(%i15)
/* Test for R^q */
for mu:    0 thru 3 do (
for sigma: 0 thru 3 do (
for nu:    0 thru 3 do (
for rho:   0 thru 3 do (
    R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
    if R_q # 0 then (
        display("==============Einstein equation R^q=0 not fulfilled! "),
        display(mu,sigma,nu,rho),
        display(R_q)
    )
))));
(%o15)  done
```

```
(%i16) /* Raising of indices,
          contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
       /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

       R0101: f(0,1);
       R0202: f(0,2);
       R0303: f(0,3);
```

$$(\%o16)\quad \frac{96\,\omega^8}{(32\,\omega^4-1)^2}$$

$$(\%o17)\quad 0$$

$$(\%o18)\quad \frac{32\,\omega^8}{(32\,\omega^4-1)^3}-\frac{1024\,\omega^{12}}{(32\,\omega^4-1)^3}$$

```
(%i19) R0101: factor(R0101);
       R0202: factor(R0202);
       R0303: factor(R0303);
```

$$(\%o19)\quad \frac{96\,\omega^8}{(32\,\omega^4-1)^2}$$

$$(\%o20)\quad 0$$

$$(\%o21)\quad -\frac{32\,\omega^8}{(32\,\omega^4-1)^2}$$

```
(%i22) R1010: f(1,0);
       R1212: f(1,2);
       R1313: f(1,3);
```

$$(\%o22)\quad \frac{32\,\omega^8}{(32\,\omega^4-1)^2}-\frac{1024\,\omega^{12}}{(32\,\omega^4-1)^2}$$

$$(\%o23)\quad 0$$

$$(\%o24)\quad \frac{4\,\omega^4(40\,\omega^4-1)}{(32\,\omega^4-1)^2}-\frac{1024\,\omega^{12}}{(32\,\omega^4-1)^2}$$

```
(%i25) R1010: factor(R1010);
       R1212: factor(R1212);
       R1313: factor(R1313);
```

$$(\%o25)\quad -\frac{32\,\omega^8}{32\,\omega^4-1}$$

$$(\%o26)\quad 0$$

$$(\%o27)\quad -\frac{4\,\omega^4(8\,\omega^4-1)}{32\,\omega^4-1}$$

```
(%i28)  R2020: f(2,0);
        R2121: f(2,1);
        R2323: f(2,3);
```

(%o28) 0

(%o29) 0

(%o30) 0

```
(%i31)  R2020: factor(R2020);
        R2121: factor(R2121);
        R2323: factor(R2323);
```

(%o31) 0

(%o32) 0

(%o33) 0

```
(%i34)  R3030: f(3,0);
        R3131: f(3,1);
        R3232: f(3,2);
```

(%o34) $$\frac{64\,\omega^8\,\%e^{-2\,x1}}{\left(32\,\omega^4-1\right)^3}-\frac{2048\,\omega^{12}\,\%e^{-2\,x1}}{\left(32\,\omega^4-1\right)^3}$$

(%o35) $$-\frac{8\,\omega^4\left(8\,\omega^4-1\right)\%e^{-2\,x1}}{\left(32\,\omega^4-1\right)^2}$$

(%o36) 0

```
(%i37)  R3030: factor(R3030);
        R3131: factor(R3131);
        R3232: factor(R3232);
```

(%o37) $$-\frac{64\,\omega^8\,\%e^{-2\,x1}}{\left(32\,\omega^4-1\right)^2}$$

(%o38) $$-\frac{8\,\omega^4\left(8\,\omega^4-1\right)\%e^{-2\,x1}}{\left(32\,\omega^4-1\right)^2}$$

(%o39) 0

```
(%i40)  /* Coulomb law */
        DivE : R0101 + R0202 + R0303;
```

(%o40) $$\frac{64\,\omega^8}{\left(32\,\omega^4-1\right)^2}$$

```
(%i41)  ratsimp(DivE);
```

(%o41) $$\frac{64\,\omega^8}{1024\,\omega^8-64\,\omega^4+1}$$

```
(%i42) /* J[r] */
       Jr : -(R1010 + R1212 + R1313);
```

$$(\%o42)\quad \frac{32\,\omega^8}{32\,\omega^4-1}+\frac{4\,\omega^4\,(8\,\omega^4-1)}{32\,\omega^4-1}$$

```
(%i43) ratsimp(Jr);
```

$$(\%o43)\quad \frac{64\,\omega^8-4\,\omega^4}{32\,\omega^4-1}$$

```
(%i44) /* J[theta] */
       Jtheta : -(R2020 + R2121 + R2323);
```

$$(\%o44)\quad 0$$

```
(%i45) ratsimp(Jtheta);
```

$$(\%o45)\quad 0$$

```
(%i46) /* J[phi] */
       Jphi : -(R3030 + R3131 + R3232);
```

$$(\%o46)\quad \frac{64\,\omega^8\,\%e^{-2\,x1}}{(32\,\omega^4-1)^2}+\frac{8\,\omega^4\,(8\,\omega^4-1)\,\%e^{-2\,x1}}{(32\,\omega^4-1)^2}$$

```
(%i47) ev(ratsimp(Jphi),r);
```

$$(\%o47)\quad \frac{(128\,\omega^8-8\,\omega^4)\,\%e^{-2\,x1}}{1024\,\omega^8-64\,\omega^4+1}$$

```
(%i48) DivE_p: ev(at(DivE,[omega=1]));
```

$$(\%o48)\quad \frac{64}{961}$$

```
(%i52) J1_p: ev(at(Jr,[omega=1]));
```

$$(\%o52)\quad \frac{60}{31}$$

```
(%i53) J2_p: ev(at(Jtheta,[omega=1]));
```

$$(\%o53)\quad 0$$

```
(%i54) J3_p: ev(at(Jphi,[omega=1]));
```

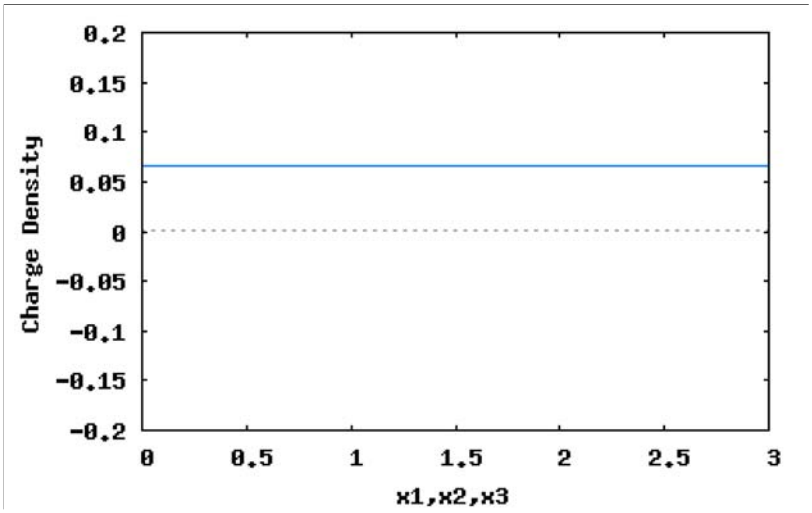
$$(\%o54)\quad \frac{120\,\%e^{-2\,x1}}{961}$$

(%i65)

```
wxplot2d([DivE_p], [r,0,3],[y,-.2,.2], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "x1,x2,x3"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t65)

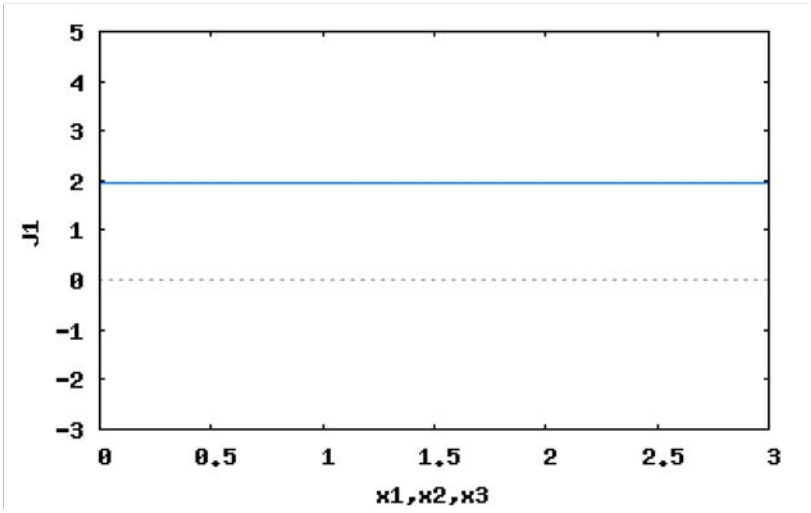


(%i59)

```
wxplot2d([J1_p], [x1,0,3],[y,-3,5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "x1,x2,x3"], [ylabel, "J1"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t59)

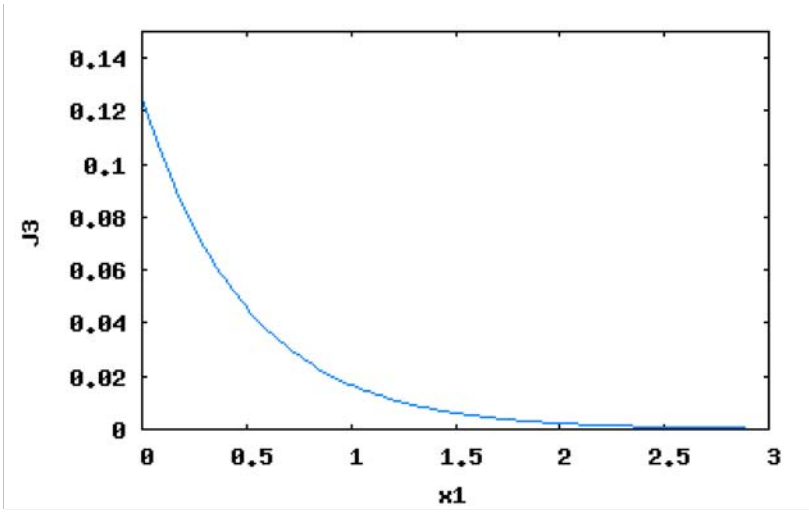


(%i62)

```
wxplot2d([J3_p], [x1,0,3],[y,0,.15], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "x1"], [ylabel, "J3"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t62)



(%i66)