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(%i1)
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);

(%o1)  f(i , j) := sum( Ri , j , σ , 0 gContri , σ gContrj , 0 , σ , 0 , 3 ) +
sum( Ri , j , σ , 1 gContri , σ gContrj , 1 , σ , 0 , 3 ) +
sum( Ri , j , σ , 2 gContri , σ gContrj , 2 , σ , 0 , 3 ) +
sum( Ri , j , σ , 3 gContri , σ gContrj , 3 , σ , 0 , 3 )

(%i2) /* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, x1, x2, x3];

(%o2) x

(%o3) [ t , x1 , x2 , x3 ]

(%i4) /* function dependencies */
depends([vs],[xs],[xs],[t],[f],[rs],[rs],[t,x1,x2,x3]);

(%o4) [ vs(xs) , xs(t) , f(rs) , rs(t , x1 , x2 , x3) ]

(%i5) /* dependency check */
diff(f,t);

(%o5)  $\left(\frac{d}{d\,rs}f\right)\left(\frac{d}{d\,t}rs\right)$ 

(%i6) vs: diff(xs,t);
rs: ((x1-xs)^2 + x2^2 + x3^2)^(1/2);

(%o6)  $\frac{d}{d\,t}xs$ 

(%o7)  $\sqrt{(x1 - xs)^2 + x3^2 + x2^2}$ 

(%i8) /* g1 is symm. metric with indices 1...4 */
g1: matrix(
  [(vs^2*f^2-1),-2*vs*f,0,0],
  [-2*vs*f,1,0,0],
  [0,0,1,0],
  [0,0,0,1]
);
```

$$(\%08) \quad \begin{bmatrix} f^2 \left(\frac{d}{dt} x^s \right)^2 - 1 & -2 f \left(\frac{d}{dt} x^s \right) & 0 & 0 \\ -2 f \left(\frac{d}{dt} x^s \right) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
(%i9) /* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

$$(\%09) \quad \begin{bmatrix} -\frac{1}{3 f^2 \left(\frac{d}{dt} x^s \right)^2 + 1} & -\frac{2 f \left(\frac{d}{dt} x^s \right)}{3 f^2 \left(\frac{d}{dt} x^s \right)^2 + 1} & 0 & 0 \\ -\frac{2 f \left(\frac{d}{dt} x^s \right)}{3 f^2 \left(\frac{d}{dt} x^s \right)^2 + 1} & -\frac{f^2 \left(\frac{d}{dt} x^s \right)^2 - 1}{3 f^2 \left(\frac{d}{dt} x^s \right)^2 + 1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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(%i10)
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```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g[mu,nu]: g1[mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

```
(%i11) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu]) +
            diff(g[rho,mu],x[nu]) -
            diff(g[mu,nu],x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

```
(%i12) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
if Gamma[i,j,k] # 0 then {
display(Gamma[i,j,k])
}}}}$
```

$$\Gamma_{0,0,0} = - \frac{2 f^2 \left(\frac{d}{d t} x s \right) \left(\frac{d^2}{d t^2} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d t} x s \right)^2}{2 \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)} -$$

$$\frac{f \left(\frac{d}{d t} x s \right) \left(- 4 f \left(\frac{d^2}{d t^2} x s \right) - 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d t} x s \right)^2 - 4 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d t} x s \right) \right)}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,0,1} = - \frac{f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,0,2} = \frac{f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x2} r s \right) \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,0,3} = \frac{f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x3} r s \right) \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,1,0} = - \frac{f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,1,1} = \frac{2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d t} x s \right)}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,1,2} = \frac{\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x2} r s \right) \left(\frac{d}{d t} x s \right)}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,1,3} = \frac{\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x3} r s \right) \left(\frac{d}{d t} x s \right)}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}$$

$$\Gamma_{0,2,0} = \frac{f\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x2}rs\right)\left(\frac{d}{d\ t}xs\right)^2}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{0,2,1} = \frac{\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x2}rs\right)\left(\frac{d}{d\ t}xs\right)}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{0,3,0} = \frac{f\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x3}rs\right)\left(\frac{d}{d\ t}xs\right)^2}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{0,3,1} = \frac{\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x3}rs\right)\left(\frac{d}{d\ t}xs\right)}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{1,0,0} = - \frac{f\left(\frac{d}{d\ t}xs\right)\left(2\ f^2\left(\frac{d}{d\ t}xs\right)\left(\frac{d^2}{d\ t^2}xs\right) + 2\ f\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ t}rs\right)\left(\frac{d}{d\ t}xs\right)^2\right)}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1} -$$

$$\frac{\left(f^2\left(\frac{d}{d\ t}xs\right)^2 - 1\right)\left(-4\ f\left(\frac{d^2}{d\ t^2}xs\right) - 2\ f\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x1}rs\right)\left(\frac{d}{d\ t}xs\right)^2 - 4\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ t}rs\right)\left(\frac{d}{d\ t}xs\right)\right)}{2\left(3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1\right)}$$

$$\Gamma_{1,0,1} = - \frac{2\ f^2\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x1}rs\right)\left(\frac{d}{d\ t}xs\right)^3}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{1,0,2} = \frac{\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x2}rs\right)\left(\frac{d}{d\ t}xs\right)\left(f^2\left(\frac{d}{d\ t}xs\right)^2 - 1\right)}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1} - \frac{2\ f^2\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x2}rs\right)\left(\frac{d}{d\ t}xs\right)^3}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{1,0,3} = \frac{\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x3}rs\right)\left(\frac{d}{d\ t}xs\right)\left(f^2\left(\frac{d}{d\ t}xs\right)^2 - 1\right)}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1} - \frac{2\ f^2\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x3}rs\right)\left(\frac{d}{d\ t}xs\right)^3}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{1,1,0} = - \frac{2\ f^2\left(\frac{d}{d\ rs}f\right)\left(\frac{d}{d\ x1}rs\right)\left(\frac{d}{d\ t}xs\right)^3}{3\ f^2\left(\frac{d}{d\ t}xs\right)^2 + 1}$$

$$\Gamma_{1,1,1} = \frac{4 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^2}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,1,2} = \frac{2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,1,3} = \frac{2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,2,0} = \frac{\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right) \left(f^2 \left(\frac{d}{d t} xs \right)^2 - 1 \right)}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1} - \frac{2 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,2,1} = \frac{2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,3,0} = \frac{\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) \left(f^2 \left(\frac{d}{d t} xs \right)^2 - 1 \right)}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1} - \frac{2 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{1,3,1} = \frac{2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2}{3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1}$$

$$\Gamma_{2,0,0} = - f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2$$

$$\Gamma_{2,0,1} = \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)$$

$$\Gamma_{2,1,0} = \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)$$

$$\Gamma_{3,0,0} = - f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2$$

$$\Gamma_{3,0,1} = \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)$$

$$\Gamma_{3,1,0} = \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)$$

```
(%i13) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
R[rho,sigma,mu,nu] :
diff(Gamma[rho,nu,sigma],x[mu]) -
diff(Gamma[rho,mu,sigma],x[nu]) +
/* lambda sums by function call: */
sum(
Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
lambda, 0, 3)
}}}}$
```

```
(%i14) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$\begin{aligned}
R_{0,0,0,1} = & - \left(2 f \left(\frac{d}{dt} x s \right) \left(2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \right. \right. \\
& \left(\frac{d}{d t} x s \right)^4 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \\
& \left(\frac{d}{d t} x s \right)^2 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right) \\
& \left. + 2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{d t} x s \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{0,0,0,2} = & - \left(2 f \left(\frac{d}{d t} x s \right) \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \right. \\
& \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \\
& \left. \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 2} r s \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{0,0,0,3} = & - \left(2 f \left(\frac{d}{dt} x s \right) \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \\
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \\
& \left. \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{0,0,1,0} = & \left(2 f \left(\frac{d}{dt} x s \right) \left(2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \right. \right. \\
& \left(\frac{d}{dt} x s \right)^4 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \left(\frac{d}{dt} x s \right)^3 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \\
& \left(\frac{d}{dt} x s \right)^2 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right) \\
& \left. + 2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \left(\frac{d}{dt} x s \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{0,0,1,2} = & \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \right. \right. \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \\
& \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

[illegible]

[illegible]

$$\left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{dx_3^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{dx_3} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2$$

$$R_{0,1,1,0} = - \left(2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - 6 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 + 6 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right) \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2$$

$$R_{0,1,1,2} = - \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\ \left. \left. \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

$$R_{0,1,1,3} = - \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\ \left. \left. \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

$$R_{0,1,2,0} = - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 \right. \\ \left. + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 \right. \\ \left. + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \right)$$

$$\left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right) \\) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2$$

$$R_{0,1,2,1} = \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\ \left. \left. \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

$$R_{0,1,3,0} = - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 \right. \\ + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\ \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\ \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\ \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\ \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2$$

$$R_{0,1,3,1} = \left(\left(\frac{d}{dt} \mathbf{xS} \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d x3} r s \right) \left(\frac{d}{dt} \mathbf{xS} \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x1} r s \right) \left(\frac{d}{d x3} r s \right) \left(\frac{d}{dt} \mathbf{xS} \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x1 d x3} r s \right) \left(\frac{d}{dt} \mathbf{xS} \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x1} r s \right) \right. \\ \left. \left. \left(\frac{d}{d x3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x1 d x3} r s \right) \right) \right) / \left(3 f^2 \left(\frac{d}{dt} \mathbf{xS} \right)^2 + 1 \right)^2$$

$$\begin{aligned} R_{0,2,0,1} = & \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x_1} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^4 + \right. \\ & 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x_1} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x_1 d x_2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \\ & \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \\ & \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x_2} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x_1} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \\ & \left. \left(\frac{d}{d x_1} rs \right) \left(\frac{d}{d x_2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x_1 d x_2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \right) \end{aligned}$$

[illegible]

[illegible]

$$\begin{aligned}
& \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{0,3,2,1} &= \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
& \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \\
& \left. \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \Big) \Big) \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{0,3,3,0} &= \left(f \left(\frac{d}{d t} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \right. \right. \\
& \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left. \left(\frac{d}{d x^3} r s \right)^2 \Big) \Big) \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{0,3,3,1} &= \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \right. \right. \\
& \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \Big) \\
& \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1,0,0,1} &= - \left(\left(f \left(\frac{d}{d t} x s \right) - 1 \right) \left(f \left(\frac{d}{d t} x s \right) + 1 \right) \left(2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x^1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \right. \right. \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x^1^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^1} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \\
& \left(\frac{d}{d x^1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x^1} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x^1} r s \right) \\
& \left(\frac{d}{d t} x s \right)^3 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x^1^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x^1} r s \right) \left(\frac{d}{d t} x s \right) + 2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x^1} r s \right) \left(\frac{d}{d t} x s \right) \Big) \Big) / \right. \\
& \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \right. \\
R_{1,0,0,2} &= - \left(\left(f \left(\frac{d}{d t} x s \right) - 1 \right) \left(f \left(\frac{d}{d t} x s \right) + 1 \right) \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d^2}{dt dx^2} rs \right) \left(\frac{d}{dt} xs \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 1, 0, 3} = & \left(2 f \left(\frac{d}{dt} xs \right) \left(\frac{d}{d rs} f \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d^2}{dt^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right) \right. \\
& \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1 dx^3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \\
& \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx^3} rs \right) \\
& \left(\frac{d}{dt} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 \\
& + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1 dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \\
& \left. \left(\frac{d^2}{dt dx^3} rs \right) \left(\frac{d}{dt} xs \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 1, 1, 0} = & - \left(2 f \left(\frac{d}{dt} xs \right) \left(2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d^2}{dt^2} xs \right) - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right) \right. \right. \\
& \left(\frac{d}{dt} xs \right)^4 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1^2} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \\
& \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 6 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - 6 f \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 + 6 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \\
& \left(\frac{d}{dt} xs \right)^2 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1^2} rs \right) \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right) \\
& \left. + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 1, 1, 2} = & - \left(2 f \left(\frac{d}{dt} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 - 3 f \right. \right. \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1 dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \\
& \left. \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^2} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1 dx^2} rs \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 1, 1, 3} = & - \left(2 f \left(\frac{d}{dt} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 - 3 f \right. \right. \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1 dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \\
& \left. \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dx^3} rs \right) \right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2
\end{aligned}$$

$$R_{1,2,0,1} = \left(\left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \right. \\ \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \\ \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \\ \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \\ \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \\ \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \\ \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \Big) \Big) / \\ \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2$$

$$R_{1,2,0,2} = \left(\left(\frac{d}{dt} x s \right) \left(3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right. \right. \\ \left. \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \right. \\ \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \right. \\ \left. \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

$$R_{1,2,0,3} = \left(\left(\frac{d}{dt} x s \right) \left(3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2$$

$$R_{1,2,1,0} = - \left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\ \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \right)$$

$$\begin{aligned}
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 2, 1, 2} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \right. \right. \\
& \left. \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + 2 f \right. \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \right)) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 2, 1, 3} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right)) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 2, 2, 0} = & - \left(\left(\frac{d}{d t} x s \right) \left(3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \right. \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + \right. \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right)) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 2, 2, 1} = & \left(\left(\frac{d}{d t} x s \right)^2 \left(6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + 2 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_2} r s \right)^2 \right)) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dx_2} rs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_2} rs \right)^2 \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 2, 3, 0} = & - \left(\left(\frac{d}{dt} xs \right) \left(3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right)^2 \right. \right. \\
& \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_2 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 4 f^2 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_2 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_2 dx_3} rs \right) \Big) \Big) / \\
& \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 2, 3, 1} = & \left(\left(\frac{d}{dt} xs \right)^2 \left(6 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \right. \right. \\
& \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_2 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 2 f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) + 2 f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_2 dx_3} rs \right) \Big) \Big) / \\
& \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 3, 0, 1} = & \left(\left(\frac{d}{dt} xs \right) \left(2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d^2}{dt^2} xs \right) + 3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \right. \right. \\
& \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^4 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 6 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 - 6 f^2 \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 4 f^2 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 2 f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right) + 2 f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_3} rs \right) \Big) \Big) / \\
& \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
R_{1, 3, 0, 2} = & \left(\left(\frac{d}{dt} xs \right) \left(3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right)^2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 3, 0, 3} = & \left(\frac{d}{d t} x s \right) \left(3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right) \right. \\
& \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right)^2 \right)) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 3, 1, 0} = & - \left(\frac{d}{d t} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \\
& \left. \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \right)) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
R_{1, 3, 1, 2} = & - \left(\frac{d}{d t} x s \right)^2 \left(6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \right. \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right)) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{1,3,1,3} = & - \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d^2}{ds^2} f \right) \right. \right. \\
& \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) + 2 f \\
& \left. \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{1,3,2,0} = & - \left(\left(\frac{d}{dt} x s \right) \left(3 f^4 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{ds} f \right)^2 \right. \right. \\
& \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2 dx^3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 4 f^2 \left(\frac{d^2}{ds^2} f \right) \\
& \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{ds} f \right) \\
& \left. \left(\frac{d^2}{dx^2 dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2 dx^3} r s \right) \right) \right) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{1,3,2,1} = & \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{ds} f \right)^2 \right. \right. \\
& \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2 dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d^2}{ds^2} f \right) \\
& \left. \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2 dx^3} r s \right) \right) \right) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{1,3,3,0} = & - \left(\left(\frac{d}{dt} x s \right) \left(3 f^4 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right. \right. \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 4 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \\
& \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) + \\
& \left. \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
R_{1,3,3,1} = & \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) \left(\frac{d}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right. \right. \\
& \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) + 2 f \left(\frac{d^2}{ds^2} f \right) \\
& \left. \left(\frac{d}{dx^3} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / (3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1) \\
R_{2, 1, 0, 2} = & - \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right)) \\
& / (3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1) \\
R_{2, 1, 0, 3} = & - \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \right. \\
& \left. \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right)) / (3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1) \\
R_{2, 1, 1, 0} = & \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right)) / (3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1) \\
R_{2, 1, 1, 2} = & \frac{\left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1} \\
R_{2, 1, 1, 3} = & \frac{\left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1} \\
R_{2, 1, 2, 0} = & \left(\left(\frac{d}{d t} x s \right) \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \right)) \\
& / (3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1) \\
R_{2, 1, 2, 1} = & - \frac{\left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2}{3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1}
\end{aligned}$$

$$R_{2,1,3,0} = \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \right. \\ \left. \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right) \\ R_{2,1,3,1} = - \frac{\left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2}{3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1}$$

$$R_{3,0,0,1} = \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + \right. \\ \left. 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \right. \\ \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \right. \\ \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \right. \\ \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\ \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)$$

$$R_{3,0,0,2} = \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \right. \\ \left. \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right) \\ R_{3,0,0,3} = \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \right. \right. \\ \left. \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \right. \\ \left. \left(\frac{d}{d x_3} r s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)$$

$$R_{3,0,1,0} = - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x^3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x^1} rs \right) \left(\frac{d}{d x^3} rs \right) \left(\frac{d}{d t} xs \right)^4 \right. \\ \left. + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x^1 d x^3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x^3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \right.$$

$$\begin{aligned} & \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\ & \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\ & \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\ &) / (3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1) \end{aligned}$$

$$R_{3,0,1,2} = - \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\ \left. \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \right. \\ \left. \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right) \\ R_{3,0,1,3} = - \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \right. \right. \\ \left. \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \right) \\ / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)$$

$$\begin{aligned}
R_{3,0,2,0} &= - \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
&\quad \left. \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \right. \\
&\quad \left. \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right) \\
R_{3,0,2,1} &= \left(\left(\frac{d}{dt} x s \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
&\quad \left. \left(\frac{d}{d x^2} r s \right) \left(\frac{d}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right) \right. \\
&\quad \left. \left(\frac{d}{d x^3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2 d x^3} r s \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right) \\
R_{3,0,3,0} &= - \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \right. \right. \\
&\quad \left. \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \right. \\
&\quad \left. \left(\frac{d}{d x^3} r s \right)^2 \right)) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)
\end{aligned}$$

$$\left(\frac{d}{dx^2}rs\right)\left(\frac{d}{dx^3}rs\right)\left(\frac{d}{dt}xs\right)^2 + 3 f^2 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^2 dx^3}rs\right)\left(\frac{d}{dt}xs\right)^2 + \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^2}rs\right)$$

$$\left(\frac{d}{dx^3}rs\right) + \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^2 dx^3}rs\right) \Big) \Big) / \left(3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1 \right)$$

$$R_{3,1,2,1} = - \frac{\left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dx^2}rs\right)\left(\frac{d}{dx^3}rs\right)\left(\frac{d}{dt}xs\right)^2}{3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1}$$

$$R_{3,1,3,0} = \left(\left(\frac{d}{dt}xs\right) \left(3 f^2 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^3^2}rs\right)\left(\frac{d}{dt}xs\right)^2 + 3 f^2 \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^3}rs\right)^2 \right. \right.$$

$$\left. \left(\frac{d}{dt}xs\right)^2 - f \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dx^3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^3^2}rs\right) + \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^3}rs\right)^2 \right) \Big)$$

$$/ \left(3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1 \right)$$

$$R_{3,1,3,1} = - \frac{\left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dx^3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2}{3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1}$$

```
(%i15) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i16) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = \left(2 f^2 \left(\frac{d}{d rs}f\right)\left(\frac{d}{dx^1}rs\right)\left(\frac{d}{dt}xs\right)^2 \left(\frac{d^2}{dt^2}xs\right) - 2 \left(\frac{d}{d rs}f\right)\left(\frac{d}{dx^1}rs\right)\left(\frac{d^2}{dt^2}xs\right) - 9 \right.$$

$$f^5 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^3^2}rs\right)\left(\frac{d}{dt}xs\right)^6 - 9 f^5 \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^3}rs\right)^2 \left(\frac{d}{dt}xs\right)^6 - 6 f^4 \left(\frac{d}{d rs}f\right)^2$$

$$\left(\frac{d}{dx^3}rs\right)^2 \left(\frac{d}{dt}xs\right)^6 - 9 f^5 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^2^2}rs\right)\left(\frac{d}{dt}xs\right)^6 - 9 f^5 \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^2}rs\right)^2$$

$$\left(\frac{d}{dt}xs\right)^6 - 6 f^4 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dx^2}rs\right)^2 \left(\frac{d}{dt}xs\right)^6 + 3 f^5 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dx^1^2}rs\right)\left(\frac{d}{dt}xs\right)^6 + 3 f^5$$

$$\left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dx^1}rs\right)^2 \left(\frac{d}{dt}xs\right)^6 + 6 f^4 \left(\frac{d^2}{d rs^2}f\right)\left(\frac{d}{dt}rs\right)\left(\frac{d}{dx^1}rs\right)\left(\frac{d}{dt}xs\right)^5 - 6 f^3 \left(\frac{d}{d rs}f\right)^2$$

$$\left(\frac{d}{dt}rs\right)\left(\frac{d}{dx^1}rs\right)\left(\frac{d}{dt}xs\right)^5 + 6 f^4 \left(\frac{d}{d rs}f\right)\left(\frac{d^2}{dt dx^1}rs\right)\left(\frac{d}{dt}xs\right)^5 - 6 f^3 \left(\frac{d}{d rs}f\right)$$

$$\begin{aligned} & \left(\frac{d^2}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^4 - 6 f^3 \left(\frac{d^2}{dx^3} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + f^2 \left(\frac{d}{dx^3} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \\ & \left(\frac{d}{dt} xs \right)^4 - 6 f^3 \left(\frac{d}{dx^3} f \right) \left(\frac{d^2}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^4 - 6 f^3 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + f^2 \\ & \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 2 f^3 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^4 - 2 f^3 \left(\frac{d^2}{dx^2} f \right) \\ & \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + f^2 \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 4 f^2 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \\ & \left(\frac{d}{dt} xs \right)^3 + 6 f \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - 4 f^2 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 \\ & - f \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d^2}{dx^3} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{dx^3} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \\ & \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{dx^3} f \right) \left(\frac{d^2}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{dx^2} f \right)^2 \\ & \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d^2}{dx^1} f \right) \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - \\ & \left(\frac{d}{dx^1} f \right)^2 \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - 2 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right) - 2 \left(\frac{d}{dx^2} f \right) \\ & \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right) \Big) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\ Ric_{0,1} = & - \left(\frac{d}{dt} xs \right) \left(4 f \left(\frac{d}{dx^2} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d^2}{dt^2} xs \right) - 9 f^4 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^3} rs \right) \right. \\ & \left(\frac{d}{dt} xs \right)^4 - 9 f^4 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 3 f^3 \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 9 \\ & f^4 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^4 - 9 f^4 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 3 f^3 \left(\frac{d}{dx^2} f \right)^2 \\ & \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 6 f^4 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^4 + 6 f^4 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^1} rs \right)^2 \\ & \left(\frac{d}{dt} xs \right)^4 + 12 f^3 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - 12 f^2 \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \\ & \left(\frac{d}{dt} xs \right)^3 + 12 f^3 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - 6 f^2 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 - 6 \\ & f^2 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - 6 f^2 \left(\frac{d}{dx^2} f \right) \\ & \left(\frac{d^2}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 - 6 f^2 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{dx^2} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 \\ & \left. + 2 f^2 \left(\frac{d}{dx^2} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^2 + 2 f^2 \left(\frac{d^2}{dx^2} f \right) \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + 2 f \left(\frac{d}{dx^2} f \right)^2 \right) \end{aligned}$$

[illegible]

$$\begin{aligned}
Ric_{1,0} = & - \left(\left(\frac{d}{dt} x s \right) \left(4 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 9 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 3^2} r s \right) \right. \right. \\
& \left(\frac{d}{d t} x s \right)^4 - 9 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 - 9 \\
& f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2^2} r s \right) \left(\frac{d}{d t} x s \right)^4 - 9 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 6 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right)^2 \\
& \left(\frac{d}{d t} x s \right)^4 + 12 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 12 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \\
& \left(\frac{d}{d t} x s \right)^3 + 12 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 - 6 \\
& f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x 2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 \\
& + 2 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 4 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right) + 4 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \\
& \left(\frac{d}{d t} x s \right) - \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 3^2} r s \right) - \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 3} r s \right)^2 - \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2^2} r s \right) - \left(\frac{d^2}{d r s^2} f \right) \\
& \left. \left(\frac{d}{d x 2} r s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{1,1} = & \left(2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 - 6 f^2 \right. \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \\
& \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{d t} x s \right)^3 - 2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 \\
& - 2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right)^2 \\
& \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d t} x s \right) + 2 \\
& \left. \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{d t} x s \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{2, 1} = & \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 \right. \\
& f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \\
& \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 2} r s \right) \left(\frac{d}{d t} x s \right)^3 - f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \\
& \left. \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 2} r s \right) \left(\frac{d}{d t} x s \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{2, 2} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(9 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 9 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 2} r s \right) \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 3 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2^2} r s \right) + 3 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x 2} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{2, 3} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(9 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right) \right. \right. \\
& \left. \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 9 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{d x 3} r s \right) + 3 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 2 d x 3} r s \right) \right) / \\
& \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \right. \\
Ric_{3, 0} = & - \left(\left(\frac{d}{d t} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \right. \\
& \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \right. \\
& \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \right. \\
& \left. \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{3,1} = & \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 \right. \\
& f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \\
& \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right)^3 - f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \\
& \left. \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right) \right) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{3,2} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(9 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 9 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d x_3} r s \right) + 3 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2 d x_3} r s \right) \right))) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
Ric_{3,3} = & - \left(\left(\frac{d}{d t} x s \right)^2 \left(9 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) \left(\frac{d}{d t} x s \right)^2 + 9 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{d t} x s \right)^2 + 3 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + 3 f \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_3} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \right))) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2
\end{aligned}$$

```
(%i17) /* Ricci Scalar */
```

```

RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
      + sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
      + sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
      + sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;

```

$$\begin{aligned}
(\%o17) \quad & - \left(2 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^2 \left(\frac{d^2}{d t^2} xs \right) - 2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 9 \right. \\
& f^5 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) \left(\frac{d}{d t} xs \right)^6 - 9 f^5 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^6 - 6 f^4 \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^6 - 9 f^5 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^6 - 9 f^5 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^6 - 6 f^4 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^6 + 3 f^5 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^6 + 3 f^5 \\
& \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^6 + 6 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^5 - 6 f^3 \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^5 + 6 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)^5 - 6 f^3 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x3^2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 6 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^4 - 6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 6 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + f^2 \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 2 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 2 f^3 \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \\
& \left(\frac{d}{d t} xs \right)^3 + 6 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - 4 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 \\
& - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right) - 2 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right) \Big) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^3 + \left(4 f \left(\frac{d}{d t} xs \right)^2 \left(4 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \right. \right. \\
& \left. \left(\frac{d^2}{d t^2} xs \right) - 9 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 9 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \right. \\
& \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 9 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 9 f^4 \left(\frac{d^2}{d rs^2} f \right) \right. \\
& \left. \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 6 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt}xs\right)^4 + 6 f^4 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x1}rs\right)^2 \left(\frac{d}{dt}xs\right)^4 + 12 f^3 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{dt}rs\right) \left(\frac{d}{d x1}rs\right) \\
& \left(\frac{d}{dt}xs\right)^3 - 12 f^2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dt}rs\right) \left(\frac{d}{d x1}rs\right) \left(\frac{d}{dt}xs\right)^3 + 12 f^3 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d t d x1}rs\right) \\
& \left(\frac{d}{dt}xs\right)^3 - 6 f^2 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x3^2}rs\right) \left(\frac{d}{dt}xs\right)^2 - 6 f^2 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 - f \\
& \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 - 6 f^2 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x2^2}rs\right) \left(\frac{d}{dt}xs\right)^2 - 6 f^2 \left(\frac{d^2}{d rs^2}f\right) \\
& \left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 - f \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + 2 f^2 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x1^2}rs\right) \left(\frac{d}{dt}xs\right)^2 \\
& + 2 f^2 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x1}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + 2 f \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x1}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + 4 f \left(\frac{d^2}{d rs^2}f\right) \\
& \left(\frac{d}{dt}rs\right) \left(\frac{d}{d x1}rs\right) \left(\frac{d}{dt}xs\right) + 4 f \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d t d x1}rs\right) \left(\frac{d}{dt}xs\right) - \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x3^2}rs\right) - \\
& \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x3}rs\right)^2 - \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x2^2}rs\right) - \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x2}rs\right)^2)) / \left(3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1\right)^3 - \\
& \left(f^2 \left(\frac{d}{dt}xs\right)^2 - 1\right) \left(2 \left(\frac{d}{d rs}f\right) \left(\frac{d}{d x1}rs\right) \left(\frac{d^2}{d t^2}xs\right) - 6 f^2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x3}rs\right)^2 \left(\frac{d}{dt}xs\right)^4 - \right. \\
& 6 f^2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^4 + 3 f^3 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x1^2}rs\right) \left(\frac{d}{dt}xs\right)^4 + 3 f^3 \left(\frac{d^2}{d rs^2}f\right) \\
& \left(\frac{d}{d x1}rs\right)^2 \left(\frac{d}{dt}xs\right)^4 + 6 f^2 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{dt}rs\right) \left(\frac{d}{d x1}rs\right) \left(\frac{d}{dt}xs\right)^3 - 6 f \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{dt}rs\right) \\
& \left(\frac{d}{d x1}rs\right) \left(\frac{d}{dt}xs\right)^3 + 6 f^2 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d t d x1}rs\right) \left(\frac{d}{dt}xs\right)^3 - 2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 \\
& - 2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + f \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x1^2}rs\right) \left(\frac{d}{dt}xs\right)^2 + f \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x1}rs\right)^2 \\
& \left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x1}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + 2 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{dt}rs\right) \left(\frac{d}{d x1}rs\right) \left(\frac{d}{dt}xs\right) + 2 \\
& \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d t d x1}rs\right) \left(\frac{d}{dt}xs\right))) / \left(3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1\right)^3 - \left(\frac{d}{dt}xs\right)^2 \left(9 f^3 \left(\frac{d}{d rs}f\right) \right. \\
& \left(\frac{d^2}{d x3^2}rs\right) \left(\frac{d}{dt}xs\right)^2 + 9 f^3 \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x3}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 - 6 f^2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x3}rs\right)^2 \\
& \left(\frac{d}{dt}xs\right)^2 + 3 f \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x3^2}rs\right) + 3 f \left(\frac{d^2}{d rs^2}f\right) \left(\frac{d}{d x3}rs\right)^2 + \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x3}rs\right)^2)) / \\
& \left(3 f^2 \left(\frac{d}{dt}xs\right)^2 + 1\right)^2 - \left(\frac{d}{dt}xs\right)^2 \left(9 f^3 \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x2^2}rs\right) \left(\frac{d}{dt}xs\right)^2 + 9 f^3 \left(\frac{d^2}{d rs^2}f\right) \right. \\
& \left.\left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 - 6 f^2 \left(\frac{d}{d rs}f\right)^2 \left(\frac{d}{d x2}rs\right)^2 \left(\frac{d}{dt}xs\right)^2 + 3 f \left(\frac{d}{d rs}f\right) \left(\frac{d^2}{d x2^2}rs\right) + 3 f \right.
\end{aligned}$$

$$\left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \Big) \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2$$

```
(%i18) ratsimp(RicSc);
```

$$\begin{aligned} & \left(4 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d^2}{d t^2} x s \right) + \left(- 18 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + \right. \right. \\ & \left(6 f^2 \left(\frac{d}{d r s} f \right)^2 - 18 f^3 \left(\frac{d^2}{d r s^2} f \right) \right) \left(\frac{d}{d x_3} r s \right)^2 - 18 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + \\ & \left(6 f^2 \left(\frac{d}{d r s} f \right)^2 - 18 f^3 \left(\frac{d^2}{d r s^2} f \right) \right) \left(\frac{d}{d x_2} r s \right)^2 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \\ & \left. \left(\frac{d}{d x_1} r s \right)^2 \right) \left(\frac{d}{d t} x s \right)^4 + \\ & \left(\left(12 f^2 \left(\frac{d^2}{d r s^2} f \right) - 12 f \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_1} r s \right) + 12 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \right) \left(\frac{d}{d t} x s \right)^3 + \\ & \left(- 6 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + \left(- 6 f \left(\frac{d^2}{d r s^2} f \right) - 4 \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d x_3} r s \right)^2 - 6 f \left(\frac{d}{d r s} f \right) \right. \\ & \left. \left(\frac{d^2}{d x_2^2} r s \right) + \left(- 6 f \left(\frac{d^2}{d r s^2} f \right) - 4 \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d x_2} r s \right)^2 + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) + \right. \\ & \left. \left(2 f \left(\frac{d^2}{d r s^2} f \right) + 2 \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d x_1} r s \right)^2 \right) \left(\frac{d}{d t} x s \right)^2 + \\ & \left(4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_1} r s \right) + 4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \right) \left(\frac{d}{d t} x s \right) \Big) / \left(9 f^4 \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \right. \\ & \left. \left(\frac{d}{d t} x s \right)^2 + 1 \right) \end{aligned}$$

```
(%i19)
```

```
/* Test for R^q */
for mu: 0 thru 3 do (
for sigma: 0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
  R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
  if R_q # 0 then (
    display("====Einstein equation R^q=0 not fulfilled! "),
    display(mu,sigma,nu,rho),
    display(R_q)
  )
))));
```

```
====Einstein equation R^q=0 not fulfilled! =
```

```
====Einstein equation R^q=0 not fulfilled!
```

```
μ = 0
```

```
σ = 0
```

$$v = 1$$

$$\rho = 2$$

$$\begin{aligned}
 R_q = & - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \right. \\
 & \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\
 & \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
 & \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
 & \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right) \\
 & \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \right. \\
 & \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \\
 & \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
 & \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \\
 & \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \\
 & \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right) \left. \right) / \\
 & \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + \left(2 f \left(\frac{d}{d t} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 - 3 f \right. \right. \\
 & \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
 & \left. \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \right) \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2
 \end{aligned}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 0$$

$$v = 1$$

$$\rho = 3$$

$$R_q = - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \right.$$

$$\begin{aligned}
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\
&) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + (\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \\
& \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \\
& \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \\
& \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)) / \\
& \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{d t} xs \right)^2 (3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right))) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 1$$

$$\begin{aligned}
R_q = & (\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_2} rs \right) \left(\frac{d}{dt} xs \right) \\
&) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d^2}{dt^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \right. \\
& \left. \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \right. \\
& \left. \left(\frac{d^2}{dx_1 dx_2} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \right. \\
& \left. \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \right. \\
& \left. \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \right. \\
& \left. \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_2} rs \right) \left(\frac{d}{dt} xs \right) \right) / \\
& \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 - 3 f \right. \right. \\
& \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
& \left. \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 0$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 1$$

$$\begin{aligned}
R_q = & \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d^2}{dt^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right)
\end{aligned}$$

$$\begin{aligned}
&) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{dr} f \right) \left(\frac{d}{dx} r s \right) \left(\frac{d^2}{dt^2} x s \right) - 3 f^3 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dx} r s \right) \right. \\
& \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^4 - 3 f^3 \left(\frac{d}{dr} f \right) \\
& \left(\frac{d^2}{dx dt} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{dr} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dt dx} r s \right) \left(\frac{d}{dt} x s \right)^3 - f \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dx} r s \right) \\
& \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dx dt dx} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dt dx} r s \right) \left(\frac{d}{dt} x s \right) \Big) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \right. \right. \\
& \left. \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dx dt dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr^2} f \right) \right. \\
& \left. \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) + \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dx dt dx} r s \right) \right) \Big) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 2$$

$$\begin{aligned}
R_q = & \left(\left(\frac{d}{dr} f \right) \left(\frac{d}{dx} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^3 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dx dt} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \\
& \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dt dx} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{dr^2} f \right) \\
& \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{dr} f \right) \\
& \left(\frac{d^2}{dx dt dx} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{dr} f \right) \left(\frac{d^2}{dt dx} r s \right) \left(\frac{d}{dt} x s \right) \Big) \\
&) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{dr} f \right) \left(\frac{d}{dx} r s \right) \left(\frac{d^2}{dt^2} x s \right) - 3 f^3 \left(\frac{d^2}{dr^2} f \right) \left(\frac{d}{dx} r s \right) \right. \\
& \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d}{dr} f \right)^2 \left(\frac{d}{dx} r s \right) \left(\frac{d}{dx} r s \right) \left(\frac{d}{dt} x s \right)^4 - 3 f^3 \left(\frac{d}{dr} f \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d^2}{dx_1 dx_2} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_2} rs \right) \left(\frac{d}{dt} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \\
& \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \\
& \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_2} rs \right) \left(\frac{d}{dt} xs \right)) / \\
& \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 - 3 f \right. \right. \\
& \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
& \left. \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_2} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_2} rs \right) \right)) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 3$$

$$\begin{aligned}
R_q = & \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d^2}{dt^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right) \\
& \left. \right) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d^2}{dt^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \right. \\
& \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx_1} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
& \left. \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx_3} rs \right) \left(\frac{d}{dt} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx_1} rs \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \\
& \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{d t} x s \right) \Big) / \\
& \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{d t} x s \right)^2 - 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f \right. \\
& \left. \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \right) \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 0$$

$$\begin{aligned}
R_q = & - \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right) \\
& \Big) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right)^3 - f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right)^2 - f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \\
& \left. \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{d t} x s \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(2 f \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \right. \\
& \left. \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \right. \\
& \left. \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled! \\
\mu &= 0 \\
\sigma &= 1 \\
v &= 3 \\
\rho &= 0 \\
R_q &= - \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \right. \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right) \\
& \left. \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 - 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right) \left. \right) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(2 f \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \right. \\
& \left. \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \right.
\end{aligned}$$


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( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_3}rs$ ) + ( $\frac{d}{drsf}$ )( $\frac{d^2}{dx_1 dx_3}rs$ ) ) / ( $3 f^2(\frac{d}{dt}xs)^2 + 1$ )^2
=====Einstein equation R^q=0 not fulfilled! =
=====Einstein equation R^q=0 not fulfilled!

μ = 0
σ = 2
ν = 0
ρ = 1

R_q = - ( ( $\frac{d}{drsf}$ )( $\frac{d}{dx_2}rs$ )( $\frac{d^2}{dt^2}xs$ ) + 3 f^3 ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^4 + 3 f^3
( $\frac{d}{drsf}$ )( $\frac{d^2}{dx_1 dx_2}rs$ )( $\frac{d}{dt}xs$ )^4 + 3 f^2 ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 - 3 f
( $\frac{d}{drsf}$ )^2 ( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 + 3 f^2 ( $\frac{d}{drsf}$ )( $\frac{d^2}{dt dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 + f ( $\frac{d^2}{drsf^2}f$ )
( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + ( $\frac{d}{drsf}$ )^2 ( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + f ( $\frac{d}{drsf}$ )
( $\frac{d^2}{dx_1 dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ ) + ( $\frac{d}{drsf}$ )( $\frac{d^2}{dt dx_2}rs$ )( $\frac{d}{dt}xs$ )
) / ( $3 f^2(\frac{d}{dt}xs)^2 + 1$ )^2 + ( ( $\frac{d}{drsf}$ )( $\frac{d}{dx_2}rs$ )( $\frac{d^2}{dt^2}xs$ ) - 3 f^3 ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dx_1}rs$ )
( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^4 + 6 f^2 ( $\frac{d}{drsf}$ )^2 ( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^4 - 3 f^3 ( $\frac{d}{drsf}$ )
( $\frac{d^2}{dx_1 dx_2}rs$ )( $\frac{d}{dt}xs$ )^4 + 3 f^2 ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 - 3 f ( $\frac{d}{drsf}$ )^2
( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 + 3 f^2 ( $\frac{d}{drsf}$ )( $\frac{d^2}{dt dx_2}rs$ )( $\frac{d}{dt}xs$ )^3 - f ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dx_1}rs$ )
( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + ( $\frac{d}{drsf}$ )^2 ( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 - f ( $\frac{d}{drsf}$ )( $\frac{d^2}{dx_1 dx_2}rs$ )
( $\frac{d}{dt}xs$ )^2 + ( $\frac{d^2}{drsf^2}f$ )( $\frac{d}{dt}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ ) + ( $\frac{d}{drsf}$ )( $\frac{d^2}{dt dx_2}rs$ )( $\frac{d}{dt}xs$ ) ) /
( $3 f^2(\frac{d}{dt}xs)^2 + 1$ )^2 + ( 2 f ( $\frac{d}{dt}xs$ )^2 (  $3 f^2(\frac{d^2}{drsf^2}f$ )( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 - 3 f
( $\frac{d}{drsf}$ )^2 ( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + 3 f^2 ( $\frac{d}{drsf}$ )( $\frac{d^2}{dx_1 dx_2}rs$ )( $\frac{d}{dt}xs$ )^2 + ( $\frac{d^2}{drsf^2}f$ )
( $\frac{d}{dx_1}rs$ )( $\frac{d}{dx_2}rs$ ) + ( $\frac{d}{drsf}$ )( $\frac{d^2}{dx_1 dx_2}rs$ ) ) ) / ( $3 f^2(\frac{d}{dt}xs)^2 + 1$ )^2
=====Einstein equation R^q=0 not fulfilled! =
=====Einstein equation R^q=0 not fulfilled!

```

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$\begin{aligned}
 R_q = & \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \right. \\
 & \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\
 & \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
 & \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
 & \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right) \\
 & \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \right. \\
 & \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \\
 & \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
 & \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \\
 & \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \\
 & \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right) \left. \right) / \\
 & \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{d t} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 - 3 f \right. \right. \\
 & \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
 & \left. \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \right) \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 \\
 & =====Einstein equation R^q=0 not fulfilled! = \\
 & =====Einstein equation R^q=0 not fulfilled!
 \end{aligned}$$

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 1$$

$$\begin{aligned}
R_q = & - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \\
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\
& \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \right. \\
& \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \\
& \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \\
& \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \left. \right) / \\
& \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 + \left(2 f \left(\frac{d}{d t} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - 3 f \right. \right. \\
& \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
& \left. \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \right) \left. \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 1$$

$$\rho = 0$$

$$\begin{aligned}
R_q = & \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \right. \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + f \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \\
&) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{d rs} f \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \right. \\
& \left. \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 - 3 f^3 \left(\frac{d}{d rs} f \right) \right. \\
& \left. \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - 3 f \left(\frac{d}{d rs} f \right)^2 \right. \\
& \left. \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right)^3 - f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \right. \\
& \left. \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \right. \\
& \left. \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x3} rs \right) \left(\frac{d}{d t} xs \right) \right) / \\
& \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{d t} xs \right)^2 \left(3 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 - 3 f \right. \right. \\
& \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + 3 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d^2}{d rs^2} f \right) \right. \\
& \left. \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x3} rs \right) + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x3} rs \right) \right)) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^2 \\
& =====Einstein equation R^q=0 not fulfilled! = \\
& =====Einstein equation R^q=0 not fulfilled!
\end{aligned}$$

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 2$$

$$\begin{aligned}
R_q = & \left(\frac{d}{d t} xs \right) \left(2 f \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d^2}{d t^2} xs \right) + 3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \right. \\
& \left. \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1 d x2} rs \right) \right. \\
& \left. \left(\frac{d}{d t} xs \right)^4 + 6 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 - 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x2} rs \right) \right. \\
& \left. \left(\frac{d}{d t} xs \right)^3 + 6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x2} rs \right) \left(\frac{d}{d t} xs \right)^3 + 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d x2} rs \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right) \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \right. \\
& \left. \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \\
& \left. \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right. \\
& \left. \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right))) / \right. \\
& \left. \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \right. \right. \\
& \left. \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \right. \\
& \left. \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \right. \\
& \left. \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \right)
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 3$$

$$\begin{aligned}
R_{-q} = & \left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \right. \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \\
& \left. \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \right)
\end{aligned}$$

$$\begin{aligned} & \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 \\ & + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\ & \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{d t} x s \right) \left(\frac{d}{d x 3} r s \right) \right. \\ & \left. \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \right. \\ & \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \\ & \left. \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \\ & \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \right. \\ & \left. \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d t} x s \right) \left(f \left(\frac{d}{d t} x s \right) - 1 \right) \left(f \left(\frac{d}{d t} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \right. \right. \\ & \left. \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \right. \right. \\ & \left. \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \right) \end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - (\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 1$$

$$\begin{aligned}
R_{-q} = & - (\left(\frac{d}{dt} x s \right) (2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{ds^2} f \right) \\
& \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{ds} f \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{ds} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{ds} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \\
& \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - (\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \\
& \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{ds} f \right) \\
& \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 2$$

$$\begin{aligned}
R_{-q} = & - (\left(\frac{d}{dt} x s \right) (2 f \left(\frac{d}{ds} f \right) \left(\frac{d}{dx_2} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^4 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_2} r s \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - (\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$\mu = 1$

$\sigma = 1$

$\nu = 0$

$\rho = 3$

$$\begin{aligned}
R_{-q} = & - (\left(\frac{d}{dt} x s \right) (2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_3} r s \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \\
& \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - (\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \\
& \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\wedge}q=0$ not fulfilled! =

=====Einstein equation $R^{\wedge}q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 0$$

$$\begin{aligned}
R_{-q} = & (\left(\frac{d}{dt} x s \right) (2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 2} r s \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{dr s^2} f \right) \\
& \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right) \left(\frac{d}{dr s} f \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d^2}{dt^2} x s \right) \right. \\
& + 3 f^3 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^4 \\
& + 3 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{dr s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \\
& \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \right. \right. \\
& \left. \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 \right. \\
& \left. + \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 0$$

$$\begin{aligned}
R_{-q} = & \left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{dr s} f \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^4 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right) \right. \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right)
\end{aligned}$$

$$\begin{aligned} & \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 \\ & + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\ & \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{d t} x s \right) \left(\frac{d}{d x 3} r s \right) \right. \\ & \left. \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \right. \\ & \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \\ & \left. \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \\ & \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \right. \\ & \left. \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d t} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{d t} x s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{d t} x s \right) \left(f \left(\frac{d}{d t} x s \right) - 1 \right) \left(f \left(\frac{d}{d t} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \right. \right. \\ & \left. \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \right. \right. \\ & \left. \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{d t} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{d t} x s \right)^2 + 1 \right)^2 \right) \end{aligned}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q{}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 2$$

$$v = 0$$

$$\rho = 1$$

$$\begin{aligned}
R_- q = & \left(\left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \right. \right. \\
& \left. \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \right. \\
& \left. \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \right. \\
& \left. \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right) \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) \right. \\
& + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 \\
& + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 \\
& + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \right. \right. \\
& - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + \left(\frac{d^2}{d x_1 d x_2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$\begin{aligned}
R_{-q} = & - \left(\frac{d}{dt} x s \right) \left(2 f \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^4 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \right. \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1 d x_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{d x_2} r s \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{dr s^2} f \right) \\
& \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{dr s} f \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^3 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{dr s} f \right) \\
& \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{dr s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \\
& \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx2} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - (\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \\
& \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{dr s} f \right) \\
& \left(\frac{d^2}{dx1 dx2} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx2} r s \right) + \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx2} r s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 1$$

$$\begin{aligned}
R_{-q} = & (\left(\frac{d}{dt} x s \right) (2 f \left(\frac{d}{dr s} f \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^4 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^4 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dx1 dx3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^3 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f^2 \left(\frac{d}{dr s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^3 \left(\frac{d}{dr s} f \right) \left(\frac{d^2}{dt dx3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 4 f^2 \left(\frac{d^2}{dr s^2} f \right) \left(\frac{d}{dx1} r s \right) \left(\frac{d}{dx3} r s \right)
\end{aligned}$$

$$\begin{aligned} & \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 \\ & + 2 f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{ds^2} f \right) \\ & \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right) \left(\frac{d}{dx_3} r s \right) \right. \\ & \left. \left(\frac{d}{ds} f \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d^2}{dt^2} x s \right) + 3 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{ds} f \right) \right. \\ & \left. \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{ds} f \right)^2 \right. \\ & \left. \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \right. \\ & \left. \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right) \right. \\ & \left. \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx_3} r s \right) \left(\frac{d}{dt} x s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) \left(3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \right. \right. \\ & \left. \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{ds} f \right) \right. \\ & \left. \left(\frac{d^2}{dx_1 dx_3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx_1} r s \right) \left(\frac{d}{dx_3} r s \right) + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx_1 dx_3} r s \right))) / \right. \\ & \left. \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \right) \end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 4 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 \\
& + 2 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right) + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + (2 f \left(\frac{d}{dt} x s \right) (\\
& \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d^2}{d t^2} x s \right) + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 - 3 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 + 3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right)^3 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \\
& \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \\
& \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 3} r s \right) \left(\frac{d}{dt} x s \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(\left(\frac{d}{dt} x s \right) \left(f \left(\frac{d}{dt} x s \right) - 1 \right) \left(f \left(\frac{d}{dt} x s \right) + 1 \right) (3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \right. \right. \\
& \left. \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d}{d r s} f \right) \right. \\
& \left. \left(\frac{d^2}{d x 1 d x 3} r s \right) \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{d x 3} r s \right) + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1 d x 3} r s \right) \right))) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

(%o19) done

(%i20) /* Raising of indices,
contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);

$$\begin{aligned}
& (%o20) \left(f^2 \left(\frac{d}{dt} x s \right)^2 - 1 \right) (2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2 \\
& \left(\frac{d}{dt} x s \right)^4 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 2} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x 1^2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \\
& \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x 1} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x 1} r s \right) \left(\frac{d}{dt} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x 1} r s \right) \left(\frac{d}{dt} x s \right)^3 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x 3} r s \right)^2
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{dt} x s \right)^2 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \\
& \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right) \\
& + 2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \left(\frac{d}{dt} x s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^4 - (4 f^2 \left(\frac{d}{dt} x s \right)^2 (2 \\
& \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) \left(\frac{d}{dt} x s \right)^4 + 3 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right)^2 \\
& \left(\frac{d}{dt} x s \right)^4 + 6 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right)^3 - 6 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \\
& \left(\frac{d}{dt} x s \right)^3 + 6 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d t d x_1} r s \right) \left(\frac{d}{dt} x s \right)^3 - \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - \left(\frac{d}{d r s} f \right)^2 \\
& \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_1^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \\
& \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_1} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d}{dt} x s \right) + 2 \left(\frac{d}{d r s} f \right) \\
& \left(\frac{d^2}{d t d x_1} r s \right) \left(\frac{d}{dt} x s \right))) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^4
\end{aligned}$$

$$\begin{aligned}
& (\%021) \quad (3 f \left(\frac{d}{dt} x s \right)^2 (3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2 \\
& \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_2^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_2} r s \right)^2) \\
&) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^3
\end{aligned}$$

$$\begin{aligned}
& (\%022) \quad (3 f \left(\frac{d}{dt} x s \right)^2 (3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right)^2 \\
& \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x_3^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x_3} r s \right)^2) \\
&) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^3
\end{aligned}$$

```

(%i23) R0101: factor(R0101);
      R0202: factor(R0202);
      R0303: factor(R0303);

```

$$\begin{aligned}
& (\%023) \quad - (2 \left(\frac{d}{d r s} f \right) \left(\frac{d}{d x_1} r s \right) \left(\frac{d^2}{d t^2} x s \right) - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x_3} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 - 3 f^2
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x2} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^4 + 3\,f^3 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x1^2} rs \right) \left(\frac{d}{d\,t} xs \right)^4 + 3\,f^3 \left(\frac{d^2}{d\,rs^2} f \right) \\
& \left(\frac{d}{d\,x1} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^4 + 6\,f^2 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,t} rs \right) \left(\frac{d}{d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 - 6\,f \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,t} rs \right) \\
& \left(\frac{d}{d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 + 6\,f^2 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,t\,d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 - \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x3} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 - \\
& \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x2} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 + f \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x1^2} rs \right) \left(\frac{d}{d\,t} xs \right)^2 + f \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x1} rs \right)^2 \\
& \left(\frac{d}{d\,t} xs \right)^2 + \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x1} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 + 2 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,t} rs \right) \left(\frac{d}{d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right) + 2 \\
& \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,t\,d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)) / \left(3\,f^2 \left(\frac{d}{d\,t} xs \right)^2 + 1 \right)^3
\end{aligned}$$

$$\begin{aligned}
& (\%024) \quad \left(3\,f \left(\frac{d}{d\,t} xs \right)^2 - \left(3\,f^2 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x2^2} rs \right) \left(\frac{d}{d\,t} xs \right)^2 + 3\,f^2 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x2} rs \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d\,t} xs \right)^2 - 3\,f \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x2} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 + \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x2^2} rs \right) + \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x2} rs \right)^2 \right) \\
& \left. \right) / \left(3\,f^2 \left(\frac{d}{d\,t} xs \right)^2 + 1 \right)^3
\end{aligned}$$

$$\begin{aligned}
& (\%025) \quad \left(3\,f \left(\frac{d}{d\,t} xs \right)^2 - \left(3\,f^2 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x3^2} rs \right) \left(\frac{d}{d\,t} xs \right)^2 + 3\,f^2 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x3} rs \right)^2 \right. \right. \\
& \left. \left(\frac{d}{d\,t} xs \right)^2 - 3\,f \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x3} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 + \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x3^2} rs \right) + \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x3} rs \right)^2 \right) \\
& \left. \right) / \left(3\,f^2 \left(\frac{d}{d\,t} xs \right)^2 + 1 \right)^3
\end{aligned}$$

```
(%i26) R1010: f(1,0);
        R1212: f(1,2);
        R1313: f(1,3);
```

$$\begin{aligned}
& (\%026) \quad \left(\left(f \left(\frac{d}{d\,t} xs \right) - 1 \right) \left(f \left(\frac{d}{d\,t} xs \right) + 1 \right) \left(f^2 \left(\frac{d}{d\,t} xs \right)^2 - 1 \right) \left(2 \left(\frac{d}{d\,rs} f \right) \left(\frac{d}{d\,x1} rs \right) \right. \right. \\
& \left. \left(\frac{d^2}{d\,t^2} xs \right) - 3\,f^2 \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x3} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^4 - 3\,f^2 \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x2} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^4 + 3\,f^3 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,x1^2} rs \right) \left(\frac{d}{d\,t} xs \right)^4 \right. \\
& \left. + 3\,f^3 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,x1} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^4 + 6\,f^2 \left(\frac{d^2}{d\,rs^2} f \right) \left(\frac{d}{d\,t} rs \right) \left(\frac{d}{d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 - 6\,f \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,t} rs \right) \left(\frac{d}{d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 \right. \\
& \left. + 6\,f^2 \left(\frac{d}{d\,rs} f \right) \left(\frac{d^2}{d\,t\,d\,x1} rs \right) \left(\frac{d}{d\,t} xs \right)^3 - \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x3} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 - \left(\frac{d}{d\,rs} f \right)^2 \left(\frac{d}{d\,x2} rs \right)^2 \left(\frac{d}{d\,t} xs \right)^2 + f \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)) / \\
& \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^4 - (4 f^2 \left(\frac{d}{d t} xs \right)^2 \left(f \left(\frac{d}{d t} xs \right) - 1 \right) \left(f \left(\frac{d}{d t} xs \right) + 1 \right) (2 \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d}{d x1} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \\
& \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - 6 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 + 6 f^2 \\
& \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \\
& \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \\
& \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \\
& \left(\frac{d}{d t} xs \right)) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^4
\end{aligned}$$

$$(\%o27) \quad \left(\left(\frac{d}{d t} xs \right)^2 \left(f^2 \left(\frac{d}{d t} xs \right)^2 - 1 \right) (6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + 6 f^3
\right.$$

$$\begin{aligned}
& \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + 2 f \left(\frac{d}{d rs} f \right) \\
& \left(\frac{d^2}{d x2^2} rs \right) + 2 f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^3 - (\\
& 2 f \left(\frac{d}{d t} xs \right)^2 (3 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^4 + 3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + \\
& 3 f^3 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 4 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) + \left(\frac{d^2}{d rs^2} f \right) \\
& \left(\frac{d}{d x2} rs \right)^2) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^3
\end{aligned}$$

$$(\%o28) \quad \left(\left(\frac{d}{d t} xs \right)^2 \left(f^2 \left(\frac{d}{d t} xs \right)^2 - 1 \right) (6 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) \left(\frac{d}{d t} xs \right)^2 + 6 f^3
\right.$$

$$\left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + 2 f \left(\frac{d}{d rs} f \right)$$

$$\left(\frac{d^2}{dx^3} rs \right) + 2 f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^3 - \left(2 f \left(\frac{d}{dt} xs \right)^2 \left(3 f^4 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^4 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 4 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^3} rs \right) \left(\frac{d}{dt} xs \right)^2 + 4 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^3} rs \right) + \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^3} rs \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^3$$

```
(%i29) R1010: factor(R1010);
      R1212: factor(R1212);
      R1313: factor(R1313);
```

$$\begin{aligned} (%o29) & - \left(\left(f \left(\frac{d}{dt} xs \right) - 1 \right) \left(f \left(\frac{d}{dt} xs \right) + 1 \right) \left(2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d^2}{dt^2} xs \right) - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^4 + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 6 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - 6 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 + 6 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right)^3 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^3} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^1} rs \right) \left(\frac{d}{dt} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^1} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dt} rs \right) \left(\frac{d}{dx^1} rs \right) \left(\frac{d}{dt} xs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dt dx^1} rs \right) \left(\frac{d}{dt} xs \right) \right) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^3 \end{aligned}$$

$$\begin{aligned} (%o30) & - \left(\left(\frac{d}{dt} xs \right)^2 \left(9 f^4 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^4 + 12 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^2} rs \right) \left(\frac{d}{dt} xs \right)^2 + 12 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 - 6 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \left(\frac{d}{dt} xs \right)^2 + 4 f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{dx^2} rs \right) + 4 f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{dx^2} rs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{dx^2} rs \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} xs \right)^2 + 1 \right)^3 \end{aligned}$$

$$\begin{aligned}
(\%031) \quad & - \left(\left(\frac{d}{dt} x s \right)^2 \left(9 f^4 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 12 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) \right. \right. \\
& \left. \left(\frac{d}{dt} x s \right)^2 + 12 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 4 \right. \\
& \left. f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) + 4 f \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \right) \right) / \\
& \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^3
\end{aligned}$$

```
(%i32) R2020: f(2,0);
      R2121: f(2,1);
      R2323: f(2,3);
```

$$\begin{aligned}
(\%032) \quad & \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \right) \\
& / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \right. \right. \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \right. \\
& \left. \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \\
& \frac{\left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 \left(f^2 \left(\frac{d}{dt} x s \right)^2 - 1 \right)}{\left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2} - \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \right. \right. \\
& \left. \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 \right. \\
& \left. + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2
\end{aligned}$$

```
(%034) 0
```

```
(%i35) R2020: factor(R2020);
      R2121: factor(R2121);
      R2323: factor(R2323);
```

$$\begin{aligned}
(\%035) \quad & - \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right. \right. \\
& \left. \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \right)
\end{aligned}$$

$$\left. \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

(%o36) $- \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right)^2 \right. \right.$

$$\left. \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2} r s \right) + 2 f \left(\frac{d^2}{ds^2} f \right) \right.$$

$$\left. \left(\frac{d}{dx^2} r s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

(%o37) 0

(%i38) R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);

(%o38) $\left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right. \right.$

$$\left. \left(\frac{d}{dt} x s \right)^2 + f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right) \right)$$

$$/ \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 - \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \right. \right.$$

$$\left. \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) + \right.$$

$$\left. \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

(%o39) $\frac{\left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 \left(f^2 \left(\frac{d}{dt} x s \right)^2 - 1 \right)}{\left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2} - \left(2 f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{ds} f \right) \right. \right.$

$$\left. \left(\frac{d^2}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - f \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 \right.$$

$$\left. + \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) + \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right) \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2$$

(%o40) 0

(%i41) R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);

(%o41) $- \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \right. \right.$

$$\left(\frac{d}{dt}xs\right)^2 - 3f\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx3^2}rs\right) + \left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx3}rs\right)^2$$

$$) / \left(3f^2\left(\frac{d}{dt}xs\right)^2 + 1\right)^2$$

$$(\%042) - \left(\left(\frac{d}{dt}xs\right)^2 - 6f^3\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx3^2}rs\right)\left(\frac{d}{dt}xs\right)^2 + 6f^3\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx3}rs\right)^2\right.$$

$$\left.\left(\frac{d}{dt}xs\right)^2 - 3f^2\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\left(\frac{d}{dt}xs\right)^2 + 2f\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx3^2}rs\right) + 2f\left(\frac{d^2}{drs^2}f\right)\right.$$

$$\left.\left(\frac{d}{dx3}rs\right)^2 + \left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\right) / \left(3f^2\left(\frac{d}{dt}xs\right)^2 + 1\right)^2$$

(%043) 0

(%i44) /* Coulomb law */
DivE : R0101 + R0202 + R0303;

$$(\%044) - \left(2\left(\frac{d}{drs}f\right)\left(\frac{d}{dx1}rs\right)\left(\frac{d^2}{dt^2}xs\right) - 3f^2\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\left(\frac{d}{dt}xs\right)^4 - 3f^2\right.$$

$$\left.\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx2}rs\right)^2\left(\frac{d}{dt}xs\right)^4 + 3f^3\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx1^2}rs\right)\left(\frac{d}{dt}xs\right)^4 + 3f^3\left(\frac{d^2}{drs^2}f\right)\right.$$

$$\left.\left(\frac{d}{dx1}rs\right)^2\left(\frac{d}{dt}xs\right)^4 + 6f^2\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dt}rs\right)\left(\frac{d}{dx1}rs\right)\left(\frac{d}{dt}xs\right)^3 - 6f\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dt}rs\right)\right.$$

$$\left.\left(\frac{d}{dx1}rs\right)\left(\frac{d}{dt}xs\right)^3 + 6f^2\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dt dx1}rs\right)\left(\frac{d}{dt}xs\right)^3 - \left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\left(\frac{d}{dt}xs\right)^2 - \right.$$

$$\left.\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx2}rs\right)^2\left(\frac{d}{dt}xs\right)^2 + f\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx1^2}rs\right)\left(\frac{d}{dt}xs\right)^2 + f\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx1}rs\right)^2\right.$$

$$\left.\left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx1}rs\right)^2\left(\frac{d}{dt}xs\right)^2 + 2\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dt}rs\right)\left(\frac{d}{dx1}rs\right)\left(\frac{d}{dt}xs\right) + 2\right.$$

$$\left.\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dt dx1}rs\right)\left(\frac{d}{dt}xs\right)\right) / \left(3f^2\left(\frac{d}{dt}xs\right)^2 + 1\right)^3 + \left(3f\left(\frac{d}{dt}xs\right)^2 - 3f^2\left(\frac{d}{drs}f\right)\right.$$

$$\left.\left(\frac{d^2}{dx3^2}rs\right)\left(\frac{d}{dt}xs\right)^2 + 3f^2\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx3}rs\right)^2\left(\frac{d}{dt}xs\right)^2 - 3f\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx3}rs\right)^2\right.$$

$$\left.\left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx3^2}rs\right) + \left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx3}rs\right)^2\right) / \left(3f^2\left(\frac{d}{dt}xs\right)^2 + 1\right)^3 + \left(3f\right.$$

$$\left.\left(\frac{d}{dt}xs\right)^2 - 3f^2\left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx2^2}rs\right)\left(\frac{d}{dt}xs\right)^2 + 3f^2\left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx2}rs\right)^2\left(\frac{d}{dt}xs\right)^2 - 3f\right.$$

$$\left.\left(\frac{d}{drs}f\right)^2\left(\frac{d}{dx2}rs\right)^2\left(\frac{d}{dt}xs\right)^2 + \left(\frac{d}{drs}f\right)\left(\frac{d^2}{dx2^2}rs\right) + \left(\frac{d^2}{drs^2}f\right)\left(\frac{d}{dx2}rs\right)^2\right) /$$

$$\left(3f^2\left(\frac{d}{dt}xs\right)^2 + 1\right)^3$$

```
(%i45) ratsimp(DivE);
```

$$\begin{aligned}
 (%o45) \quad & - \left(2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d^2}{d t^2} xs \right) + \left(- 9 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) + \right. \right. \\
 & \left. \left(6 f^2 \left(\frac{d}{d rs} f \right)^2 - 9 f^3 \left(\frac{d^2}{d rs^2} f \right) \right) \left(\frac{d}{d x3} rs \right)^2 - 9 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) + \right. \\
 & \left. \left(6 f^2 \left(\frac{d}{d rs} f \right)^2 - 9 f^3 \left(\frac{d^2}{d rs^2} f \right) \right) \left(\frac{d}{d x2} rs \right)^2 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \right. \\
 & \left. \left(\frac{d}{d x1} rs \right)^2 \right) \left(\frac{d}{d t} xs \right)^4 + \\
 & \left(\left(6 f^2 \left(\frac{d^2}{d rs^2} f \right) - 6 f \left(\frac{d}{d rs} f \right)^2 \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) + 6 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \right) \left(\frac{d}{d t} xs \right)^3 + \left(- \right. \\
 & 3 f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x3^2} rs \right) + \left(- 3 f \left(\frac{d^2}{d rs^2} f \right) - \left(\frac{d}{d rs} f \right)^2 \right) \left(\frac{d}{d x3} rs \right)^2 - 3 f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x2^2} rs \right) + \\
 & \left(- 3 f \left(\frac{d^2}{d rs^2} f \right) - \left(\frac{d}{d rs} f \right)^2 \right) \left(\frac{d}{d x2} rs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) + \left(f \left(\frac{d^2}{d rs^2} f \right) + \left(\frac{d}{d rs} f \right)^2 \right) \\
 & \left. \left(\frac{d}{d x1} rs \right)^2 \right) \left(\frac{d}{d t} xs \right)^2 + \left(2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \right) \left(\frac{d}{d t} xs \right) \right) / \left(\right. \\
 & \left. 27 f^6 \left(\frac{d}{d t} xs \right)^6 + 27 f^4 \left(\frac{d}{d t} xs \right)^4 + 9 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)
 \end{aligned}$$

```
(%i46) /* J[r] */
        Jr : -(R1010 + R1212 + R1313);
```

$$\begin{aligned}
 (%o46) \quad & \left(\left(f \left(\frac{d}{d t} xs \right) - 1 \right) \left(f \left(\frac{d}{d t} xs \right) + 1 \right) \left(2 \left(\frac{d}{d rs} f \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d^2}{d t^2} xs \right) - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \right. \right. \\
 & \left. \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 - 3 f^2 \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \right. \\
 & \left. \left(\frac{d}{d t} xs \right)^4 + 3 f^3 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^4 + 6 f^2 \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 \right. \\
 & \left. - 6 f \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 + 6 f^2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right)^3 - \right. \\
 & \left. \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x3} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 - \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x2} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d x1^2} rs \right) \right. \\
 & \left. \left(\frac{d}{d t} xs \right)^2 + f \left(\frac{d^2}{d rs^2} f \right) \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + \left(\frac{d}{d rs} f \right)^2 \left(\frac{d}{d x1} rs \right)^2 \left(\frac{d}{d t} xs \right)^2 + 2 \left(\frac{d^2}{d rs^2} f \right) \right. \\
 & \left. \left(\frac{d}{d t} rs \right) \left(\frac{d}{d x1} rs \right) \left(\frac{d}{d t} xs \right) + 2 \left(\frac{d}{d rs} f \right) \left(\frac{d^2}{d t d x1} rs \right) \left(\frac{d}{d t} xs \right) \right) \right) / \left(3 f^2 \left(\frac{d}{d t} xs \right)^2 + 1 \right)^3 + \left(\right.
 \end{aligned}$$

$$\begin{aligned} & \left(\frac{d}{dt} x s \right)^2 \left(9 f^4 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 12 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 12 \right. \\ & f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 6 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 4 f \left(\frac{d}{ds} f \right) \\ & \left. \left(\frac{d^2}{dx^3^2} r s \right) + 4 f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^3} r s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 + 1 \right)^3 + \left(\right. \\ & \left. \left(\frac{d}{dt} x s \right)^2 \left(9 f^4 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^4 + 12 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 12 \right. \right. \\ & \left. f^3 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 6 f^2 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 4 f \left(\frac{d}{ds} f \right) \right. \\ & \left. \left. \left(\frac{d^2}{dx^2^2} r s \right) + 4 f \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dx^2} r s \right)^2 + \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 \right) \right) \left(\frac{d}{dt} x s \right)^2 + 1 \right)^3 \end{aligned}$$

(%i47) ratsimp(Jr);

$$\begin{aligned} & \left(2 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d}{dx^1} r s \right) \left(\frac{d}{dt} x s \right)^2 - 2 \left(\frac{d}{ds} f \right) \left(\frac{d}{dx^1} r s \right) \right) \left(\frac{d^2}{dt^2} x s \right) + \left(6 f^4 \right. \\ & \left. \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^3} r s \right)^2 + 6 f^4 \left(\frac{d}{ds} f \right)^2 \left(\frac{d}{dx^2} r s \right)^2 + 3 f^5 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^1^2} r s \right) + 3 f^5 \left(\frac{d^2}{ds^2} f \right) \right. \\ & \left. \left(\frac{d}{dx^1} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^6 + \\ & \left(\left(6 f^4 \left(\frac{d^2}{ds^2} f \right) - 6 f^3 \left(\frac{d}{ds} f \right)^2 \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx^1} r s \right) + 6 f^4 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx^1} r s \right) \right) \left(\frac{d}{dt} x s \right)^5 + \left(\right. \\ & 12 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3^2} r s \right) + \left(12 f^3 \left(\frac{d^2}{ds^2} f \right) - 4 f^2 \left(\frac{d}{ds} f \right)^2 \right) \left(\frac{d}{dx^3} r s \right)^2 + 12 f^3 \left(\frac{d}{ds} f \right) \\ & \left. \left(\frac{d^2}{dx^2^2} r s \right) + \left(12 f^3 \left(\frac{d^2}{ds^2} f \right) - 4 f^2 \left(\frac{d}{ds} f \right)^2 \right) \left(\frac{d}{dx^2} r s \right)^2 - 2 f^3 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^1^2} r s \right) + \right. \\ & \left. \left(f^2 \left(\frac{d}{ds} f \right)^2 - 2 f^3 \left(\frac{d^2}{ds^2} f \right) \right) \left(\frac{d}{dx^1} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^4 + \\ & \left(\left(6 f \left(\frac{d}{ds} f \right)^2 - 4 f^2 \left(\frac{d^2}{ds^2} f \right) \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx^1} r s \right) - 4 f^2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx^1} r s \right) \right) \left(\frac{d}{dt} x s \right)^3 + \left(4 \right. \\ & f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^3^2} r s \right) + \left(4 f \left(\frac{d^2}{ds^2} f \right) + 2 \left(\frac{d}{ds} f \right)^2 \right) \left(\frac{d}{dx^3} r s \right)^2 + 4 f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^2^2} r s \right) + \\ & \left. \left(4 f \left(\frac{d^2}{ds^2} f \right) + 2 \left(\frac{d}{ds} f \right)^2 \right) \left(\frac{d}{dx^2} r s \right)^2 - f \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dx^1^2} r s \right) + \left(- f \left(\frac{d^2}{ds^2} f \right) - \left(\frac{d}{ds} f \right)^2 \right) \right. \\ & \left. \left(\frac{d}{dx^1} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 + \left(- 2 \left(\frac{d^2}{ds^2} f \right) \left(\frac{d}{dt} r s \right) \left(\frac{d}{dx^1} r s \right) - 2 \left(\frac{d}{ds} f \right) \left(\frac{d^2}{dt dx^1} r s \right) \right) \left(\frac{d}{dt} x s \right) \right) / \end{aligned}$$

$$\left(27 f^6 \left(\frac{d}{dt} x s \right)^6 + 27 f^4 \left(\frac{d}{dt} x s \right)^4 + 9 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)$$

```
(%i48) /* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

$$\begin{aligned} & \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right. \right. \\ & \left. \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + 2 f \left(\frac{d^2}{d r s^2} f \right) \right. \\ & \left. \left(\frac{d}{d x^2} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^2} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \right. \right. \\ & \left. \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \\ & \left. \left(\frac{d}{d x^2} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \end{aligned}$$

```
(%i49) ratsimp(Jtheta);
```

$$\begin{aligned} & \left(\left(9 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(9 f^3 \left(\frac{d^2}{d r s^2} f \right) - 6 f^2 \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^4 + \right. \\ & \left. \left(3 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^2} r s \right) + \left(3 f \left(\frac{d^2}{d r s^2} f \right) + \left(\frac{d}{d r s} f \right)^2 \right) \left(\frac{d}{d x^2} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 \right) / \left(9 f^4 \left(\frac{d}{dt} x s \right)^4 \right. \\ & \left. + 6 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right) \end{aligned}$$

```
(%i50) /* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

$$\begin{aligned} & \left(\left(\frac{d}{dt} x s \right)^2 \left(6 f^3 \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 6 f^3 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \right. \right. \\ & \left. \left(\frac{d}{dt} x s \right)^2 - 3 f^2 \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + 2 f \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) + 2 f \left(\frac{d^2}{d r s^2} f \right) \right. \\ & \left. \left(\frac{d}{d x^3} r s \right)^2 + \left(\frac{d}{d r s} f \right)^2 \left(\frac{d}{d x^3} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 + \left(f \left(\frac{d}{dt} x s \right)^2 \left(3 f^2 \right. \right. \\ & \left. \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) \left(\frac{d}{dt} x s \right)^2 + 3 f^2 \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 - 3 f \left(\frac{d}{d r s} f \right)^2 \right. \\ & \left. \left(\frac{d}{d x^3} r s \right)^2 \left(\frac{d}{dt} x s \right)^2 + \left(\frac{d}{d r s} f \right) \left(\frac{d^2}{d x^3} r s \right) + \left(\frac{d^2}{d r s^2} f \right) \left(\frac{d}{d x^3} r s \right)^2 \right) \left(\frac{d}{dt} x s \right)^2 \right) / \left(3 f^2 \left(\frac{d}{dt} x s \right)^2 + 1 \right)^2 \end{aligned}$$

```
(%i51) ev(ratsimp(Jphi),r);
```

$$\begin{aligned}
& (\%o51) \quad \left(\left(9 f^3 \left(\frac{d}{d \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2}} f \right) \left(\frac{d}{d t} x s \right)^4 + 3 f \right. \right. \\
& \left. \left(\frac{d}{d \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2}} f \right) \left(\frac{d}{d t} x s \right)^2 \right) \left(\frac{d^2}{d x_3^2} \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2} \right) + \\
& \left. \left(9 f^3 \left(\frac{d^2}{d (x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2)} f \right) - 6 f^2 \left(\frac{d}{d \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2}} f \right)^2 \right) \right. \\
& \left. \left(\frac{d}{d t} x s \right)^4 + \left(3 f \left(\frac{d^2}{d (x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2)} f \right) + \left(\frac{d}{d \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2}} f \right)^2 \right) \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 \right) \left(\frac{d}{d x_3} \sqrt{x s^2 - 2 x_1 x s + x_3^2 + x_2^2 + x_1^2} \right)^2 \Big) / \left(9 f^4 \left(\frac{d}{d t} x s \right)^4 + 6 f^2 \right. \\
& \left. \left(\frac{d}{d t} x s \right)^2 + 1 \right)
\end{aligned}$$

(%i52)