(%i1)
```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```
(%o1) $$f(i,j) := sum(R_{i,j,\sigma,0}\,gContr_{i,\sigma}\,gContr_{j,0},\sigma,0,3) + sum(R_{i,j,\sigma,1}\,gContr_{i,\sigma}\,gContr_{j,1},\sigma,0,3) + sum(R_{i,j,\sigma,2}\,gContr_{i,\sigma}\,gContr_{j,2},\sigma,0,3) + sum(R_{i,j,\sigma,3}\,gContr_{i,\sigma}\,gContr_{j,3},\sigma,0,3)$$

(%i2)
```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```
(%o2) x

(%o3) $[t, r, \theta, \varphi]$

(%i4)
```
depends([C],[r]);
```
(%o4) $[C(r)]$

(%i5)
```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
 [-A*C^(1/2),0,0,0],
 [0,B*C^(1/2),0,0],
 [0,0,C,0],
 [0,0,0,C*sin(theta)^2]
);
```
(%o5)
$$\begin{bmatrix} -A\sqrt{C} & 0 & 0 & 0 \\ 0 & B\sqrt{C} & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & \sin(\theta)^2 C \end{bmatrix}$$

(%i6)
```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o6)
$$\begin{bmatrix} -\frac{1}{A\sqrt{C}} & 0 & 0 & 0 \\ 0 & \frac{1}{B\sqrt{C}} & 0 & 0 \\ 0 & 0 & \frac{1}{C} & 0 \\ 0 & 0 & 0 & \frac{1}{\sin(\theta)^2\, C} \end{bmatrix}$$

(%i7)
```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g     [mu,nu]: g1     [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

(%i8)
```
/* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
        diff(g[nu,rho],x[mu] ) +
        diff(g[rho,mu],x[nu] ) -
        diff(g[mu,nu] ,x[rho])),
    rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

(%i9)
```
/* dislplay Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
}}}}$
```

$$\Gamma_{0,0,1} = \frac{\frac{d}{dr}C}{4\,C}$$

$$\Gamma_{0,1,0} = \frac{\frac{d}{dr}C}{4\,C}$$

$$\Gamma_{1,0,0} = \frac{A\left(\frac{d}{dr}C\right)}{4\,B\,C}$$

$$\Gamma_{1,1,1} = \frac{\frac{d}{dr}C}{4C}$$

$$\Gamma_{1,2,2} = -\frac{\frac{d}{dr}C}{2B\sqrt{C}}$$

$$\Gamma_{1,3,3} = -\frac{sin(\theta)^2\left(\frac{d}{dr}C\right)}{2B\sqrt{C}}$$

$$\Gamma_{2,1,2} = \frac{\frac{d}{dr}C}{2C}$$

$$\Gamma_{2,2,1} = \frac{\frac{d}{dr}C}{2C}$$

$$\Gamma_{2,3,3} = -cos(\theta)sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{\frac{d}{dr}C}{2C}$$

$$\Gamma_{3,2,3} = \frac{cos(\theta)}{sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{\frac{d}{dr}C}{2C}$$

$$\Gamma_{3,3,2} = \frac{cos(\theta)}{sin(\theta)}$$

```
(%i10)  /* compute Riemann tensor elements */
        for rho:0 thru 3 do {
        for sigma:0 thru 3 do {
        for mu:0 thru 3 do {
        for nu:0 thru 3 do {
            R[rho,sigma,mu,nu] :
            diff(Gamma[rho,nu,sigma],x[mu]) -
            diff(Gamma[rho,mu,sigma],x[nu]) +
            /* lambda sums by function call: */
            sum(
                Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
                Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
                lambda, 0, 3)
        }}}}$
```

```
(%i11)  /* dislplay R's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
        for k:0 thru 3 do {
        for l:0 thru 3 do {
            R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
            if R[i,j,k,l] # 0 then display(R[i,j,k,l])
            }}}}$
```

$$R_{0,1,0,1} = -\frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4C^2}$$

$$R_{0,1,1,0} = \frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4C^2}$$

$$R_{0,2,0,2} = -\frac{\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{0,2,2,0} = \frac{\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{0,3,0,3} = -\frac{sin(\theta)^2\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{0,3,3,0} = \frac{sin(\theta)^2\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{1,0,0,1} = -\frac{A\left(C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2\right)}{4BC^2}$$

$$R_{1,0,1,0} = \frac{A\left(C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2\right)}{4BC^2}$$

$$R_{1,2,1,2} = -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{1,2,2,1} = \frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8BC^{3/2}}$$

$$R_{1,3,1,3} = -\frac{\sin(\theta)^2\left(4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2\right)}{8\,B\,C^{3/2}}$$

$$R_{1,3,3,1} = \frac{\sin(\theta)^2\left(4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2\right)}{8\,B\,C^{3/2}}$$

$$R_{2,0,0,2} = -\frac{A\left(\frac{d}{dr}C\right)^2}{8\,B\,C^2}$$

$$R_{2,0,2,0} = \frac{A\left(\frac{d}{dr}C\right)^2}{8\,B\,C^2}$$

$$R_{2,1,1,2} = \frac{4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2}{8\,C^2}$$

$$R_{2,1,2,1} = -\frac{4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2}{8\,C^2}$$

$$R_{2,3,2,3} = -\frac{\sin(\theta)^2\left(\left(\frac{d}{dr}C\right)^2 - 4\,B\,C^{3/2}\right)}{4\,B\,C^{3/2}}$$

$$R_{2,3,3,2} = \frac{\sin(\theta)^2\left(\left(\frac{d}{dr}C\right)^2 - 4\,B\,C^{3/2}\right)}{4\,B\,C^{3/2}}$$

$$R_{3,0,0,3} = -\frac{A\left(\frac{d}{dr}C\right)^2}{8\,B\,C^2}$$

$$R_{3,0,3,0} = \frac{A\left(\frac{d}{dr}C\right)^2}{8\,B\,C^2}$$

$$R_{3,1,1,3} = \frac{4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2}{8\,C^2}$$

$$R_{3,1,3,1} = -\frac{4\,C\left(\frac{d^2}{dr^2}C\right) - 3\left(\frac{d}{dr}C\right)^2}{8\,C^2}$$

$$R_{3,2,2,3} = \frac{\left(\frac{d}{dr}C\right)^2 - 4\,B\,C^{3/2}}{4\,B\,C^{3/2}}$$

$$R_{3,2,3,2} = -\frac{\left(\frac{d}{dr}C\right)^2 - 4BC^{3/2}}{4BC^{3/2}}$$

```
(%i12)  /* Ricci tensor Ric[mu,nu] */
        for mu:0 thru 3 do {
        for nu:0 thru 3 do {
            Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
        }}$
```

```
(%i13)  /* dislplay Ric's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
            Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
            if Ric[i,j] # 0 then display(Ric[i,j])
            }}$
```

$$Ric_{0,0} = \frac{A\left(\frac{d^2}{dr^2}C\right)}{4BC}$$

$$Ric_{1,1} = -\frac{5C\left(\frac{d^2}{dr^2}C\right) - 4\left(\frac{d}{dr}C\right)^2}{4C^2}$$

$$Ric_{2,2} = -\frac{\frac{d^2}{dr^2}C - 2B\sqrt{C}}{2B\sqrt{C}}$$

$$Ric_{3,3} = -\frac{sin(\theta)^2\left(\frac{d^2}{dr^2}C - 2B\sqrt{C}\right)}{2B\sqrt{C}}$$

```
(%i14)  /* Ricci Scalar */
        RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
            +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
            +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
            +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
        ;
```

$$(\%o14)\quad -\frac{5C\left(\frac{d^2}{dr^2}C\right) - 4\left(\frac{d}{dr}C\right)^2}{4BC^{5/2}} - \frac{\frac{d^2}{dr^2}C - 2B\sqrt{C}}{BC^{3/2}} - \frac{\frac{d^2}{dr^2}C}{4BC^{3/2}}$$

```
(%i15)  ratsimp(RicSc);
```

$$(\%o15)\quad \frac{-5C\left(\frac{d^2}{dr^2}C\right) + 2\left(\frac{d}{dr}C\right)^2 + 4BC^{3/2}}{2BC^{5/2}}$$

```
(%i16) /* Raising of indices,
          contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
       /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

       R0101: f(0,1);
       R0202: f(0,2);
       R0303: f(0,3);
```

$$(\%o16)\quad \frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4ABC^3}$$

$$(\%o17)\quad \frac{\left(\frac{d}{dr}C\right)^2}{8ABC^3}$$

$$(\%o18)\quad \frac{\left(\frac{d}{dr}C\right)^2}{8ABC^3}$$

```
(%i19) R0101: factor(R0101);
       R0202: factor(R0202);
       R0303: factor(R0303);
```

$$(\%o19)\quad \frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4ABC^3}$$

$$(\%o20)\quad \frac{\left(\frac{d}{dr}C\right)^2}{8ABC^3}$$

$$(\%o21)\quad \frac{\left(\frac{d}{dr}C\right)^2}{8ABC^3}$$

```
(%i22) R1010: f(1,0);
       R1212: f(1,2);
       R1313: f(1,3);
```

$$(\%o22)\quad -\frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4B^2C^3}$$

$$(\%o23)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8B^2C^3}$$

$$(\%o24)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8B^2C^3}$$

```
(%i25) R1010: factor(R1010);
       R1212: factor(R1212);
       R1313: factor(R1313);
```

$$(\%o25)\quad -\frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4B^2C^3}$$

$$(\%o26)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8B^2C^3}$$

$$(\%o27)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8B^2C^3}$$

```
(%i28) R2020: f(2,0);
       R2121: f(2,1);
       R2323: f(2,3);
```

$$(\%o28)\quad -\frac{\left(\frac{d}{dr}C\right)^2}{8BC^{7/2}}$$

$$(\%o29)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8BC^{7/2}}$$

$$(\%o30)\quad -\frac{\left(\frac{d}{dr}C\right)^2-4BC^{3/2}}{4BC^{7/2}}$$

```
(%i31) R2020: factor(R2020);
       R2121: factor(R2121);
       R2323: factor(R2323);
```

$$(\%o31)\quad -\frac{\left(\frac{d}{dr}C\right)^2}{8BC^{7/2}}$$

$$(\%o32)\quad -\frac{4C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8BC^{7/2}}$$

$$(\%o33)\quad -\frac{\left(\frac{d}{dr}C\right)^2-4BC^{3/2}}{4BC^{7/2}}$$

```
(%i34) R3030: f(3,0);
       R3131: f(3,1);
       R3232: f(3,2);
```

$$(\%o34)\quad -\frac{\left(\frac{d}{dr}C\right)^2}{8\,sin(\theta)^2\,B\,C^{7/2}}$$

$$(\%o35)\quad -\frac{4\,C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8\,sin(\theta)^2\,B\,C^{7/2}}$$

$$(\%o36)\quad -\frac{\left(\frac{d}{dr}C\right)^2-4\,B\,C^{3/2}}{4\,sin(\theta)^2\,B\,C^{7/2}}$$

```
(%i37) R3030: factor(R3030);
       R3131: factor(R3131);
       R3232: factor(R3232);
```

$$(\%o37)\quad -\frac{\left(\frac{d}{dr}C\right)^2}{8\,sin(\theta)^2\,B\,C^{7/2}}$$

$$(\%o38)\quad -\frac{4\,C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8\,sin(\theta)^2\,B\,C^{7/2}}$$

$$(\%o39)\quad -\frac{\left(\frac{d}{dr}C\right)^2-4\,B\,C^{3/2}}{4\,sin(\theta)^2\,B\,C^{7/2}}$$

```
(%i40) /* Coulomb law */
       DivE : R0101 + R0202 + R0303;
```

$$(\%o40)\quad \frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4\,A\,B\,C^3}+\frac{\left(\frac{d}{dr}C\right)^2}{4\,A\,B\,C^3}$$

```
(%i41) ev(ratsimp(DivE),diff);
```

$$(\%o41)\quad \frac{\frac{d^2}{dr^2}C}{4\,A\,B\,C^2}$$

```
(%i42) /* J[r] */
       Jr : -(R1010 + R1212 + R1313);
```

(%o42) $$\frac{4\,C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{4\,B^2\,C^3}+\frac{C\left(\frac{d^2}{dr^2}C\right)-\left(\frac{d}{dr}C\right)^2}{4\,B^2\,C^3}$$

(%i43)
```
ratsimp(Jr);
```

(%o43) $$\frac{5\,C\left(\frac{d^2}{dr^2}C\right)-4\left(\frac{d}{dr}C\right)^2}{4\,B^2\,C^3}$$

(%i44)
```
/* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

(%o44) $$\frac{4\,C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8\,B\,C^{7/2}}+\frac{\left(\frac{d}{dr}C\right)^2-4\,B\,C^{3/2}}{4\,B\,C^{7/2}}+\frac{\left(\frac{d}{dr}C\right)^2}{8\,B\,C^{7/2}}$$

(%i45)
```
ratsimp(Jtheta);
```

(%o45) $$-\frac{2\,B\sqrt{C}-\frac{d^2}{dr^2}C}{2\,B\,C^{5/2}}$$

(%i46)
```
/* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

(%o46) $$\frac{4\,C\left(\frac{d^2}{dr^2}C\right)-3\left(\frac{d}{dr}C\right)^2}{8\,\sin(\theta)^2\,B\,C^{7/2}}+\frac{\left(\frac{d}{dr}C\right)^2-4\,B\,C^{3/2}}{4\,\sin(\theta)^2\,B\,C^{7/2}}+\frac{\left(\frac{d}{dr}C\right)^2}{8\,\sin(\theta)^2\,B\,C^{7/2}}$$

(%i47)
```
ratsimp(Jphi);
```

(%o47) $$-\frac{2\,B\sqrt{C}-\frac{d^2}{dr^2}C}{2\,\sin(\theta)^2\,B\,C^{5/2}}$$

(%i48)