(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $f(i,j) := sum(R_{i,j,\sigma,0}\, gContr_{i,\sigma}\, gContr_{j,0}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,1}\, gContr_{i,\sigma}\, gContr_{j,1}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,2}\, gContr_{i,\sigma}\, gContr_{j,2}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,3}\, gContr_{i,\sigma}\, gContr_{j,3}, \sigma, 0, 3)$

(%i2)

```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

(%o2) x

(%o3) $[t, r, \theta, \varphi]$

(%i4)

```
/* define coordinate dependent functions */
depends([alpha,beta], [t,r]);
```

(%o4) $[\alpha(t,r), \beta(t,r)]$

(%i5)

```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
 [-exp(2*alpha),0,0,0],
 [0,exp(2*beta),0,0],
 [0,0,r^2,0],
 [0,0,0,r^2*sin(theta)^2]
);
```

(%o5) $$\begin{bmatrix} -\%e^{2\alpha} & 0 & 0 & 0 \\ 0 & \%e^{2\beta} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

(%i6)

```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o6) $$\begin{bmatrix} -\%e^{-2\alpha} & 0 & 0 & 0 \\ 0 & \%e^{-2\beta} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2 \sin(\theta)^2} \end{bmatrix}$$

(%i7)

```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g      [mu,nu]: g1      [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

(%i8)

```
/* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
        diff(g[nu,rho],x[mu] ) +
        diff(g[rho,mu],x[nu] ) -
        diff(g[mu,nu] ,x[rho])),
    rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

(%i9)

```
/* dislplay Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$
```

$$\Gamma_{0,0,0} = \frac{d}{d\,t}\alpha$$

$$\Gamma_{0,0,1} = \frac{d}{d\,r}\alpha$$

$$\Gamma_{0,1,0} = \frac{d}{d\,r}\alpha$$

$$\Gamma_{0,1,1} = \%e^{2\,\beta-2\,\alpha}\left(\frac{d}{d\,t}\beta\right)$$

$$\Gamma_{1,0,0} = \left(\frac{d}{d\,r}\alpha\right)\%e^{2\,\alpha-2\,\beta}$$

$$\Gamma_{1,0,1} = \frac{d}{d\,t}\beta$$

$$\Gamma_{1,1,0} = \frac{d}{d\,t}\beta$$

$$\Gamma_{1,1,1} = \frac{d}{d\,r}\beta$$

$$\Gamma_{1,2,2} = - \%e^{-2\beta}\, r$$

$$\Gamma_{1,3,3} = - \%e^{-2\beta}\, r\, \sin(\theta)^2$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,3} = - \cos(\theta) \sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{1}{r}$$

$$\Gamma_{3,2,3} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{1}{r}$$

$$\Gamma_{3,3,2} = \frac{\cos(\theta)}{\sin(\theta)}$$

```
(%i10) /* compute Riemann tensor elements */
       for rho:0 thru 3 do {
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           R[rho,sigma,mu,nu] :
           diff(Gamma[rho,nu,sigma],x[mu]) -
           diff(Gamma[rho,mu,sigma],x[nu]) +
           /* lambda sums by function call: */
           sum(
               Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
               Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
               lambda, 0, 3)
       }}}}$
```

```
(%i11) /* dislplay R's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
       for k:0 thru 3 do {
       for l:0 thru 3 do {
           R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
           if R[i,j,k,l] # 0 then display(R[i,j,k,l])
           }}}}$
```

$$R_{0,1,0,1} = \%e^{-2\alpha} \left(\%e^{2\beta} \left(\frac{d^2}{d t^2}\beta\right) + \%e^{2\beta} \left(\frac{d}{d t}\beta\right)^2 - \left(\frac{d}{d t}\alpha\right) \%e^{2\beta} \left(\frac{d}{d t}\beta\right) + \%e^{2\alpha} \left(\frac{d}{d r}\alpha\right)\left(\frac{d}{d r}\beta\right) - \%e^{2\alpha} \left(\frac{d^2}{d r^2}\alpha\right) - \%e^{2\alpha} \left(\frac{d}{d r}\alpha\right)^2 \right)$$

$$R_{0,1,1,0} = - \%e^{-2\alpha} \left(\%e^{2\beta} \left(\frac{d^2}{d t^2}\beta\right) + \%e^{2\beta} \left(\frac{d}{d t}\beta\right)^2 - \left(\frac{d}{d t}\alpha\right) \%e^{2\beta} \left(\frac{d}{d t}\beta\right) + \%e^{2\alpha} \right.$$

$$\left(\frac{d}{d r}\alpha\right)\left(\frac{d}{d r}\beta\right) - \%e^{2\alpha}\left(\frac{d^2}{d r^2}\alpha\right) - \%e^{2\alpha}\left(\frac{d}{d r}\alpha\right)^2)$$

$$R_{0,2,0,2} = -\left(\frac{d}{d r}\alpha\right)\%e^{-2\beta}\, r$$

$$R_{0,2,1,2} = -\%e^{-2\alpha}\left(\frac{d}{d t}\beta\right) r$$

$$R_{0,2,2,0} = \left(\frac{d}{d r}\alpha\right)\%e^{-2\beta}\, r$$

$$R_{0,2,2,1} = \%e^{-2\alpha}\left(\frac{d}{d t}\beta\right) r$$

$$R_{0,3,0,3} = -\left(\frac{d}{d r}\alpha\right)\%e^{-2\beta}\, r\, sin(\theta)^2$$

$$R_{0,3,1,3} = -\%e^{-2\alpha}\left(\frac{d}{d t}\beta\right) r\, sin(\theta)^2$$

$$R_{0,3,3,0} = \left(\frac{d}{d r}\alpha\right)\%e^{-2\beta}\, r\, sin(\theta)^2$$

$$R_{0,3,3,1} = \%e^{-2\alpha}\left(\frac{d}{d t}\beta\right) r\, sin(\theta)^2$$

$$R_{1,0,0,1} = \%e^{-2\beta}\ (\%e^{2\beta}\left(\frac{d^2}{d t^2}\beta\right) + \%e^{2\beta}\left(\frac{d}{d t}\beta\right)^2 - \left(\frac{d}{d t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d t}\beta\right) + \%e^{2\alpha}$$

$$\left(\frac{d}{d r}\alpha\right)\left(\frac{d}{d r}\beta\right) - \%e^{2\alpha}\left(\frac{d^2}{d r^2}\alpha\right) - \%e^{2\alpha}\left(\frac{d}{d r}\alpha\right)^2)$$

$$R_{1,0,1,0} = -\%e^{-2\beta}\ (\%e^{2\beta}\left(\frac{d^2}{d t^2}\beta\right) + \%e^{2\beta}\left(\frac{d}{d t}\beta\right)^2 - \left(\frac{d}{d t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d t}\beta\right) + \%e^{2\alpha}$$

$$\left(\frac{d}{d r}\alpha\right)\left(\frac{d}{d r}\beta\right) - \%e^{2\alpha}\left(\frac{d^2}{d r^2}\alpha\right) - \%e^{2\alpha}\left(\frac{d}{d r}\alpha\right)^2)$$

$$R_{1,2,0,2} = \%e^{-2\beta}\left(\frac{d}{d t}\beta\right) r$$

$$R_{1,2,1,2} = \%e^{-2\beta}\left(\frac{d}{d r}\beta\right) r$$

$$R_{1,2,2,0} = -\%e^{-2\beta}\left(\frac{d}{d t}\beta\right) r$$

$$R_{1,2,2,1} = -\%e^{-2\beta}\left(\frac{d}{d r}\beta\right) r$$

$$R_{1,3,0,3} = \%e^{-2\beta}\left(\frac{d}{d t}\beta\right) r\, sin(\theta)^2$$

$$R_{1,3,1,3} = \%e^{-2\beta}\left(\frac{d}{d r}\beta\right) r\, sin(\theta)^2$$

$$R_{1,3,3,0} = -\%e^{-2\beta}\left(\frac{d}{d t}\beta\right) r\, sin(\theta)^2$$

$$R_{1,3,3,1} = -\%e^{-2\beta}\left(\frac{d}{d\,r}\beta\right) r\,sin(\theta)^2$$

$$R_{2,0,0,2} = -\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{2\alpha-2\beta}}{r}$$

$$R_{2,0,1,2} = -\frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{2,0,2,0} = \frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{2\alpha-2\beta}}{r}$$

$$R_{2,0,2,1} = \frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{2,1,0,2} = -\frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{2,1,1,2} = -\frac{\frac{d}{d\,r}\beta}{r}$$

$$R_{2,1,2,0} = \frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{2,1,2,1} = \frac{\frac{d}{d\,r}\beta}{r}$$

$$R_{2,3,2,3} = \%e^{-2\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)\,sin(\theta)^2$$

$$R_{2,3,3,2} = -\%e^{-2\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)\,sin(\theta)^2$$

$$R_{3,0,0,3} = -\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{2\alpha-2\beta}}{r}$$

$$R_{3,0,1,3} = -\frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{3,0,3,0} = \frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{2\alpha-2\beta}}{r}$$

$$R_{3,0,3,1} = \frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{3,1,0,3} = -\frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{3,1,1,3} = -\frac{\frac{d}{d\,r}\beta}{r}$$

$$R_{3,1,3,0} = \frac{\frac{d}{d\,t}\beta}{r}$$

$$R_{3,1,3,1} = \frac{\frac{d}{d\,r}\beta}{r}$$

$$R_{3,2,2,3} = -\%e^{-2\,\beta}\,(\%e^{\beta}-1)(\%e^{\beta}+1)$$

$$R_{3,2,3,2} = \%e^{-2\,\beta}\,(\%e^{\beta}-1)(\%e^{\beta}+1)$$

```
(%i12) /* Ricci tensor Ric[mu,nu] */
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
       }}$
```

Maxima encountered a Lisp error:
 Console interrupt.
Automatically continuing.
To reenable the Lisp debugger set *debugger-hook* to nil.

(%i12)

```
(%i13) /* dislplay Ric's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
           Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
           if Ric[i,j] # 0 then display(Ric[i,j])
           }}$
```

$$Ric_{0,0} = -\left(\%e^{-2\,\beta}\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)r + \%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2 r - \left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)r + \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)r - \%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)r - \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2 r - 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\right)\right)/r$$

$$Ric_{0,1} = \frac{2\left(\frac{d}{d\,t}\beta\right)}{r}$$

$$Ric_{1,0} = \frac{2\left(\frac{d}{d\,t}\beta\right)}{r}$$

$$Ric_{1,1} = \left(\%e^{-2\,\alpha}\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)r + \%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2 r - \left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)r + \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)r - \%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)r - \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2 r + 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\beta\right)\right)\right)/r$$

$$Ric_{2,2} = \%e^{-2\,\beta}\left(\left(\frac{d}{d\,r}\beta\right)r - \left(\frac{d}{d\,r}\alpha\right)r + \%e^{2\,\beta} - 1\right)$$

$$Ric_{3,3} = \%e^{-2\,\beta}\left(\left(\frac{d}{d\,r}\beta\right)r - \left(\frac{d}{d\,r}\alpha\right)r + \%e^{2\,\beta} - 1\right)\sin(\theta)^2$$

(%i14)
```
/* Ricci Scalar */
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
     +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
     +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
     +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

(%o14)
$$\left(\%e^{-2\,\beta-2\,\alpha}\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)r + \%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2 r - \left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)r + \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)r - \%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)r - \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2 r + 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\beta\right)\right)\right)/r + \left(\%e^{-2\,\beta-2\,\alpha}\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)r + \%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2 r - \left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)r + \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)r - \%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)r - \%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2 r - 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\right)\right)/r + \frac{2\,\%e^{-2\,\beta}\left(\left(\frac{d}{d\,r}\beta\right)r - \left(\frac{d}{d\,r}\alpha\right)r + \%e^{2\,\beta} - 1\right)}{r^2}$$

(%i15)
```
ratsimp(RicSc);
```

(%o15)
$$\left(\%e^{-2\,\beta-2\,\alpha}\left(\left(2\,\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right) + 2\,\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2 - 2\left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right) + 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right) - 2\,\%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right) - 2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)r^2 + \left(4\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\beta\right) - 4\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\right)r + 2\,\%e^{2\,\beta+2\,\alpha} - 2\,\%e^{2\,\alpha}\right)\right)/r^2$$

(%i16)
```
/* Test for R^q */
for mu:    0 thru 3 do (
for sigma:0 thru 3 do (
for nu:    0 thru 3 do (
for rho:   0 thru 3 do (
    R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
    if R_q # 0 then (
        display("=============Einstein equation R^q=0 not fulfilled! "),
        display(mu,sigma,nu,rho),
        display(R_q)
    )
))));
```

(%o16) *done*

(%i17)
```
/* Raising of indices,
   contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);
```

(%o17) $$-\%e^{-2\beta-4\alpha}\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%o18) $$\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta-2\alpha}}{r}$$

(%o19) $$\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta-2\alpha}}{r}$$

(%i20)
```
R0101: factor(R0101);
R0202: factor(R0202);
R0303: factor(R0303);
```

(%o20) $$-\%e^{-2\beta-4\alpha}\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%o21) $$\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta-2\alpha}}{r}$$

(%o22) $$\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta-2\alpha}}{r}$$

(%i23)
```
R1010: f(1,0);
R1212: f(1,2);
R1313: f(1,3);
```

(%o23) $$\%e^{-4\beta-2\alpha}\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%o24) $$\frac{\%e^{-4\beta}\left(\frac{d}{d\,r}\beta\right)}{r}$$

(%o25) $$\frac{\%e^{-4\beta}\left(\frac{d}{d\,r}\beta\right)}{r}$$

```
(%i26) R1010: factor(R1010);
       R1212: factor(R1212);
       R1313: factor(R1313);
```

(%o26) $$\%e^{-4\beta-2\alpha}\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%o27) $$\frac{\%e^{-4\beta}\left(\frac{d}{d\,r}\beta\right)}{r}$$

(%o28) $$\frac{\%e^{-4\beta}\left(\frac{d}{d\,r}\beta\right)}{r}$$

```
(%i29) R2020: f(2,0);
       R2121: f(2,1);
       R2323: f(2,3);
```

(%o29) $$-\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta}}{r^3}$$

(%o30) $$\frac{\%e^{-2\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3}$$

(%o31) $$\frac{\%e^{-2\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)}{r^4}$$

```
(%i32) R2020: factor(R2020);
       R2121: factor(R2121);
       R2323: factor(R2323);
```

(%o32) $$-\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta}}{r^3}$$

(%o33) $$\frac{\%e^{-2\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3}$$

(%o34) $$\frac{\%e^{-2\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)}{r^4}$$

(%i35) R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);

(%o35) $$-\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\,\beta}}{r^3\,\sin(\theta)^2}$$

(%o36) $$\frac{\%e^{-2\,\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3\,\sin(\theta)^2}$$

(%o37) $$\frac{\%e^{-2\,\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)}{r^4\,\sin(\theta)^2}$$

(%i38) R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);

(%o38) $$-\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\,\beta}}{r^3\,\sin(\theta)^2}$$

(%o39) $$\frac{\%e^{-2\,\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3\,\sin(\theta)^2}$$

(%o40) $$\frac{\%e^{-2\,\beta}(\%e^{\beta}-1)(\%e^{\beta}+1)}{r^4\,\sin(\theta)^2}$$

(%i41) /* Coulomb law */
DivE : R0101 + R0202 + R0303;

(%o41) $$\frac{2\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\,\beta-2\,\alpha}}{r}-\%e^{-2\,\beta-4\,\alpha}\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%i42) ratsimp(DivE);

(%o42) $$-\left(\%e^{-2\,\beta-4\,\alpha}\left(\left(\%e^{2\,\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\,\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\,\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)r-2\,\%e^{2\,\alpha}\left(\frac{d}{d\,r}\alpha\right)\right)\right)/r$$

(%i43) /* J[r] */
Jr : -(R1010 + R1212 + R1313);

(%o43) $$-\frac{2\,\%e^{-4\beta}\left(\frac{d}{d\,r}\beta\right)}{r}-\%e^{-4\beta-2\alpha}\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)$$

(%i44) `ratsimp(Jr);`

(%o44) $$-\left(\%e^{-4\beta-2\alpha}\left(\left(\%e^{2\beta}\left(\frac{d^2}{d\,t^2}\beta\right)+\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)^2-\left(\frac{d}{d\,t}\alpha\right)\%e^{2\beta}\left(\frac{d}{d\,t}\beta\right)+\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)\left(\frac{d}{d\,r}\beta\right)-\%e^{2\alpha}\left(\frac{d^2}{d\,r^2}\alpha\right)-\%e^{2\alpha}\left(\frac{d}{d\,r}\alpha\right)^2\right)r+2\,\%e^{2\alpha}\left(\frac{d}{d\,r}\beta\right)\right)\right)/\,r$$

(%i45) `/* J[theta] */`
`Jtheta : -(R2020 + R2121 + R2323);`

(%o45) $$-\frac{\%e^{-2\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3}+\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta}}{r^3}-\frac{\%e^{-2\beta}\left(\%e^{\beta}-1\right)\left(\%e^{\beta}+1\right)}{r^4}$$

(%i46) `ratsimp(Jtheta);`

(%o46) $$-\frac{\%e^{-2\beta}\left(\left(\frac{d}{d\,r}\beta-\frac{d}{d\,r}\alpha\right)r+\%e^{2\beta}-1\right)}{r^4}$$

(%i47) `/* J[phi] */`
`Jphi : -(R3030 + R3131 + R3232);`

(%o47) $$-\frac{\%e^{-2\beta}\left(\frac{d}{d\,r}\beta\right)}{r^3\,\sin(\theta)^2}+\frac{\left(\frac{d}{d\,r}\alpha\right)\%e^{-2\beta}}{r^3\,\sin(\theta)^2}-\frac{\%e^{-2\beta}\left(\%e^{\beta}-1\right)\left(\%e^{\beta}+1\right)}{r^4\,\sin(\theta)^2}$$

(%i48) `ratsimp(Jphi);`

(%o48) $$-\frac{\%e^{-2\beta}\left(\left(\frac{d}{d\,r}\beta-\frac{d}{d\,r}\alpha\right)r+\%e^{2\beta}-1\right)}{r^4\,\sin(\theta)^2}$$

(%i99) `alpha: 1/r;`

(%o99) $$\frac{1}{r}$$

(%i100) `beta: r;`

(%o100) r

(%i101) `DivE_p: ev(DivE,diff);`

(%o101) $$-\frac{2\,\%e^{-2r-\frac{2}{r}}}{r^3}-\left(-\frac{\%e^{2/r}}{r^2}-\frac{2\,\%e^{2/r}}{r^3}-\frac{\%e^{2/r}}{r^4}\right)\%e^{-2r-\frac{4}{r}}$$

(%i102) `Jr_p: ev(Jr,diff);`

(%o102) $$-\frac{2\,\%e^{-4r}}{r}-\left(-\frac{\%e^{2/r}}{r^2}-\frac{2\,\%e^{2/r}}{r^3}-\frac{\%e^{2/r}}{r^4}\right)\%e^{-4r-\frac{2}{r}}$$

(%i103) `Jtheta_p: ev(Jtheta,diff);`

(%o103) $$-\frac{\%e^{-2r}(\%e^{r}-1)(\%e^{r}+1)}{r^4}-\frac{\%e^{-2r}}{r^3}-\frac{\%e^{-2r}}{r^5}$$

(%i104) `Jphi_p: ev(at(Jphi,[theta=%pi/2]),diff);`

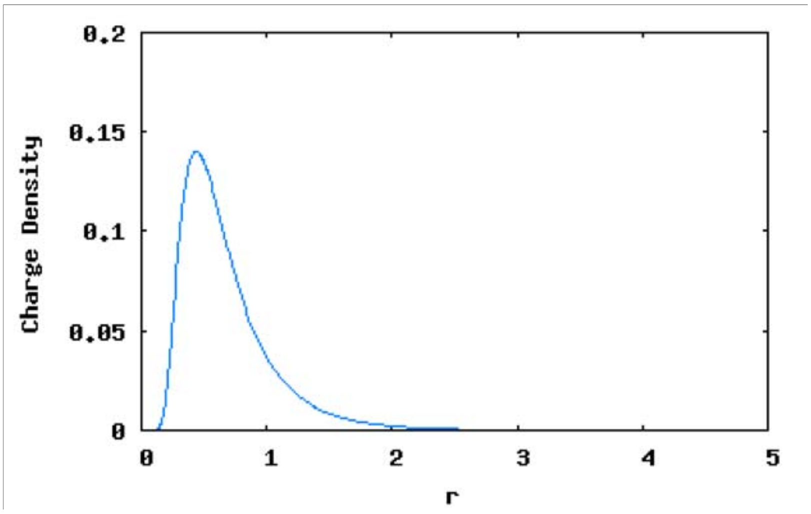
(%o104) $$-\frac{\%e^{-2r}(\%e^{r}-1)(\%e^{r}+1)}{r^4}-\frac{\%e^{-2r}}{r^3}-\frac{\%e^{-2r}}{r^5}$$

(%i111)

```
wxplot2d([DivE_p], [r,0,5],[y,0,.2], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

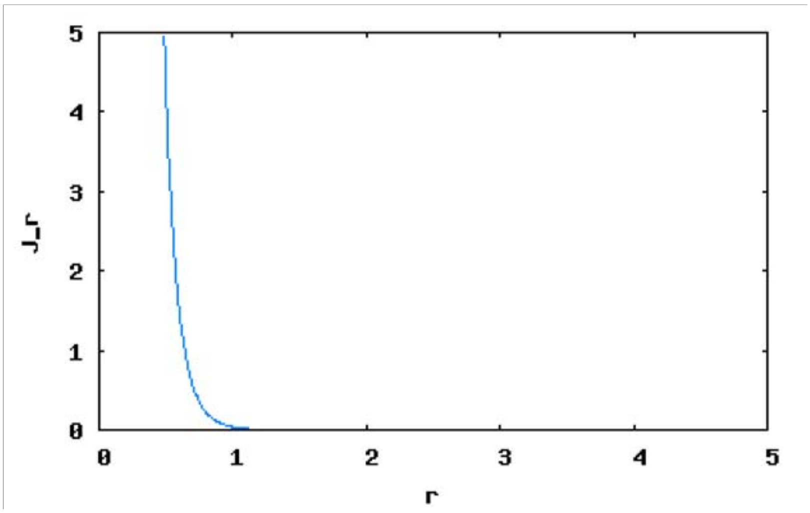
(%t111)



(%i114)

```
wxplot2d([Jr_p], [r,0,5],[y,0,5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

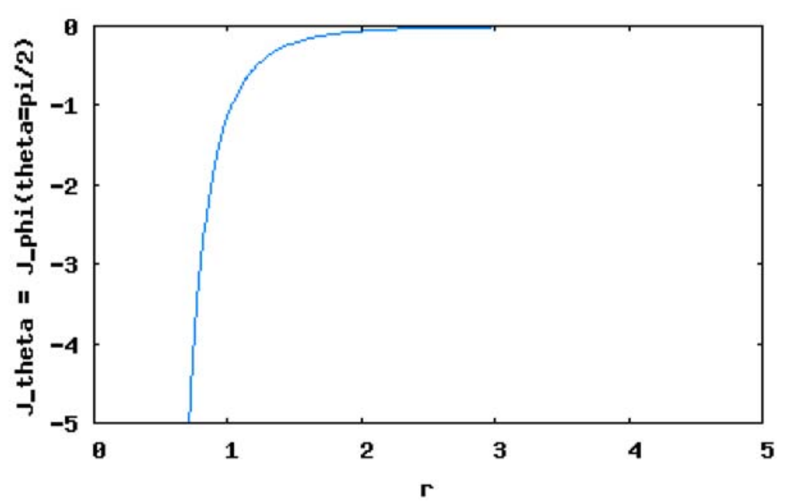
Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t114) 

(%i117)

```
wxplot2d([Jtheta_p], [r,0,5],[y,-5,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t117) 

(%i118)