(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $$f(i,j) := sum(R_{i,j,\sigma,0}\,gContr_{i,\sigma}\,gContr_{j,0},\sigma,0,3) + sum(R_{i,j,\sigma,1}\,gContr_{i,\sigma}\,gContr_{j,1},\sigma,0,3) + sum(R_{i,j,\sigma,2}\,gContr_{i,\sigma}\,gContr_{j,2},\sigma,0,3) + sum(R_{i,j,\sigma,3}\,gContr_{i,\sigma}\,gContr_{j,3},\sigma,0,3)$$

(%i2)

```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

(%o2) x

(%o3) $[\,t\,,\,r\,,\,\theta\,,\,\varphi\,]$

(%i4)

```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
 [-(1-r^2/alpha^2),0,0,0],
 [0,(1-r^2/alpha^2)^-1,0,0],
 [0,0,r^2,0],
 [0,0,0,r^2*sin(theta)^2]
);
```

(%o4) $$\begin{bmatrix} \frac{r^2}{\alpha^2}-1 & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{r^2}{\alpha^2}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,sin(\theta)^2 \end{bmatrix}$$

(%i5)

```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o5)
$$\begin{bmatrix} \frac{\alpha^2}{r^2-\alpha^2} & 0 & 0 & 0 \\ 0 & -\frac{r^2-\alpha^2}{\alpha^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\,sin(\theta)^2} \end{bmatrix}$$

(%i6)
```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g     [mu,nu]: g1     [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

(%i7)
```
/* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
        diff(g[nu,rho],x[mu] ) +
        diff(g[rho,mu],x[nu] ) -
        diff(g[mu,nu] ,x[rho])),
    rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

(%i8)
```
/* dislplay Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$
```

$$\Gamma_{0,0,1} = \frac{r}{r^2-\alpha^2}$$

$$\Gamma_{0,1,0} = \frac{r}{r^2-\alpha^2}$$

$$\Gamma_{1,0,0} = \frac{r(r^2-\alpha^2)}{\alpha^4}$$

$$\Gamma_{1,1,1} = -\frac{r(r^2-\alpha^2)}{\alpha^4\left(1-\frac{r^2}{\alpha^2}\right)^2}$$

$$\Gamma_{1,2,2} = \frac{r(r^2-\alpha^2)}{\alpha^2}$$

$$\Gamma_{1,3,3} = \frac{r(r^2-\alpha^2)\,sin(\theta)^2}{\alpha^2}$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,3} = -cos(\theta)\,sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{1}{r}$$

$$\Gamma_{3,2,3} = \frac{cos(\theta)}{sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{1}{r}$$

$$\Gamma_{3,3,2} = \frac{cos(\theta)}{sin(\theta)}$$

```
(%i9)  /* compute Riemann tensor elements */
       for rho:0 thru 3 do {
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           R[rho,sigma,mu,nu] :
           diff(Gamma[rho,nu,sigma],x[mu]) -
           diff(Gamma[rho,mu,sigma],x[nu]) +
           /* lambda sums by function call: */
           sum(
               Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
               Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
               lambda, 0, 3)
       }}}}$
```

```
(%i10)  /* dislplay R's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
        for k:0 thru 3 do {
        for l:0 thru 3 do {
            R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
            if R[i,j,k,l] # 0 then display(R[i,j,k,l])
            }}}}$
```

$$R_{0,1,0,1} = -\frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{0,1,1,0} = \frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{0,2,0,2} = \frac{r^2}{\alpha^2}$$

$$R_{0,2,2,0} = -\frac{r^2}{\alpha^2}$$

$$R_{0,3,0,3} = \frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{0,3,3,0} = -\frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{1,0,0,1} = -\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{1,0,1,0} = \frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{1,2,1,2} = \frac{r^2}{\alpha^2}$$

$$R_{1,2,2,1} = -\frac{r^2}{\alpha^2}$$

$$R_{1,3,1,3} = \frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{1,3,3,1} = -\frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{2,0,0,2} = -\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{2,0,2,0} = \frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{2,1,1,2} = \frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{2,1,2,1} = -\frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{2,3,2,3} = \frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{2,3,3,2} = -\frac{r^2 \sin(\theta)^2}{\alpha^2}$$

$$R_{3,0,0,3} = -\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{3,0,3,0} = \frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$R_{3,1,1,3} = \frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{3,1,3,1} = -\frac{1}{(r-\alpha)(r+\alpha)}$$

$$R_{3,2,2,3} = -\frac{r^2}{\alpha^2}$$

$$R_{3,2,3,2} = \frac{r^2}{\alpha^2}$$

```
(%i11)  /* Ricci tensor Ric[mu,nu] */
        for mu:0 thru 3 do {
        for nu:0 thru 3 do {
            Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
        }}$
```

```
(%i12)  /* dislplay Ric's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
            Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
            if Ric[i,j] # 0 then display(Ric[i,j])
            }}$
```

$$Ric_{0,0} = \frac{3(r-\alpha)(r+\alpha)}{\alpha^4}$$

$$Ric_{1,1} = -\frac{3}{(r-\alpha)(r+\alpha)}$$

$$Ric_{2,2} = \frac{3r^2}{\alpha^2}$$

$$Ric_{3,3} = \frac{3r^2\sin(\theta)^2}{\alpha^2}$$

```
(%i13)  /* Ricci Scalar */
        RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
            +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
            +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
            +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
        ;
```

$$(\%o13)\quad \frac{3(r^2-\alpha^2)}{\alpha^2(r-\alpha)(r+\alpha)} + \frac{3(r-\alpha)(r+\alpha)}{\alpha^2(r^2-\alpha^2)} + \frac{6}{\alpha^2}$$

```
(%i14)  ratsimp(RicSc);
```

$$(\%o14)\quad \frac{12}{\alpha^2}$$

```
(%i15)
```

```
/* Test for R^q */
for mu:   0 thru 3 do (
for sigma:0 thru 3 do (
for nu:   0 thru 3 do (
for rho:  0 thru 3 do (
    R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
    if R_q # 0 then (
        display("=============Einstein equation R^q=0 not fulfilled! "),
        display(mu,sigma,nu,rho),
        display(R_q)
    )
))));
```

(%o15) *done*

```
(%i16) /* Raising of indices,
          contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
       /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

       R0101: f(0,1);
       R0202: f(0,2);
       R0303: f(0,3);
```

(%o16) $$\frac{1}{(r-\alpha)(r+\alpha)}$$

(%o17) $$\frac{1}{r^2-\alpha^2}$$

(%o18) $$\frac{1}{r^2-\alpha^2}$$

```
(%i19) R0101: factor(R0101);
       R0202: factor(R0202);
       R0303: factor(R0303);
```

(%o19) $$\frac{1}{(r-\alpha)(r+\alpha)}$$

(%o20) $$\frac{1}{(r-\alpha)(r+\alpha)}$$

(%o21) $$\frac{1}{(r-\alpha)(r+\alpha)}$$

```
(%i22) R1010: f(1,0);
       R1212: f(1,2);
       R1313: f(1,3);
```

(%o22) $$-\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

(%o23) $$-\frac{r^2-\alpha^2}{\alpha^4}$$

(%o24) $$-\frac{r^2-\alpha^2}{\alpha^4}$$

(%i25)
```
R1010: factor(R1010);
R1212: factor(R1212);
R1313: factor(R1313);
```

(%o25) $$-\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

(%o26) $$-\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

(%o27) $$-\frac{(r-\alpha)(r+\alpha)}{\alpha^4}$$

(%i28)
```
R2020: f(2,0);
R2121: f(2,1);
R2323: f(2,3);
```

(%o28) $$\frac{(r-\alpha)(r+\alpha)}{\alpha^2 r^2 (r^2-\alpha^2)}$$

(%o29) $$\frac{r^2-\alpha^2}{\alpha^2 r^2 (r-\alpha)(r+\alpha)}$$

(%o30) $$\frac{1}{\alpha^2 r^2}$$

(%i31)
```
R2020: factor(R2020);
R2121: factor(R2121);
R2323: factor(R2323);
```

(%o31) $$\frac{1}{\alpha^2 r^2}$$

(%o32) $$\frac{1}{\alpha^2 r^2}$$

(%o33) $$\frac{1}{\alpha^2 r^2}$$

(%i34)
```
R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);
```

(%o34) $$\frac{(r-\alpha)(r+\alpha)}{\alpha^2 r^2 (r^2-\alpha^2)\sin(\theta)^2}$$

(%o35) $$\frac{r^2-\alpha^2}{\alpha^2 r^2 (r-\alpha)(r+\alpha)\sin(\theta)^2}$$

(%o36) $$\frac{1}{\alpha^2 r^2 \sin(\theta)^2}$$

```
(%i37) R3030: factor(R3030);
       R3131: factor(R3131);
       R3232: factor(R3232);
```

$$(\%o37)\quad \frac{1}{\alpha^2\, r^2\, sin(\theta)^2}$$

$$(\%o38)\quad \frac{1}{\alpha^2\, r^2\, sin(\theta)^2}$$

$$(\%o39)\quad \frac{1}{\alpha^2\, r^2\, sin(\theta)^2}$$

```
(%i40) /* Coulomb law */
       DivE : R0101 + R0202 + R0303;
```

$$(\%o40)\quad \frac{3}{(r-\alpha)(r+\alpha)}$$

```
(%i41) ratsimp(DivE);
```

$$(\%o41)\quad \frac{3}{r^2-\alpha^2}$$

```
(%i42) /* J[r] */
       Jr : -(R1010 + R1212 + R1313);
```

$$(\%o42)\quad \frac{3(r-\alpha)(r+\alpha)}{\alpha^4}$$

```
(%i43) ratsimp(Jr);
```

$$(\%o43)\quad \frac{3\,r^2-3\,\alpha^2}{\alpha^4}$$

```
(%i44) /* J[theta] */
       Jtheta : -(R2020 + R2121 + R2323);
```

$$(\%o44)\quad -\frac{3}{\alpha^2\, r^2}$$

```
(%i45) ratsimp(Jtheta);
```

$$(\%o45)\quad -\frac{3}{\alpha^2\, r^2}$$

```
(%i46) /* J[phi] */
       Jphi : -(R3030 + R3131 + R3232);
```

$$(\%o46)\quad -\frac{3}{\alpha^2\, r^2\, sin(\theta)^2}$$

(%i47) `ratsimp(Jphi);`

(%o47) $$-\frac{3}{\alpha^2 r^2 \sin(\theta)^2}$$

(%i48) `DivE_p: ev(at(DivE,[alpha=1]));`

(%o48) $$\frac{3}{(r-1)(r+1)}$$

(%i49) `Jr_p: ev(at(Jr,[alpha=1]));`

(%o49) $3(r-1)(r+1)$

(%i50) `Jtheta_p: ev(at(Jtheta,[alpha=1]));`

(%o50) $$-\frac{3}{r^2}$$

(%i52) `Jphi_p: ev(at(Jphi,[alpha=1, theta=%pi/2]));`

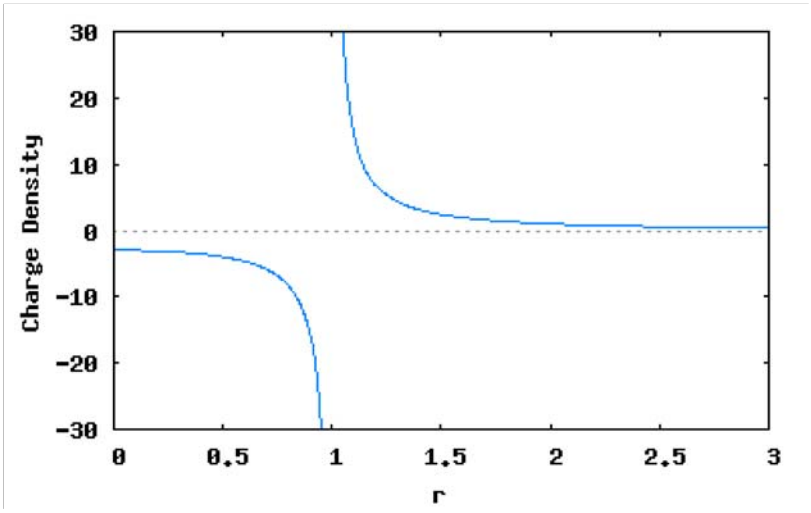
(%o52) $$-\frac{3}{r^2}$$

(%i54)

```
wxplot2d([DivE_p], [r,0,3],[y,-30,30], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t54)

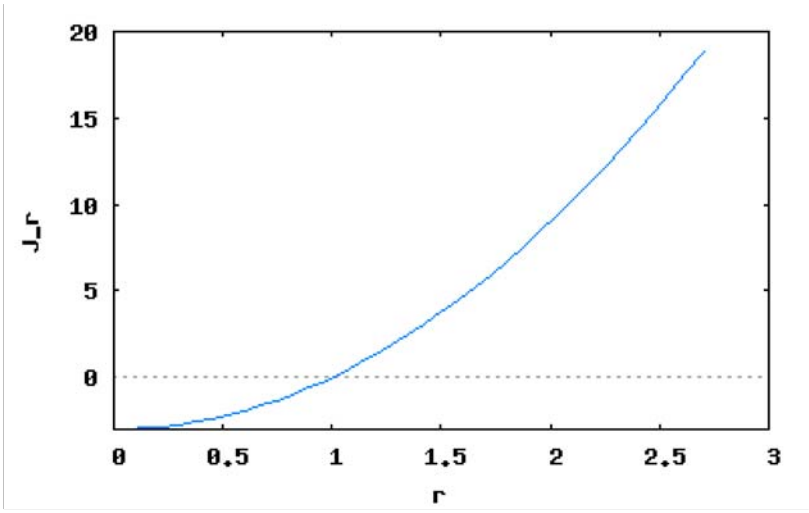


(%i57)

```
wxplot2d([Jr_p], [r,0,3],[y,-3,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t57)

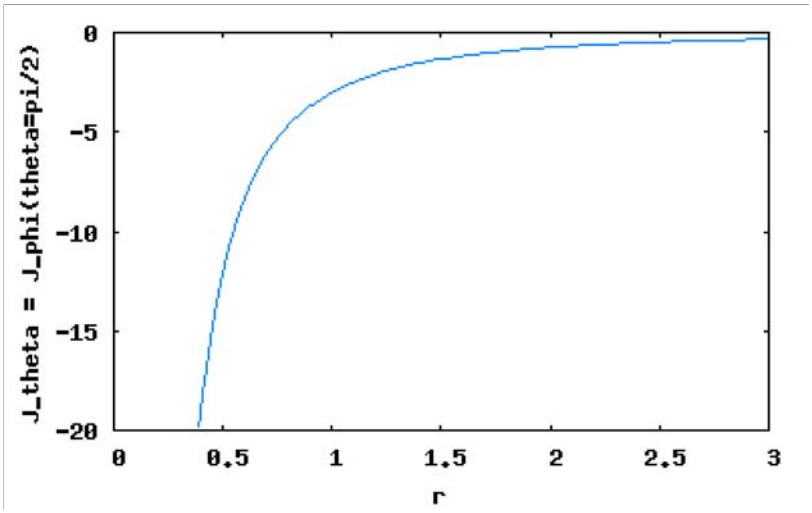


(%i59)

```
wxplot2d([Jtheta_p], [r,0,3],[y,-20,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t59)



(%i60)