(%i1)
```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1)
$$f(i,j):=\mathrm{sum}(R_{i,j,\sigma,0}\,gContr_{i,\sigma}\,gContr_{j,0},\sigma,0,3)+\mathrm{sum}(R_{i,j,\sigma,1}\,gContr_{i,\sigma}\,gContr_{j,1},\sigma,0,3)+\mathrm{sum}(R_{i,j,\sigma,2}\,gContr_{i,\sigma}\,gContr_{j,2},\sigma,0,3)+\mathrm{sum}(R_{i,j,\sigma,3}\,gContr_{i,\sigma}\,gContr_{j,3},\sigma,0,3)$$

(%i2)
```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

(%o2) x

(%o3) $[\,t\,,\,r\,,\,\theta\,,\,\varphi\,]$

(%i4)
```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
 [-(1-2*M/r+Q^2/r^2),0,0,0],
 [0,(1-2*M/r+Q^2/r^2)^(-1),0,0],
 [0,0,r^2,0],
 [0,0,0,r^2*sin(theta)^2]
);
```

(%o4)
$$\begin{bmatrix} -\frac{Q^2}{r^2}+\frac{2M}{r}-1 & 0 & 0 & 0 \\ 0 & \frac{1}{\frac{Q^2}{r^2}-\frac{2M}{r}+1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 \end{bmatrix}$$

(%i5)
```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o5)
$$\begin{bmatrix} -\frac{r^2}{Q^2 - 2\,r\,M + r^2} & 0 & 0 & 0 \\ 0 & \frac{Q^2 - 2\,r\,M + r^2}{r^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\,\sin(\theta)^2} \end{bmatrix}$$

(%i6)
```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g     [mu,nu]: g1     [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

(%i7)
```
/* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
        diff(g[nu,rho],x[mu] ) +
        diff(g[rho,mu],x[nu] ) -
        diff(g[mu,nu] ,x[rho])),
    rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

(%i8)
```
/* dislplay Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$
```

$$\Gamma_{0,0,1} = -\frac{r^2\left(\frac{2\,Q^2}{r^3} - \frac{2\,M}{r^2}\right)}{2\,(Q^2 - 2\,r\,M + r^2)}$$

$$\Gamma_{0,1,0} = -\frac{r^2\left(\frac{2\,Q^2}{r^3} - \frac{2\,M}{r^2}\right)}{2\,(Q^2 - 2\,r\,M + r^2)}$$

$$\Gamma_{1,0,0} = \frac{(Q^2 - 2rM + r^2)\left(\frac{2M}{r^2} - \frac{2Q^2}{r^3}\right)}{2r^2}$$

$$\Gamma_{1,1,1} = -\frac{(Q^2 - 2rM + r^2)\left(\frac{2M}{r^2} - \frac{2Q^2}{r^3}\right)}{2r^2\left(\frac{Q^2}{r^2} - \frac{2M}{r} + 1\right)^2}$$

$$\Gamma_{1,2,2} = -\frac{Q^2 - 2rM + r^2}{r}$$

$$\Gamma_{1,3,3} = -\frac{\sin(\theta)^2(Q^2 - 2rM + r^2)}{r}$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,3} = -\cos(\theta)\sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{1}{r}$$

$$\Gamma_{3,2,3} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{1}{r}$$

$$\Gamma_{3,3,2} = \frac{\cos(\theta)}{\sin(\theta)}$$

```
(%i9)  /* compute Riemann tensor elements */
       for rho:0 thru 3 do {
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           R[rho,sigma,mu,nu] :
           diff(Gamma[rho,nu,sigma],x[mu]) -
           diff(Gamma[rho,mu,sigma],x[nu]) +
           /* lambda sums by function call: */
           sum(
               Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
               Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
               lambda, 0, 3)
       }}}}$
```

```
(%i10)  /* dislplay R's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
        for k:0 thru 3 do {
        for l:0 thru 3 do {
            R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
            if R[i,j,k,l] # 0 then display(R[i,j,k,l])
            }}}}$
```

$$R_{0,1,0,1} = -\frac{3\,Q^2 - 2\,r\,M}{r^2\,(Q^2 - 2\,r\,M + r^2)}$$

$$R_{0,1,1,0} = \frac{3\,Q^2 - 2\,r\,M}{r^2\,(Q^2 - 2\,r\,M + r^2)}$$

$$R_{0,2,0,2} = \frac{Q^2 - r\,M}{r^2}$$

$$R_{0,2,2,0} = -\frac{Q^2 - r\,M}{r^2}$$

$$R_{0,3,0,3} = \frac{sin(\theta)^2\,(Q^2 - r\,M)}{r^2}$$

$$R_{0,3,3,0} = -\frac{sin(\theta)^2\,(Q^2 - r\,M)}{r^2}$$

$$R_{1,0,0,1} = -\frac{(Q^2 - 2\,r\,M + r^2)(3\,Q^2 - 2\,r\,M)}{r^6}$$

$$R_{1,0,1,0} = \frac{(Q^2 - 2\,r\,M + r^2)(3\,Q^2 - 2\,r\,M)}{r^6}$$

$$R_{1,2,1,2} = \frac{Q^2 - r\,M}{r^2}$$

$$R_{1,2,2,1} = -\frac{Q^2 - r\,M}{r^2}$$

$$R_{1,3,1,3} = \frac{sin(\theta)^2\,(Q^2 - r\,M)}{r^2}$$

$$R_{1,3,3,1} = -\frac{sin(\theta)^2\,(Q^2 - r\,M)}{r^2}$$

$$R_{2,0,0,2} = \frac{(Q^2 - 2\,r\,M + r^2)(Q^2 - r\,M)}{r^6}$$

$$R_{2,0,2,0} = -\frac{(Q^2 - 2\,r\,M + r^2)(Q^2 - r\,M)}{r^6}$$

$$R_{2,1,1,2} = -\frac{Q^2 - r\,M}{r^2\,(Q^2 - 2\,r\,M + r^2)}$$

$$R_{2,1,2,1} = \frac{Q^2 - rM}{r^2(Q^2 - 2rM + r^2)}$$

$$R_{2,3,2,3} = -\frac{\sin(\theta)^2(Q^2 - 2rM)}{r^2}$$

$$R_{2,3,3,2} = \frac{\sin(\theta)^2(Q^2 - 2rM)}{r^2}$$

$$R_{3,0,0,3} = \frac{(Q^2 - 2rM + r^2)(Q^2 - rM)}{r^6}$$

$$R_{3,0,3,0} = -\frac{(Q^2 - 2rM + r^2)(Q^2 - rM)}{r^6}$$

$$R_{3,1,1,3} = -\frac{Q^2 - rM}{r^2(Q^2 - 2rM + r^2)}$$

$$R_{3,1,3,1} = \frac{Q^2 - rM}{r^2(Q^2 - 2rM + r^2)}$$

$$R_{3,2,2,3} = \frac{Q^2 - 2rM}{r^2}$$

$$R_{3,2,3,2} = -\frac{Q^2 - 2rM}{r^2}$$

```
(%i11)  /* Ricci tensor Ric[mu,nu] */
        for mu:0 thru 3 do {
        for nu:0 thru 3 do {
            Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
        }}$
```

```
(%i12)  /* dislplay Ric's being different from zero */
        for i:0 thru 3 do {
        for j:0 thru 3 do {
            Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
            if Ric[i,j] # 0 then display(Ric[i,j])
            }}$
```

$$Ric_{0,0} = \frac{Q^2(Q^2 - 2rM + r^2)}{r^6}$$

$$Ric_{1,1} = -\frac{Q^2}{r^2(Q^2 - 2rM + r^2)}$$

$$Ric_{2,2} = \frac{Q^2}{r^2}$$

$$Ric_{3,3} = \frac{\sin(\theta)^2 Q^2}{r^2}$$

```
(%i13)  /* Ricci Scalar */
        RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
            +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
            +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
            +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
        ;
(%o13)  0

(%i14)  ratsimp(RicSc);
(%o14)  0

(%i15)
/* Test for R^q */
for mu:   0 thru 3 do (
for sigma:0 thru 3 do (
for nu:   0 thru 3 do (
for rho:  0 thru 3 do (
    R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
    if R_q # 0 then (
        display("=============Einstein equation R^q=0 not fulfilled! "),
        display(mu,sigma,nu,rho),
        display(R_q)
    )
))));
(%o15)  done

(%i16)  /* Raising of indices,
           contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
        /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

        R0101: f(0,1);
        R0202: f(0,2);
        R0303: f(0,3);

(%o16)  (3 Q^2 - 2 r M)/(r^2 (Q^2 - 2 r M + r^2))

(%o17)  -(Q^2 - r M)/(r^2 (Q^2 - 2 r M + r^2))

(%o18)  -(Q^2 - r M)/(r^2 (Q^2 - 2 r M + r^2))

(%i19)  R0101: factor(R0101);
        R0202: factor(R0202);
        R0303: factor(R0303);

(%o19)  (3 Q^2 - 2 r M)/(r^2 (Q^2 - 2 r M + r^2))
```

(%o20) $$-\frac{Q^2 - r M}{r^2 (Q^2 - 2 r M + r^2)}$$

(%o21) $$-\frac{Q^2 - r M}{r^2 (Q^2 - 2 r M + r^2)}$$

(%i22)
```
R1010: f(1,0);
R1212: f(1,2);
R1313: f(1,3);
```

(%o22) $$-\frac{(Q^2 - 2 r M + r^2)(3 Q^2 - 2 r M)}{r^6}$$

(%o23) $$\frac{(Q^2 - 2 r M + r^2)(Q^2 - r M)}{r^6}$$

(%o24) $$\frac{(Q^2 - 2 r M + r^2)(Q^2 - r M)}{r^6}$$

(%i25)
```
R1010: factor(R1010);
R1212: factor(R1212);
R1313: factor(R1313);
```

(%o25) $$-\frac{(Q^2 - 2 r M + r^2)(3 Q^2 - 2 r M)}{r^6}$$

(%o26) $$\frac{(Q^2 - 2 r M + r^2)(Q^2 - r M)}{r^6}$$

(%o27) $$\frac{(Q^2 - 2 r M + r^2)(Q^2 - r M)}{r^6}$$

(%i28)
```
R2020: f(2,0);
R2121: f(2,1);
R2323: f(2,3);
```

(%o28) $$\frac{Q^2 - r M}{r^6}$$

(%o29) $$\frac{Q^2 - r M}{r^6}$$

(%o30) $$-\frac{Q^2 - 2 r M}{r^6}$$

(%i31)
```
R2020: factor(R2020);
R2121: factor(R2121);
R2323: factor(R2323);
```

(%o31) $$\frac{Q^2 - r M}{r^6}$$

(%o32) $$\frac{Q^2 - r M}{r^6}$$

(%o33) $$-\frac{Q^2 - 2 r M}{r^6}$$

(%i34)
```
R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);
```

(%o34) $$\frac{Q^2 - r M}{r^6 \sin(\theta)^2}$$

(%o35) $$\frac{Q^2 - r M}{r^6 \sin(\theta)^2}$$

(%o36) $$-\frac{Q^2 - 2 r M}{r^6 \sin(\theta)^2}$$

(%i37)
```
R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);
```

(%o37) $$\frac{Q^2 - r M}{r^6 \sin(\theta)^2}$$

(%o38) $$\frac{Q^2 - r M}{r^6 \sin(\theta)^2}$$

(%o39) $$-\frac{Q^2 - 2 r M}{r^6 \sin(\theta)^2}$$

(%i40)
```
/* Coulomb law */
DivE : R0101 + R0202 + R0303;
```

(%o40) $$\frac{3 Q^2 - 2 r M}{r^2 (Q^2 - 2 r M + r^2)} - \frac{2 (Q^2 - r M)}{r^2 (Q^2 - 2 r M + r^2)}$$

(%i41)
```
ratsimp(DivE);
```

(%o41) $$\frac{Q^2}{r^2 Q^2 - 2 r^3 M + r^4}$$

(%i42)
```
/* J[r] */
Jr : -(R1010 + R1212 + R1313);
```

(%o42) $$\frac{(Q^2 - 2 r M + r^2)(3 Q^2 - 2 r M)}{r^6} - \frac{2 (Q^2 - 2 r M + r^2)(Q^2 - r M)}{r^6}$$

(%i43)
```
ratsimp(Jr);
```

(%o43) $$\frac{Q^4+(r^2-2rM)Q^2}{r^6}$$

(%i44)
```
/* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

(%o44) $$\frac{Q^2-2rM}{r^6}-\frac{2(Q^2-rM)}{r^6}$$

(%i45)
```
ratsimp(Jtheta);
```

(%o45) $$-\frac{Q^2}{r^6}$$

(%i46)
```
/* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

(%o46) $$\frac{Q^2-2rM}{r^6\,sin(\theta)^2}-\frac{2(Q^2-rM)}{r^6\,sin(\theta)^2}$$

(%i47)
```
ev(ratsimp(Jphi),r);
```

(%o47) $$-\frac{Q^2}{r^6\,sin(\theta)^2}$$

(%i48)
```
DivE_p: ratsimp(ev(DivE,[Q=2, M=1]));
```

(%o48) $$\frac{4}{r^4-2r^3+4r^2}$$

(%i49)
```
Jr_p: ratsimp(ev(Jr,[Q=2, M=1]));
```

(%o49) $$\frac{4r^2-8r+16}{r^6}$$

(%i50)
```
Jtheta_p: ratsimp(ev(Jtheta,[Q=2, M=1]));
```

(%o50) $$-\frac{4}{r^6}$$

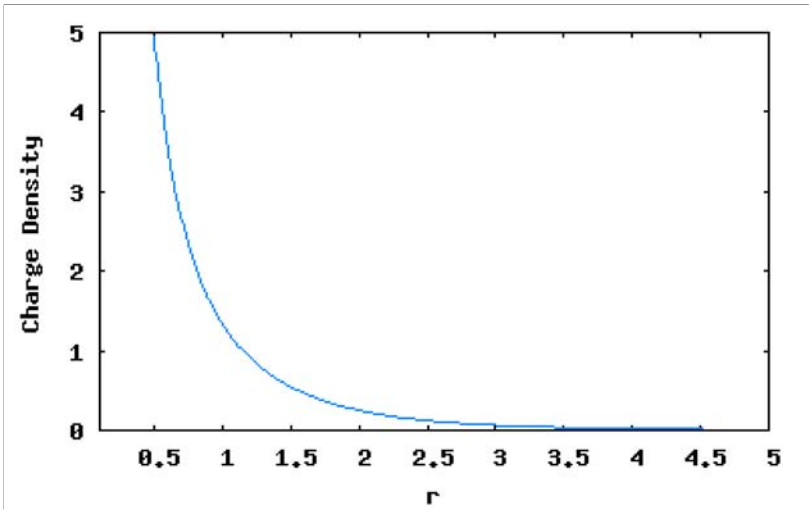
(%i51)
```
Jphi_p: ratsimp(ev(Jphi,[Q=2, M=1, theta=%pi/2]));
```

(%o51) $$-\frac{4}{r^6}$$

(%i52)
```
wxplot2d([DivE_p], [r,.1,5],[y,0,5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t52)

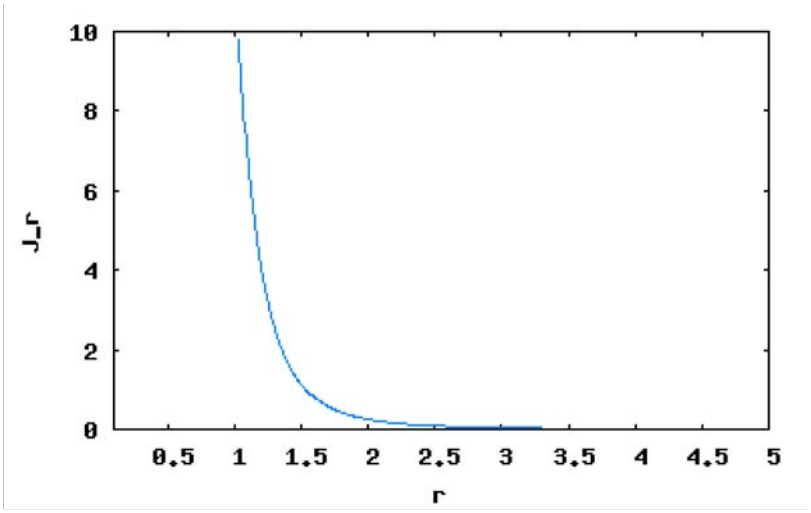


(%i53)

```
wxplot2d([Jr_p], [r,.1,5],[y,0,10], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t53)

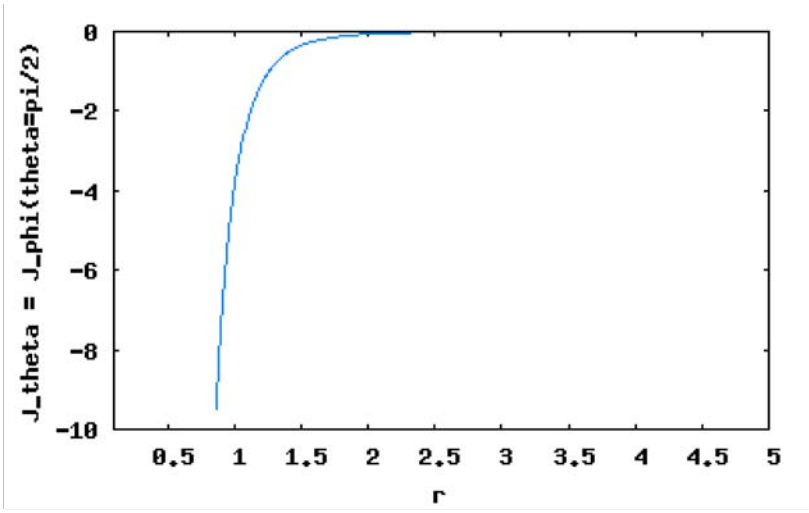


(%i54)

```
wxplot2d([Jtheta_p], [r,.1,5],[y,-10,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t54)



(%i55) `DivE_p: ratsimp(ev(DivE,[Q=1, M=2]));`

(%o55) $$\frac{1}{r^4 - 4\,r^3 + r^2}$$

(%i56) `Jr_p: ratsimp(ev(Jr,[Q=1, M=2]));`

(%o56) $$\frac{r^2 - 4\,r + 1}{r^6}$$

(%i57) `Jtheta_p: ratsimp(ev(Jtheta,[Q=1, M=2]));`

(%o57) $$-\frac{1}{r^6}$$

(%i58) `Jphi_p: ratsimp(ev(Jphi,[Q=1, M=2, theta=%pi/2]));`

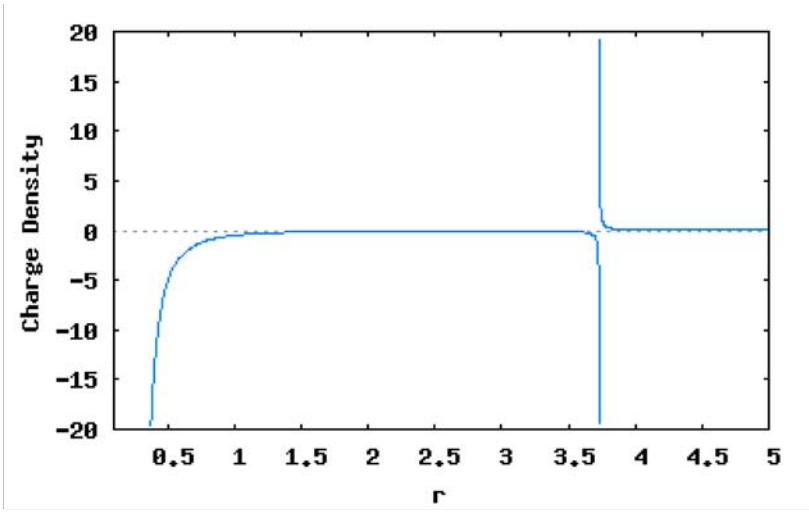
(%o58) $$-\frac{1}{r^6}$$

(%i59)

```
wxplot2d([DivE_p], [r,.1,5],[y,-20,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t59)

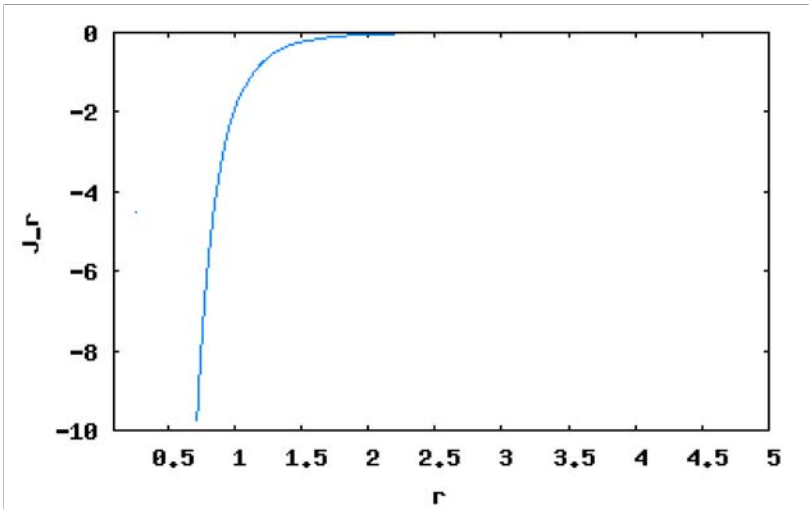


(%i60)

```
wxplot2d([Jr_p], [r,.1,5],[y,-10,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

```
Output file "C:/Documents and Settings/Administrator/maxout.png".
```

(%t60)

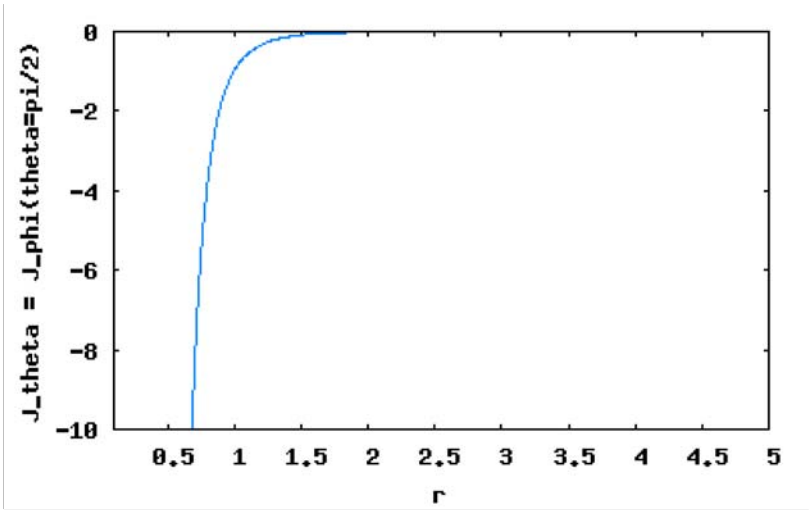


(%i61)

```
wxplot2d([Jtheta_p], [r,.1,5],[y,-10,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])$
```

```
Output file "C:/Documents and Settings/Administrator/maxout.png".
```

(%t61)



(%i62)