(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $f(i,j) := sum(R_{i,j,\sigma,0}\, gContr_{i,\sigma}\, gContr_{j,0}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,1}\, gContr_{i,\sigma}\, gContr_{j,1}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,2}\, gContr_{i,\sigma}\, gContr_{j,2}, \sigma, 0, 3) +$
$sum(R_{i,j,\sigma,3}\, gContr_{i,\sigma}\, gContr_{j,3}, \sigma, 0, 3)$

(%i2)

```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, l, theta, phi];
```

(%o2) x

(%o3) $[t, l, \theta, \varphi]$

(%i4)

```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
 [-1,0,0,0],
 [0,1,0,0],
 [0,0,k^2+l^2,0],
 [0,0,0,(k^2+l^2)*sin(theta)^2]
);
```

(%o4) $$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & l^2+k^2 & 0 \\ 0 & 0 & 0 & (l^2+k^2)\sin(\theta)^2 \end{bmatrix}$$

(%i5)

```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o5) $$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{l^2+k^2} & 0 \\ 0 & 0 & 0 & \frac{1}{(l^2+k^2)\sin(\theta)^2} \end{bmatrix}$$

(%i6)

```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g      [mu,nu]: g1      [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

```
(%i7) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
      for sigma:0 thru 3 do {
      for mu:0 thru 3 do {
      for nu:0 thru 3 do {
          Gamma[sigma,mu,nu] :
          /* rho sum by function call: */
          sum(
              1/2 * gContr[sigma,rho]*(
              diff(g[nu,rho],x[mu] ) +
              diff(g[rho,mu],x[nu] ) -
              diff(g[mu,nu] ,x[rho])),
          rho, 0, 3),
          /* evaluate differentiation dy/dr */
          Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
      }}}$
```

```
(%i8) /* dislplay Gamma's being different from zero */
      for i:0 thru 3 do {
      for j:0 thru 3 do {
      for k:0 thru 3 do {
          if Gamma[i,j,k] # 0 then {
              display(Gamma[i,j,k])
          }}}}$
```

$$\Gamma_{1,2,2} = -l$$

$$\Gamma_{1,3,3} = -l\,\sin(\theta)^2$$

$$\Gamma_{2,1,2} = \frac{l}{l^2+k^2}$$

$$\Gamma_{2,2,1} = \frac{l}{l^2+k^2}$$

$$\Gamma_{2,3,3} = -\cos(\theta)\,\sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{l}{l^2+k^2}$$

$$\Gamma_{3,2,3} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{l}{l^2+k^2}$$

$$\Gamma_{3,3,2} = \frac{\cos(\theta)}{\sin(\theta)}$$

```
(%i9) /* compute Riemann tensor elements */
      for rho:0 thru 3 do {
      for sigma:0 thru 3 do {
      for mu:0 thru 3 do {
      for nu:0 thru 3 do {
          R[rho,sigma,mu,nu] :
          diff(Gamma[rho,nu,sigma],x[mu]) -
          diff(Gamma[rho,mu,sigma],x[nu]) +
          /* lambda sums by function call: */
          sum(
              Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
              Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
              lambda, 0, 3)
      }}}}$
```

```
(%i10) /* dislplay R's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
       for k:0 thru 3 do {
       for l:0 thru 3 do {
           R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
           if R[i,j,k,l] # 0 then display(R[i,j,k,l])
           }}}}$
```

$$R_{1,2,1,2} = -\frac{k^2}{l^2+k^2}$$

$$R_{1,2,2,1} = \frac{k^2}{l^2+k^2}$$

$$R_{1,3,1,3} = -\frac{k^2 \sin(\theta)^2}{l^2+k^2}$$

$$R_{1,3,3,1} = \frac{k^2 \sin(\theta)^2}{l^2+k^2}$$

$$R_{2,1,1,2} = \frac{k^2}{(l^2+k^2)^2}$$

$$R_{2,1,2,1} = -\frac{k^2}{(l^2+k^2)^2}$$

$$R_{2,3,2,3} = \frac{k^2 \sin(\theta)^2}{l^2+k^2}$$

$$R_{2,3,3,2} = -\frac{k^2 \sin(\theta)^2}{l^2+k^2}$$

$$R_{3,1,1,3} = \frac{k^2}{(l^2+k^2)^2}$$

$$R_{3,1,3,1} = -\frac{k^2}{(l^2+k^2)^2}$$

$$R_{3,2,2,3} = -\frac{k^2}{l^2+k^2}$$

$$R_{3,2,3,2} = \frac{k^2}{l^2+k^2}$$

(%i11)
```
/* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

(%i12)
```
/* dislplay Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
    Ric[i,j]  : /*ratsimp*/(factor(Ric[i,j])),
    if Ric[i,j] # 0 then display(Ric[i,j])
    }}$
```

$$Ric_{1,1} = -\frac{2\,k^2}{\left(l^2+k^2\right)^2}$$

(%i13)
```
/* Ricci Scalar */
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
    +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
    +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
    +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

(%o13) $-\frac{2\,k^2}{\left(l^2+k^2\right)^2}$

(%i14)
```
ratsimp(RicSc);
```

(%o14) $-\frac{2\,k^2}{l^4+2\,k^2\,l^2+k^4}$

(%i15)

```
/* Test for R^q */
for mu:    0 thru 3 do (
for sigma:0 thru 3 do (
for nu:    0 thru 3 do (
for rho:   0 thru 3 do (
    R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
    if R_q # 0 then (
        display("============Einstein equation R^q=0 not fulfilled! "),
        display(mu,sigma,nu,rho),
        display(R_q)
    )
))));
```

(%o15) *done*

(%i16)
```
/* Raising of indices,
   contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);
```

(%o16) 0

(%o17) 0

(%o18) 0

(%i19)
```
R0101: factor(R0101);
R0202: factor(R0202);
R0303: factor(R0303);
```

(%o19) 0

(%o20) 0

(%o21) 0

(%i22)
```
R1010: f(1,0);
R1212: f(1,2);
R1313: f(1,3);
```

(%o22) 0

(%o23) $-\frac{k^2}{(l^2+k^2)^2}$

(%o24) $-\frac{k^2}{(l^2+k^2)^2}$

(%i25)
```
R1010: factor(R1010);
R1212: factor(R1212);
R1313: factor(R1313);
```

(%o25) 0

(%o26) $-\frac{k^2}{(l^2+k^2)^2}$

(%o27) $-\frac{k^2}{(l^2+k^2)^2}$

(%i28)
```
R2020: f(2,0);
R2121: f(2,1);
R2323: f(2,3);
```

(%o28) 0

(%o29) $$-\frac{k^2}{(l^2+k^2)^3}$$

(%o30) $$\frac{k^2}{(l^2+k^2)^3}$$

(%i31)
```
R2020: factor(R2020);
R2121: factor(R2121);
R2323: factor(R2323);
```

(%o31) 0

(%o32) $$-\frac{k^2}{(l^2+k^2)^3}$$

(%o33) $$\frac{k^2}{(l^2+k^2)^3}$$

(%i34)
```
R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);
```

(%o34) 0

(%o35) $$-\frac{k^2}{(l^2+k^2)^3\sin(\theta)^2}$$

(%o36) $$\frac{k^2}{(l^2+k^2)^3\sin(\theta)^2}$$

(%i37)
```
R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);
```

(%o37) 0

(%o38) $$-\frac{k^2}{(l^2+k^2)^3\sin(\theta)^2}$$

(%o39) $$\frac{k^2}{(l^2+k^2)^3\sin(\theta)^2}$$

(%i40)
```
/* Coulomb law */
DivE : R0101 + R0202 + R0303;
```

(%o40) 0

(%i41)
```
ratsimp(DivE);
```

(%o41) 0

```
(%i42) /* J[r] */
       Jr : -(R1010 + R1212 + R1313);
```

(%o42) $\dfrac{2\,k^2}{(l^2+k^2)^2}$

```
(%i43) ratsimp(Jr);
```

(%o43) $\dfrac{2\,k^2}{l^4+2\,k^2\,l^2+k^4}$

```
(%i44) /* J[theta] */
       Jtheta : -(R2020 + R2121 + R2323);
```

(%o44) 0

```
(%i45) ratsimp(Jtheta);
```

(%o45) 0

```
(%i46) /* J[phi] */
       Jphi : -(R3030 + R3131 + R3232);
```

(%o46) 0

```
(%i47) ev(ratsimp(Jphi),r);
```

(%o47) 0

```
(%i48)
```