

(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $f(i, j) := \text{sum}(R_{i, j, \sigma, 0} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 0}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 1} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 1}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 2} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 2}, \sigma, 0, 3) +$

$\text{sum}(R_{i, j, \sigma, 3} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 3}, \sigma, 0, 3)$

(%i2) /* define coordinate vector */

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array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
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(%o2) x

(%o3) [t , r , θ , ϕ]

(%i4) /* g1 is symm. metric with indices 1...4 */

/* ***** Ciufolini - Wheeler (6.1.1)

weak field, slow motion limit 2nd variant ***** */

```
g1: matrix(
  [-(1-2*M/r), 0, 0, -4*J/r*sin(theta)^2],
  [0, (1-2*M/r)^-1, 0, 0],
  [0, 0, r^2, 0],
  [-4*J/r*sin(theta)^2, 0, 0, r^2*sin(theta)^2]
);
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(%o4)

$$\begin{bmatrix} \frac{2M}{r} - 1 & 0 & 0 & -\frac{4 \sin(\theta)^2 J}{r} \\ 0 & \frac{1}{1 - \frac{2M}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ -\frac{4 \sin(\theta)^2 J}{r} & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

(%i5) /* contravariant g is inverse of g */

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gContr1: ratsimp(invert(g1));
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$$\begin{pmatrix}
 \frac{r^4}{2r^3M - 16\sin(\theta)^2J^2 - r^4} & 0 & 0 & \frac{4rJ}{2r^3M - 16\sin(\theta)^2J^2 - r^4} \\
 0 & -\frac{2M-r}{r} & 0 & 0 \\
 0 & 0 & \frac{1}{r^2} & 0 \\
 \frac{4rJ}{2r^3M - 16\sin(\theta)^2J^2 - r^4} & 0 & 0 & \frac{2rM - r^2}{2r^3\sin(\theta)^2M - 16\sin(\theta)^4J^2 - r^4\sin(\theta)^2}
 \end{pmatrix}$$

(%i6)

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/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g[mu,nu]: g1[mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$

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(%i7) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu]) +
            diff(g[rho,mu],x[nu]) -
            diff(g[mu,nu],x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$

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(%i8) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$

```

$$\Gamma_{0,0,1} = \frac{8\sin(\theta)^2J^2}{r(2r^3M - 16\sin(\theta)^2J^2 - r^4)} - \frac{r^2M}{2r^3M - 16\sin(\theta)^2J^2 - r^4}$$

$$\Gamma_{0,0,2} = -\frac{16\cos(\theta)\sin(\theta)J^2}{2r^3M - 16\sin(\theta)^2J^2 - r^4}$$

$$\Gamma_{0,1,0} = \frac{8\sin(\theta)^2J^2}{r(2r^3M - 16\sin(\theta)^2J^2 - r^4)} - \frac{r^2M}{2r^3M - 16\sin(\theta)^2J^2 - r^4}$$

$$\Gamma_{0,1,3} = \frac{6 r^2 \sin(\theta)^2 J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,2,0} = - \frac{16 \cos(\theta) \sin(\theta) J^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{0,3,1} = \frac{6 r^2 \sin(\theta)^2 J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{1,0,0} = - \frac{M(2M - r)}{r^3}$$

$$\Gamma_{1,0,3} = \frac{2 \sin(\theta)^2 J(2M - r)}{r^3}$$

$$\Gamma_{1,1,1} = \frac{M(2M - r)}{r^3 \left(1 - \frac{2M}{r}\right)^2}$$

$$\Gamma_{1,2,2} = 2M - r$$

$$\Gamma_{1,3,0} = \frac{2 \sin(\theta)^2 J(2M - r)}{r^3}$$

$$\Gamma_{1,3,3} = \sin(\theta)^2 (2M - r)$$

$$\Gamma_{2,0,3} = \frac{4 \cos(\theta) \sin(\theta) J}{r^3}$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,0} = \frac{4 \cos(\theta) \sin(\theta) J}{r^3}$$

$$\Gamma_{2,3,3} = - \cos(\theta) \sin(\theta)$$

$$\Gamma_{3,0,1} = \frac{2 \sin(\theta)^2 J(2rM - r^2)}{r^2(2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \frac{4JM}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,0,2} = - \frac{4 \cos(\theta) \sin(\theta) J(2rM - r^2)}{r(2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

$$\Gamma_{3,1,0} = \frac{2 \sin(\theta)^2 J(2rM - r^2)}{r^2(2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} - \frac{4JM}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,1,3} = \frac{r \sin(\theta)^2 (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} + \frac{8 \sin(\theta)^2 J^2}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,2,0} = - \frac{4 \cos(\theta) \sin(\theta) J(2rM - r^2)}{r(2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

$$\Gamma_{3,2,3} = \frac{r^2 \cos(\theta) \sin(\theta) (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$\Gamma_{3,3,1} = \frac{r \sin(\theta)^2 (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} + \frac{8 \sin(\theta)^2 J^2}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$\Gamma_{3,3,2} = \frac{r^2 \cos(\theta) \sin(\theta) (2rM - r^2)}{2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

```
(%i9) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
  R[rho,sigma,mu,nu] :
  diff(Gamma[rho,nu,sigma],x[mu]) -
  diff(Gamma[rho,mu,sigma],x[nu]) +
  /* lambda sums by function call: */
  sum(
    Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
    Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
    lambda, 0, 3)
}}}}$
```

```
(%i10) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
  R[i,j,k,l] : /ratsimp*/(factor(R[i,j,k,l])),
  if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$R_{0,0,0,3} = \frac{4 \sin(\theta)^2 J (2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{r^4 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,0,1,2} = \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2M - r)}{(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,0,2,1} = - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2M - r)}{(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,0,3,0} = - \frac{4 \sin(\theta)^2 J (2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{r^4 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,1,0,1} = - (2 (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 56 r^4 \sin(\theta)^2 J^2 M + r^8 M - 96 r \sin(\theta)^4 J^4 - 14 r^5 \sin(\theta)^2 J^2)) / (r^2 (2M - r) (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,1,0,2} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,1,0} = (2 (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 56 r^4 \sin(\theta)^2 J^2 M + r^8 M - 96 r \sin(\theta)^4 J^4 - 14 r^5 \sin(\theta)^2 J^2)) / (r^2 (2 M - r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,1,1,3} = - \frac{6 r \sin(\theta)^2 J (4 r^3 M^2 + 16 \sin(\theta)^2 J^2 M - 4 r^4 M - 16 r \sin(\theta)^2 J^2 + r^5)}{(2 M - r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,2,0} = \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,2,3} = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,3,1} = \frac{6 r \sin(\theta)^2 J (4 r^3 M^2 + 16 \sin(\theta)^2 J^2 M - 4 r^4 M - 16 r \sin(\theta)^2 J^2 + r^5)}{(2 M - r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,1,3,2} = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,0,1} = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,0,2} = - (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 80 r^4 \sin(\theta)^2 J^2 M - 32 r^4 \cos(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) / (r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,2,1,0} = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,2,1,3} = \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{0,2,2,0} = (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 80 r^4 \sin(\theta)^2 J^2 M - 32 r^4 \cos(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) / (r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{0,2,2,3} = - \frac{6 r^2 \sin(\theta)^2 J (2 M - r)}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{0,2,3,1} = - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{0,2,3,2} = \frac{6 r^2 \sin(\theta)^2 J(2 M - r)}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{0,3,0,3} = - \frac{\sin(\theta)^2 (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{r(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{0,3,1,2} = - \frac{6 r^5 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,3,2,1} = \frac{6 r^5 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{0,3,3,0} = \frac{\sin(\theta)^2 (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{r(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,0,1} = - \frac{2(4 r^3 M^3 - 32 \sin(\theta)^2 J^2 M^2 - 4 r^4 M^2 + 16 r \sin(\theta)^2 J^2 M + r^5 M - 2 r^2 \sin(\theta)^2 J^2)}{r^4(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,0,2} = \frac{8 \cos(\theta) \sin(\theta) J^2(2 M - r)}{r^2(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,1,0} = \frac{2(4 r^3 M^3 - 32 \sin(\theta)^2 J^2 M^2 - 4 r^4 M^2 + 16 r \sin(\theta)^2 J^2 M + r^5 M - 2 r^2 \sin(\theta)^2 J^2)}{r^4(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,1,3} = - \frac{2 \sin(\theta)^2 J(8 r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10 r^4 M + 24 r \sin(\theta)^2 J^2 + 3 r^5)}{r^4(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,2,0} = - \frac{8 \cos(\theta) \sin(\theta) J^2(2 M - r)}{r^2(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,2,3} = \frac{12 \cos(\theta) \sin(\theta) J(2 M - r)(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,3,1} = \frac{2 \sin(\theta)^2 J(8 r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10 r^4 M + 24 r \sin(\theta)^2 J^2 + 3 r^5)}{r^4(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,0,3,2} = - \frac{12 \cos(\theta) \sin(\theta) J(2 M - r)(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,2,0,3} = \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{1,2,1,2} = - \frac{M}{r}$$

$$R_{1,2,2,1} = \frac{M}{r}$$

$$R_{1,2,3,0} = -\frac{6 \cos(\theta) \sin(\theta) J(2M - r)^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4}$$

$$R_{1,3,0,1} = \frac{2 \sin(\theta)^2 J(8r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10r^4 M + 24r \sin(\theta)^2 J^2 + 3r^5)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,0,2} = -\frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

$$R_{1,3,1,0} = -\frac{2 \sin(\theta)^2 J(8r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10r^4 M + 24r \sin(\theta)^2 J^2 + 3r^5)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,1,3} = -\frac{\sin(\theta)^2(2r^3 M^2 + 56 \sin(\theta)^2 J^2 M - r^4 M - 36r \sin(\theta)^2 J^2)}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{1,3,2,0} = \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

$$R_{1,3,3,1} = \frac{\sin(\theta)^2(2r^3 M^2 + 56 \sin(\theta)^2 J^2 M - r^4 M - 36r \sin(\theta)^2 J^2)}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,0,1} = -\frac{8 \cos(\theta) \sin(\theta) J^2}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,0,2} = \frac{(2M - r)(2r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 16r \cos(\theta)^2 J^2)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,1,0} = \frac{8 \cos(\theta) \sin(\theta) J^2}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,1,3} = -\frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

$$R_{2,0,2,0} = -\frac{(2M - r)(2r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 16r \cos(\theta)^2 J^2)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,0,2,3} = \frac{2 \sin(\theta)^2 J(2M - 3r)}{r^4}$$

$$R_{2,0,3,1} = \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

$$R_{2,0,3,2} = -\frac{2 \sin(\theta)^2 J(2M - 3r)}{r^4}$$

$$R_{2,1,0,3} = \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,1,1,2} = -\frac{M}{r^2(2M - r)}$$

$$R_{2,1,2,1} = \frac{M}{r^2(2M - r)}$$

$$R_{2,1,3,0} = -\frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,0,1} = \frac{12 \cos(\theta) \sin(\theta) J(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,0,2} = -\frac{2 \sin(\theta)^2 J(2M - 3r)}{r^4}$$

$$R_{2,3,1,0} = -\frac{12 \cos(\theta) \sin(\theta) J(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{2,3,2,0} = \frac{2 \sin(\theta)^2 J(2M - 3r)}{r^4}$$

$$R_{2,3,2,3} = \frac{2 \sin(\theta)^2 M}{r}$$

$$R_{2,3,3,2} = -\frac{2 \sin(\theta)^2 M}{r}$$

$$R_{3,0,0,3} = \frac{(2M - r)(2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,0,1,2} = \frac{6r^2 \cos(\theta) J(2M - r)^2}{\sin(\theta)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,0,2,1} = -\frac{6r^2 \cos(\theta) J(2M - r)^2}{\sin(\theta)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,0,3,0} = -\frac{(2M - r)(2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{r^4(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,1,0,1} = -\frac{2J(12r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12r^4 M + 16r \sin(\theta)^2 J^2 + 3r^5)}{r(2M - r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,0,2} = -\frac{2 \cos(\theta) J(12r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12r^4 M + 32r \sin(\theta)^2 J^2 + 3r^5)}{r \sin(\theta)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,1,0} = \frac{2J(12r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12r^4 M + 16r \sin(\theta)^2 J^2 + 3r^5)}{r(2M - r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,1,3} = -\left(4r^6 M^3 + 176r^3 \sin(\theta)^2 J^2 M^2 - 4r^7 M^2 - 512 \sin(\theta)^4 J^4 M - 208r^4 \sin(\theta)^2 J^2 M + r^8 M + 192r \sin(\theta)^4 J^4 + 60r^5 \sin(\theta)^2 J^2\right) / \left(r^2(2M - r)(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2\right)$$

$$R_{3,1,2,0} = \frac{2 \cos(\theta) J(12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,2,3} = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,1,3,1} = (4 r^6 M^3 + 176 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 - 512 \sin(\theta)^4 J^4 M - 208 r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2) / (r^2 (2 M - r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$R_{3,1,3,2} = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,0,1} = - \frac{4 \cos(\theta) J(12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,0,2} = - \frac{2 J(2 M - r) (6 r^3 M - 48 \sin(\theta)^2 J^2 - 32 \cos(\theta)^2 J^2 - 3 r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,1,0} = \frac{4 \cos(\theta) J(12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,1,3} = \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,2,0} = \frac{2 J(2 M - r) (6 r^3 M - 48 \sin(\theta)^2 J^2 - 32 \cos(\theta)^2 J^2 - 3 r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,2,2,3} = - \frac{2 (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,3,1} = - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,2,3,2} = \frac{2 (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,3,0,3} = - \frac{4 \sin(\theta)^2 J(2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$R_{3,3,1,2} = - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,3,2,1} = \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$R_{3,3,3,0} = \frac{4 \sin(\theta)^2 J (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

```
(%i11) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i12) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = - \frac{8 J^2 (6 \sin(\theta)^2 M^2 - 4 r \sin(\theta)^2 M - 8 r \cos(\theta)^2 M + r^2 \sin(\theta)^2 + 4 r^2 \cos(\theta)^2)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$Ric_{0,3} = \frac{32 \sin(\theta)^2 J^3 (3 \sin(\theta)^2 M + r \sin(\theta)^2 - 2 r \cos(\theta)^2)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$Ric_{1,1} = \frac{8 \sin(\theta)^2 J^2 (30 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 36 r^4 M + 48 r \sin(\theta)^2 J^2 + 11 r^5)}{r^2 (2 M - r) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$Ric_{1,2} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (9 r^3 M - 48 \sin(\theta)^2 J^2 - 5 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$Ric_{2,1} = - \frac{16 \cos(\theta) \sin(\theta) J^2 (9 r^3 M - 48 \sin(\theta)^2 J^2 - 5 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$Ric_{2,2} = (16 J^2 (6 r^3 \sin(\theta)^2 M^2 - 48 \sin(\theta)^4 J^2 M - 9 r^4 \sin(\theta)^2 M + 2 r^4 \cos(\theta)^2 M + 48 r \sin(\theta)^4 J^2 + 3 r^5 \sin(\theta)^2 - r^5 \cos(\theta)^2)) / (r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2)$$

$$Ric_{3,0} = \frac{32 \sin(\theta)^2 J^3 (3 \sin(\theta)^2 M + r \sin(\theta)^2 - 2 r \cos(\theta)^2)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

$$Ric_{3,3} = - \frac{8 \sin(\theta)^2 J^2 (12 \sin(\theta)^2 M - 5 r \sin(\theta)^2 - 2 r \cos(\theta)^2)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

```
(%i13) /* Ricci Scalar */
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
+ sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
+ sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
+ sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

```
(%o13) ( 16 J^2 ( 6 r^3 sin(theta)^2 M^2 - 48 sin(theta)^4 J^2 M - 9 r^4 sin(theta)^2 M + 2 r^4
cos(theta)^2 M + 48 r sin(theta)^4 J^2 + 3 r^5 sin(theta)^2 - r^5 cos(theta)^2 ) ) / ( r^3
(2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2 ) -
8 J^2 (6 sin(theta)^2 M^2 - 4 r sin(theta)^2 M - 8 r cos(theta)^2 M + r^2 sin(theta)^2 + 4 r^2 cos(theta)^2 )
-----
(2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2
8 sin(theta)^2 J^2 (30 r^3 M^2 - 96 sin(theta)^2 J^2 M - 36 r^4 M + 48 r sin(theta)^2 J^2 + 11 r^5 )
-----
r^3 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2
8 sin(theta)^2 J^2 (2 r M - r^2) (12 sin(theta)^2 M - 5 r sin(theta)^2 - 2 r cos(theta)^2 )
-----
r (2 r^3 M - 16 sin(theta)^2 J^2 - r^4) (2 r^3 sin(theta)^2 M - 16 sin(theta)^4 J^2 - r^4 sin(theta)^2 )
+
256 sin(theta)^2 J^4 (3 sin(theta)^2 M + r sin(theta)^2 - 2 r cos(theta)^2 )
-----
r^3 (2 r^3 M - 16 sin(theta)^2 J^2 - r^4)^2
```

```
(%i14) ratsimp(RicSc);
```

```
(%o14) - ( 384 r^3 sin(theta)^2 J^2 M^2 +
((- 352 r^4 sin(theta)^2 - 128 r^4 cos(theta)^2) J^2 - 768 sin(theta)^4 J^4 ) M +
(512 r cos(theta)^2 sin(theta)^2 - 640 r sin(theta)^4 ) J^4 + (88 r^5 sin(theta)^2 + 64 r^5 cos(theta)^2 ) J^2 ) / (
4 r^9 M^2 + (- 64 r^6 sin(theta)^2 J^2 - 4 r^10 ) M + 256 r^3 sin(theta)^4 J^4 + 32 r^7 sin(theta)^2 J^2 +
r^11 )
```

```
(%i15)
```

```
/* Test for R^q */
for mu: 0 thru 3 do (
for sigma:0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
if R_q # 0 then (
display("====Einstein equation R^q=0 not fulfilled! "),
display(mu,sigma,nu,rho),
display(R_q)
)
))));
```

```
====Einstein equation R^q=0 not fulfilled! =
```

```
====Einstein equation R^q=0 not fulfilled!
```

```
μ = 0
```

```
σ = 0
```

```
ν = 1
```

```
ρ = 2
```

$$R_q = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 2$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled! =

=====Einstein equation $R^{\hat{q}}_q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 0$$

$$R_q = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 3$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 2$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} +$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = - \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$R_q = \frac{8 \cos(\theta) \sin(\theta) J^2 (12 r^3 M - 48 \sin(\theta)^2 J^2 - 7 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{16 \cos(\theta) \sin(\theta) J^2 (3 r^3 M - 24 \sin(\theta)^2 J^2 - 2 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} + \frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} - \frac{6 r^5 \cos(\theta) \sin(\theta) J (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = \frac{12 r^2 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 0$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = - \frac{12 r^2 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) \sin(\theta) J}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} +$$

$$\frac{6 r^5 \cos(\theta) \sin(\theta) J(2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 2$$

$$\rho = 3$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 M - r)(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} -$$

$$\frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r^3}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 1$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 2$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2 M - r)(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)^2}{2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4} +$$

$$\frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r^3}$$

=====*Einstein equation $R^q=0$ not fulfilled!* =

=====*Einstein equation $R^q=0$ not fulfilled!*

$$\mu = 1$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 3$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2M - r)(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J(2M - r)^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4} + \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 0$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2M - r)(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2M - r)^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4} - \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 2$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2M - r)(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2M - r)^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4} - \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 1$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 0$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2M - r)(2r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^3(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J(2M - r)^2}{2r^3 M - 16 \sin(\theta)^2 J^2 - r^4} + \frac{6 \cos(\theta) \sin(\theta) J(2M - r)}{r^3}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 0$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 1$$

$$\nu = 0$$

$$\rho = 3$$

$$R_q = - \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 0$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J(2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J(2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 2$$

$$\sigma = 3$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = \frac{12 \cos(\theta) \sin(\theta) J (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{6 \cos(\theta) \sin(\theta) J (2 M - r)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

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$$\sigma = 3$$

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$$\frac{6 \cos(\theta) \sin(\theta) J}{r^4}$$

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=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 0$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

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$$\sigma = 0$$

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$$\rho = 1$$

$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

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$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

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=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 0$$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} +$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 1$$

$$\nu = 2$$

$$\rho = 3$$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 1$$

$$\nu = 3$$

$$\rho = 2$$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 0$$

$$\rho = 1$$

$$R_q = - \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 0$$

$$R_q = \frac{4 \cos(\theta) J (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{2 \cos(\theta) J (12 r^3 M^2 - 96 \sin(\theta)^2 J^2 M - 12 r^4 M + 32 r \sin(\theta)^2 J^2 + 3 r^5)}{r \sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} -$$

$$\frac{6 r^2 \cos(\theta) J (2 M - r)^2}{\sin(\theta) (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 1$$

$$\rho = 3$$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} +$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 2$$

$$\nu = 3$$

$$\rho = 1$$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 3$$

$$\nu = 1$$

$$\rho = 2$$

$$R_q = \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} - \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

=====Einstein equation $R^q=0$ not fulfilled! =

=====Einstein equation $R^q=0$ not fulfilled!

$$\mu = 3$$

$$\sigma = 3$$

$$\nu = 2$$

$$\rho = 1$$

$$R_q = - \frac{48 \cos(\theta) \sin(\theta) J^2 (2 r^3 M - 8 \sin(\theta)^2 J^2 - r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \frac{24 \cos(\theta) \sin(\theta) J^2}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} + \frac{24 r^2 \cos(\theta) \sin(\theta) J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

(%o15) done

```
(%i16) /* Raising of indices,
        contravariant metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
        /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

        R0101: f(0,1);
        R0202: f(0,2);
        R0303: f(0,3);
```

$$(\%o16) \quad (2 r (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 56 r^4 \sin(\theta)^2 J^2 M + r^8 M - 96 r \sin(\theta)^4 J^4 - 14 r^5 \sin(\theta)^2 J^2)) /$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 -$$

$$\frac{24 r \sin(\theta)^2 J^2 (4 r^3 M^2 + 16 \sin(\theta)^2 J^2 M - 4 r^4 M - 16 r \sin(\theta)^2 J^2 + r^5)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

$$(\%o17) \quad \frac{24 r \sin(\theta)^2 J^2 (2 M - r)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - (r (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7$$

$$M^2 + 256 \sin(\theta)^4 J^4 M + 80 r^4 \sin(\theta)^2 J^2 M - 32 r^4 \cos(\theta)^2 J^2 M + r^8 M - 384 r \sin(\theta)^4 J^4 - 24 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2)) /$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3$$

$$(\%o18) \quad \frac{16 r \sin(\theta)^2 J^2 (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3} -$$

$$\frac{r^3 \sin(\theta)^2 (2 r M - r^2) (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

```
(%i19) R0101: factor(R0101);
        R0202: factor(R0202);
        R0303: factor(R0303);
```

$$\begin{aligned} & (2 r (4 r^6 M^3 - 112 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 64 \sin(\theta)^4 J^4 M + \\ & 104 r^4 \sin(\theta)^2 J^2 M + r^8 M + 96 r \sin(\theta)^4 J^4 - 26 r^5 \sin(\theta)^2 J^2)) / \\ & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 \end{aligned}$$

$$\begin{aligned} & - (r (4 r^6 M^3 - 160 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 1024 \sin(\theta)^4 J^4 M + \\ & 176 r^4 \sin(\theta)^2 J^2 M - 32 r^4 \cos(\theta)^2 J^2 M + r^8 M - 768 r \sin(\theta)^4 J^4 - 48 r^5 \\ & \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2)) / (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 \end{aligned}$$

$$\begin{aligned} & - \frac{r(2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} \end{aligned}$$

```
(%i22) R1010: f(1,0);
        R1212: f(1,2);
        R1313: f(1,3);
```

$$\begin{aligned} & \frac{8 \sin(\theta)^2 J^2 (2 M - r) (8 r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10 r^4 M + 24 r \sin(\theta)^2 J^2 + 3 r^5)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} - \end{aligned}$$

$$\begin{aligned} & \frac{2 (2 M - r) (4 r^3 M^3 - 32 \sin(\theta)^2 J^2 M^2 - 4 r^4 M^2 + 16 r \sin(\theta)^2 J^2 M + r^5 M - 2 r^2 \sin(\theta)^2 J^2)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} \end{aligned}$$

$$\begin{aligned} & \frac{M (2 M - r)}{r^4} \end{aligned}$$

$$\begin{aligned} & \frac{8 \sin(\theta)^2 J^2 (2 M - r) (8 r^3 M^2 - 64 \sin(\theta)^2 J^2 M - 10 r^4 M + 24 r \sin(\theta)^2 J^2 + 3 r^5)}{r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2} + \end{aligned}$$

$$\begin{aligned} & \frac{\sin(\theta)^2 (2 M - r) (2 r M - r^2) (2 r^3 M^2 + 56 \sin(\theta)^2 J^2 M - r^4 M - 36 r \sin(\theta)^2 J^2)}{r^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4) (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} \end{aligned}$$

```
(%i25) R1010: factor(R1010);
        R1212: factor(R1212);
        R1313: factor(R1313);
```

$$\begin{aligned} & - (2 (2 M - r) (4 r^6 M^3 - 64 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 256 \sin(\theta)^4 \\ & J^4 M + 56 r^4 \sin(\theta)^2 J^2 M + r^8 M - 96 r \sin(\theta)^4 J^4 - 14 r^5 \sin(\theta)^2 J^2)) / (\\ & r^4 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2) \end{aligned}$$

$$(\%o26) \quad \frac{M(2M - r)}{r^4}$$

$$(\%o27) \quad \left((2M - r) (4r^6 M^3 + 176r^3 \sin(\theta)^2 J^2 M^2 - 4r^7 M^2 - 512 \sin(\theta)^4 J^4 M - 208r^4 \sin(\theta)^2 J^2 M + r^8 M + 192r \sin(\theta)^4 J^4 + 60r^5 \sin(\theta)^2 J^2) \right) / \left(r^4 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 \right)$$

(%i28) R2020: f(2,0);
 R2121: f(2,1);
 R2323: f(2,3);

$$(\%o28) \quad \frac{8 \sin(\theta)^2 J^2 (2M - 3r)}{r^5 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)} -$$

$$\frac{(2M - r)(2r^3 M^2 - 16 \sin(\theta)^2 J^2 M - r^4 M - 16r \cos(\theta)^2 J^2)}{r^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$(\%o29) \quad - \frac{M}{r^5}$$

$$(\%o30) \quad \frac{2 \sin(\theta)^2 M (2rM - r^2)}{r^3 (2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} + \frac{8 \sin(\theta)^2 J^2 (2M - 3r)}{r^5 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

(%i31) R2020: factor(R2020);
 R2121: factor(R2121);
 R2323: factor(R2323);

$$(\%o31) \quad - \left(4r^6 M^3 - 64r^3 \sin(\theta)^2 J^2 M^2 - 4r^7 M^2 + 256 \sin(\theta)^4 J^4 M + 80r^4 \sin(\theta)^2 J^2 M - 32r^4 \cos(\theta)^2 J^2 M + r^8 M - 384r \sin(\theta)^4 J^4 - 24r^5 \sin(\theta)^2 J^2 + 16r^5 \cos(\theta)^2 J^2 \right) / \left(r^5 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 \right)$$

$$(\%o32) \quad - \frac{M}{r^5}$$

$$(\%o33) \quad \frac{2(2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 12r \sin(\theta)^2 J^2)}{r^5 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)}$$

(%i34) R3030: f(3,0);
 R3131: f(3,1);
 R3232: f(3,2);

$$(\%o34) \quad \frac{16 J^2 (2M - r)(2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{r^2 (2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3} -$$

$$\frac{(2M - r)(2rM - r^2)(2r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4r \sin(\theta)^2 J^2 - 16r \cos(\theta)^2 J^2)}{(2r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)}$$

$$(\%o35) \quad \frac{8 J^2 (12 r^3 M^2 - 48 \sin(\theta)^2 J^2 M - 12 r^4 M + 16 r \sin(\theta)^2 J^2 + 3 r^5)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3} - ((2 r M - r^2) ($$

$$4 r^6 M^3 + 176 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 - 512 \sin(\theta)^4 J^4 M - 208 r^4 \sin(\theta)^2 J^2 M + r^8 M + 192 r \sin(\theta)^4 J^4 + 60 r^5 \sin(\theta)^2 J^2) / (r^3$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2))$$

$$(\%o36) \quad \frac{2 (2 r M - r^2) (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 12 r \sin(\theta)^2 J^2)}{r^3 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4) (2 r^3 \sin(\theta)^2 M - 16 \sin(\theta)^4 J^2 - r^4 \sin(\theta)^2)} -$$

$$\frac{8 J^2 (2 M - r) (6 r^3 M - 48 \sin(\theta)^2 J^2 - 32 \cos(\theta)^2 J^2 - 3 r^4)}{r (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3}$$

```
(%i37) R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);
```

$$(\%o37) \quad - \frac{(2 M - r) (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{r^2 \sin(\theta)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

$$(\%o38) \quad - (8 r^6 M^4 + 352 r^3 \sin(\theta)^2 J^2 M^3 - 12 r^7 M^3 - 1024 \sin(\theta)^4 J^4 M^2 - 688 r^4 \sin(\theta)^2 J^2 M^2 + 6 r^8 M^2 + 1280 r \sin(\theta)^4 J^4 M + 424 r^5 \sin(\theta)^2 J^2 M - r^9 M - 320 r^2 \sin(\theta)^4 J^4 - 84 r^6 \sin(\theta)^2 J^2) / (r^2 \sin(\theta)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3)$$

$$(\%o39) \quad (2 (2 M - r) (4 r^6 M^3 - 16 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 - 128 \sin(\theta)^4 J^4 M - 40 r^4 \sin(\theta)^2 J^2 M + r^8 M + 384 r \sin(\theta)^4 J^4 + 128 r \cos(\theta)^2 \sin(\theta)^2 J^4 + 24 r^5 \sin(\theta)^2 J^2) / (r^2 \sin(\theta)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3)$$

```
(%i40) /* Coulomb law */
DivE : R0101 + R0202 + R0303;
```

$$(\%o40) \quad (2 r (4 r^6 M^3 - 112 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 64 \sin(\theta)^4 J^4 M + 104 r^4 \sin(\theta)^2 J^2 M + r^8 M + 96 r \sin(\theta)^4 J^4 - 26 r^5 \sin(\theta)^2 J^2) /$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 - (r (4 r^6 M^3 - 160 r^3 \sin(\theta)^2 J^2 M^2 - 4 r^7 M^2 + 1024 \sin(\theta)^4 J^4 M + 176 r^4 \sin(\theta)^2 J^2 M - 32 r^4 \cos(\theta)^2 J^2 M + r^8 M - 768 r \sin(\theta)^4 J^4 - 48 r^5 \sin(\theta)^2 J^2 + 16 r^5 \cos(\theta)^2 J^2) /$$

$$(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3 -$$

$$\frac{r (2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2)}{(2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2}$$

```
(%i41) ratsimp(DivE);
```

```
(%o41) - ( 48 r^4 sin(θ)^2 J^2 M^2 +
(768 r sin(θ)^4 J^4 + (- 32 r^5 sin(θ)^2 - 64 r^5 cos(θ)^2) J^2 ) M +
(256 r^2 cos(θ)^2 sin(θ)^2 - 896 r^2 sin(θ)^4) J^4 + (8 r^6 sin(θ)^2 + 32 r^6 cos(θ)^2) J^2 ) /
( 8 r^9 M^3 + (- 192 r^6 sin(θ)^2 J^2 - 12 r^10) M^2 +
(1536 r^3 sin(θ)^4 J^4 + 192 r^7 sin(θ)^2 J^2 + 6 r^11) M - 4096 sin(θ)^6 J^6 - 768 r^4
sin(θ)^4 J^4 - 48 r^8 sin(θ)^2 J^2 - r^12 )
```

```
(%i42) /* J[r] */
```

```
Jr : -(R1010 + R1212 + R1313);
```

```
(%o42) - ( (2 M - r) ( 4 r^6 M^3 + 176 r^3 sin(θ)^2 J^2 M^2 - 4 r^7 M^2 - 512 sin(θ)^4
J^4 M - 208 r^4 sin(θ)^2 J^2 M + r^8 M + 192 r sin(θ)^4 J^4 + 60 r^5 sin(θ)^2 J^2 ) ) /
( r^4 (2 r^3 M - 16 sin(θ)^2 J^2 - r^4)^2 ) + ( 2 (2 M - r) ( 4 r^6 M^3 - 64 r^3 sin(θ)^2 J^2
M^2 - 4 r^7 M^2 + 256 sin(θ)^4 J^4 M + 56 r^4 sin(θ)^2 J^2 M + r^8 M - 96 r sin(θ)^4 J^4
- 14 r^5 sin(θ)^2 J^2 ) ) / ( r^4 (2 r^3 M - 16 sin(θ)^2 J^2 - r^4)^2 ) - \frac{M(2 M - r)}{r^4}
```

```
(%i43) ratsimp(Jr);
```

```
(%o43) - ( 480 r^3 sin(θ)^2 J^2 M^3 + (- 1536 sin(θ)^4 J^4 - 816 r^4 sin(θ)^2 J^2) M^2 +
(1536 r sin(θ)^4 J^4 + 464 r^5 sin(θ)^2 J^2) M - 384 r^2 sin(θ)^4 J^4 - 88 r^6 sin(θ)^2 J^2
) / ( 4 r^10 M^2 + (- 64 r^7 sin(θ)^2 J^2 - 4 r^11) M + 256 r^4 sin(θ)^4 J^4 + 32 r^8
sin(θ)^2 J^2 + r^12 )
```

```
(%i44) /* J[theta] */
```

```
Jtheta : -(R2020 + R2121 + R2323);
```

```
(%o44) ( 4 r^6 M^3 - 64 r^3 sin(θ)^2 J^2 M^2 - 4 r^7 M^2 + 256 sin(θ)^4 J^4 M + 80 r^4
sin(θ)^2 J^2 M - 32 r^4 cos(θ)^2 J^2 M + r^8 M - 384 r sin(θ)^4 J^4 - 24 r^5 sin(θ)^2
J^2 + 16 r^5 cos(θ)^2 J^2 ) / ( r^5 (2 r^3 M - 16 sin(θ)^2 J^2 - r^4)^2 ) -
\frac{2(2 r^3 M^2 + 8 sin(θ)^2 J^2 M - r^4 M - 12 r sin(θ)^2 J^2)}{r^5 (2 r^3 M - 16 sin(θ)^2 J^2 - r^4)} + \frac{M}{r^5}
```

```
(%i45) ratsimp(Jtheta);
```

```
(%o45) - ( 96 r^3 sin(θ)^2 J^2 M^2 +
((32 r^4 cos(θ)^2 - 144 r^4 sin(θ)^2) J^2 - 768 sin(θ)^4 J^4) M + 768 r sin(θ)^4 J^4 +
(48 r^5 sin(θ)^2 - 16 r^5 cos(θ)^2) J^2 ) / ( 4 r^11 M^2 + (- 64 r^8 sin(θ)^2 J^2 - 4 r^12) M +
```

$$256 r^5 \sin(\theta)^4 J^4 + 32 r^9 \sin(\theta)^2 J^2 + r^{13})$$

(%i46) /* J[phi] */

Jphi : -(R3030 + R3131 + R3232);

$$\begin{aligned} & (8 r^6 M^4 + 352 r^3 \sin(\theta)^2 J^2 M^3 - 12 r^7 M^3 - 1024 \sin(\theta)^4 J^4 M^2 - \\ & 688 r^4 \sin(\theta)^2 J^2 M^2 + 6 r^8 M^2 + 1280 r \sin(\theta)^4 J^4 M + 424 r^5 \sin(\theta)^2 J^2 M - \\ & r^9 M - 320 r^2 \sin(\theta)^4 J^4 - 84 r^6 \sin(\theta)^2 J^2) / (r^2 \sin(\theta)^2 \\ & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3) - (2 (2 M - r) (4 r^6 M^3 - 16 r^3 \sin(\theta)^2 J^2 M^2 - 4 \\ & r^7 M^2 - 128 \sin(\theta)^4 J^4 M - 40 r^4 \sin(\theta)^2 J^2 M + r^8 M + 384 r \sin(\theta)^4 J^4 + 128 \\ & r \cos(\theta)^2 \sin(\theta)^2 J^4 + 24 r^5 \sin(\theta)^2 J^2) / (r^2 \sin(\theta)^2 \\ & (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^3) + \\ & (2 M - r)(2 r^3 M^2 + 8 \sin(\theta)^2 J^2 M - r^4 M - 4 r \sin(\theta)^2 J^2 - 16 r \cos(\theta)^2 J^2) \\ & \hline & r^2 \sin(\theta)^2 (2 r^3 M - 16 \sin(\theta)^2 J^2 - r^4)^2 \end{aligned}$$

(%i47) ev(ratsimp(Jphi),r);

$$\begin{aligned} & (384 r^3 \sin(\theta)^2 J^2 M^3 + \\ & ((-544 r^4 \sin(\theta)^2 - 64 r^4 \cos(\theta)^2) J^2 - 768 \sin(\theta)^4 J^4) M^2 + \\ & ((256 r^5 \sin(\theta)^2 + 64 r^5 \cos(\theta)^2) J^2 - 256 r \sin(\theta)^4 J^4) M + 384 r^2 \sin(\theta)^4 J^4 + \\ & (-40 r^6 \sin(\theta)^2 - 16 r^6 \cos(\theta)^2) J^2) / (8 r^{11} \sin(\theta)^2 M^3 + \\ & (-192 r^8 \sin(\theta)^4 J^2 - 12 r^{12} \sin(\theta)^2) M^2 + \\ & (1536 r^5 \sin(\theta)^6 J^4 + 192 r^9 \sin(\theta)^4 J^2 + 6 r^{13} \sin(\theta)^2) M - 4096 r^2 \sin(\theta)^8 J^6 - \\ & 768 r^6 \sin(\theta)^6 J^4 - 48 r^{10} \sin(\theta)^4 J^2 - r^{14} \sin(\theta)^2) \end{aligned}$$

(%i48) DivE_p: ratsimp(ev(at(DivE, [M=2, J=1, theta=%pi/2])));

$$\begin{aligned} & (8 r^6 - 64 r^5 + 192 r^4 - 896 r^2 + 1536 r \\ & \hline & r^{12} - 12 r^{11} + 48 r^{10} - 64 r^9 + 48 r^8 - 384 r^7 + 768 r^6 + 768 r^4 - 3072 r^3 + 4096 \end{aligned}$$

(%i49) Jr_p: ratsimp(ev(at(Jr, [M=2, J=1, theta=%pi/2])));

$$\begin{aligned} & (88 r^6 - 928 r^5 + 3264 r^4 - 3840 r^3 + 384 r^2 - 3072 r + 6144 \\ & \hline & r^{12} - 8 r^{11} + 16 r^{10} + 32 r^8 - 128 r^7 + 256 r^4 \end{aligned}$$

(%i50) Jtheta_p: ratsimp(ev(at(Jtheta, [M=2, J=1, theta=%pi/2])));

$$\begin{aligned} & (48 \\ & \hline & r^8 - 2 r^7 - 4 r^6 - 8 r^5 \end{aligned}$$

(%i51) Jphi_p: ratsimp(ev(at(Jphi, [M=2, J=1, theta=%pi/2])));

```
(%o51) 
$$\frac{40 r^6 - 512 r^5 + 2176 r^4 - 3072 r^3 - 384 r^2 + 512 r + 3072}{r^{14} - 12 r^{13} + 48 r^{12} - 64 r^{11} + 48 r^{10} - 384 r^9 + 768 r^8 + 768 r^6 - 3072 r^5 + 4096 r^2}$$

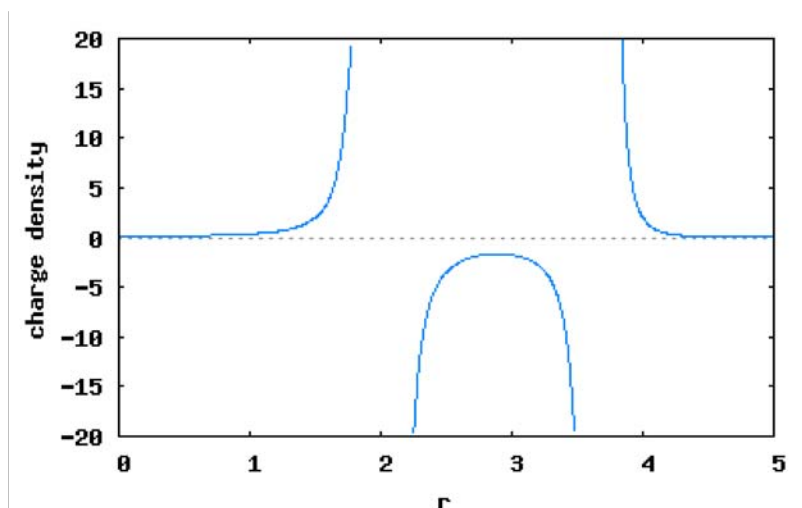
```

```
(%i52)
```

```
wxplot2d([DivE_p], [r,0,5],[y,-20,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "charge density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t52)
```

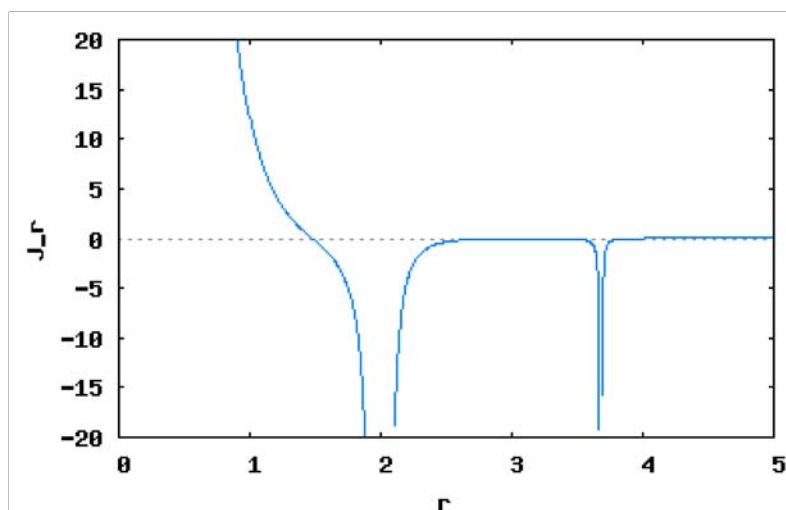


```
(%i53)
```

```
wxplot2d([Jr_p], [r,0,5],[y,-20,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t53)
```

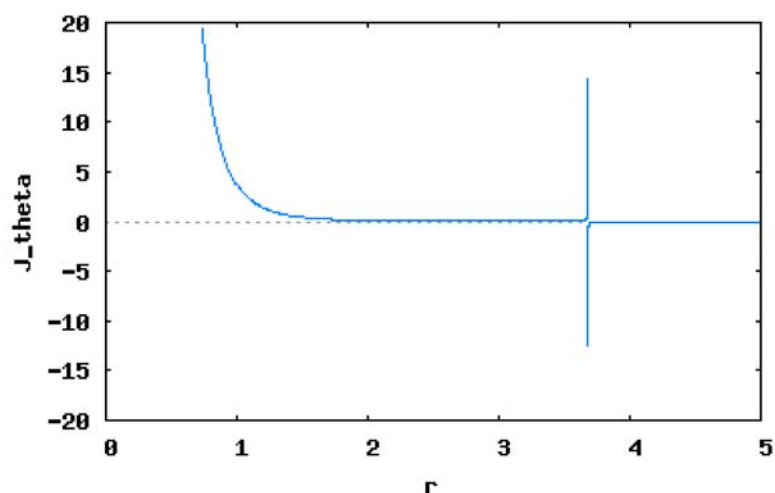


```
(%i54)
```

```
wxplot2d([Jtheta_p], [r,0,5],[y,-20,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t54)

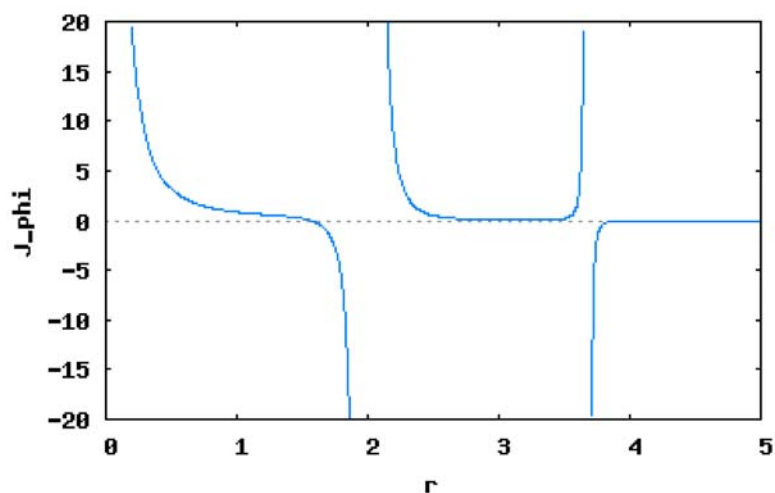


(%i55)

```
wxplot2d([Jphi_p], [r,0,5],[y,-20,20], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_phi"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t55)



```
(%i56) DivE_p: ratsimp(ev(at(DivE, [M=1, J=2, theta=%pi/2])));
```

```
(%o56) 
$$\frac{32 r^6 - 128 r^5 + 192 r^4 - 14336 r^2 + 12288 r}{r^{12} - 6 r^{11} + 12 r^{10} - 8 r^9 + 192 r^8 - 768 r^7 + 768 r^6 + 12288 r^4 - 24576 r^3 + 262144}$$

```

```
(%i57) Jr_p: ratsimp(ev(at(Jr, [M=1, J=2, theta=%pi/2])));
```

```
(%o57) 
$$\frac{352 r^6 - 1856 r^5 + 3264 r^4 - 1920 r^3 + 6144 r^2 - 24576 r + 24576}{r^{12} - 4 r^{11} + 4 r^{10} + 128 r^8 - 256 r^7 + 4096 r^4}$$

```

```
(%i58) Jtheta_p: ratsimp(ev(at(Jtheta, [M=1, J=2, theta=%pi/2])));
```

```
(%o58) 
$$-\frac{192 r - 192}{r^9 - 2 r^8 + 64 r^5}$$

```

```
(%i59) Jphi_p: ratsimp(ev(at(Jphi, [M=1, J=2, theta=%pi/2])));
```

```
(%o59) 
$$\frac{160 r^6 - 1024 r^5 + 2176 r^4 - 1536 r^3 - 6144 r^2 + 4096 r + 12288}{r^{14} - 6 r^{13} + 12 r^{12} - 8 r^{11} + 192 r^{10} - 768 r^9 + 768 r^8 + 12288 r^6 - 24576 r^5 + 262144 r^2}$$

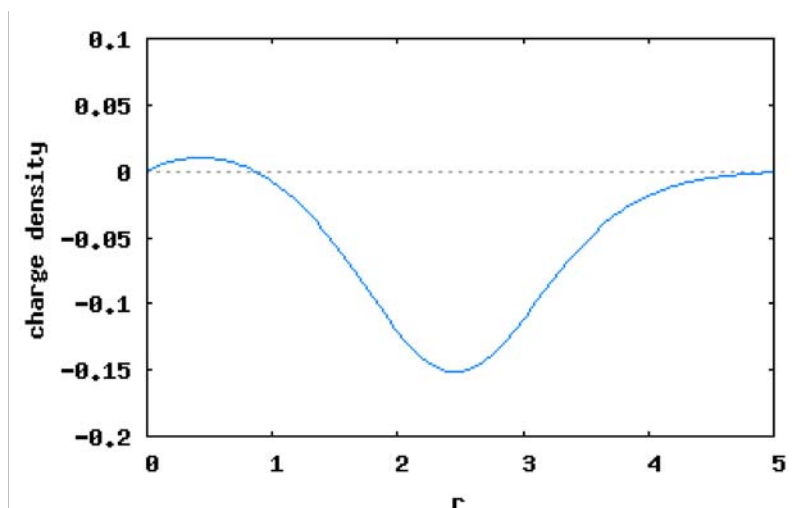
```

```
(%i60)
```

```
wxplot2d([DivE_p], [r,0,5],[y,-.2,.1], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "charge density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t60)
```

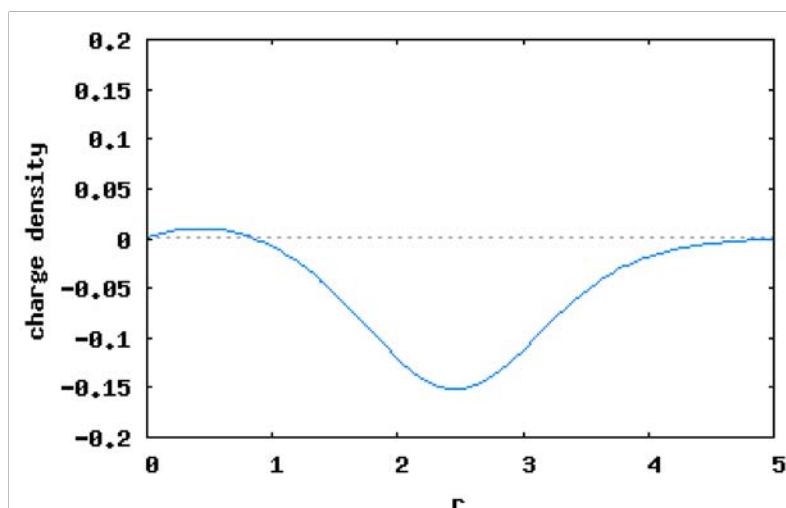


```
(%i61)
```

```
wxplot2d([DivE_p], [r,0,5],[y,-.2,.2], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "charge density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t61)
```

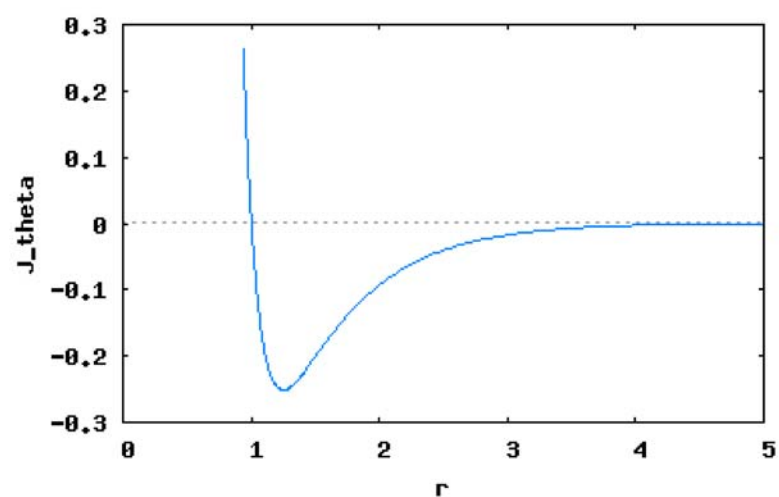


```
(%i62)
```

```
wxplot2d([Jtheta_p], [r,0,5],[y,-.3,.3], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t62)

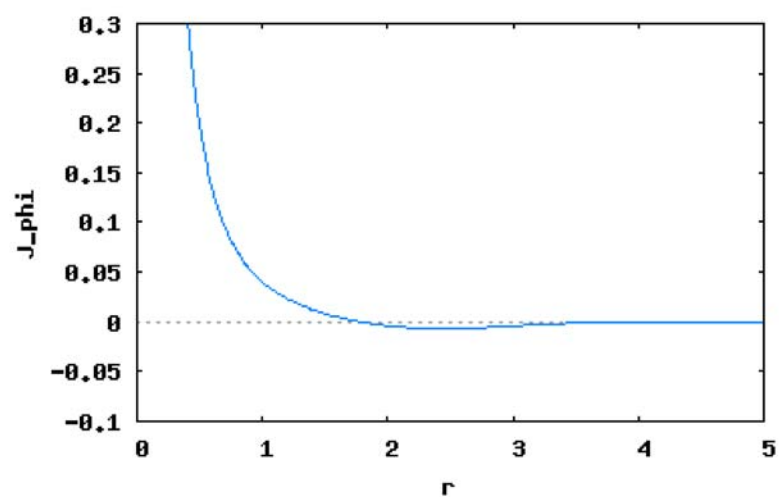


(%i63)

```
wxplot2d([Jphi_p], [r,0,5],[y,-.1,.3], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_phi"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t63)



(%i64)