(%i1)
```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1)
$$f(i,j) := \mathrm{sum}(R_{i,j,\sigma,0}\,gContr_{i,\sigma}\,gContr_{j,0},\sigma,0,3) + \mathrm{sum}(R_{i,j,\sigma,1}\,gContr_{i,\sigma}\,gContr_{j,1},\sigma,0,3) + \mathrm{sum}(R_{i,j,\sigma,2}\,gContr_{i,\sigma}\,gContr_{j,2},\sigma,0,3) + \mathrm{sum}(R_{i,j,\sigma,3}\,gContr_{i,\sigma}\,gContr_{j,3},\sigma,0,3)$$

(%i2)
```
/* define coordinate vector */
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

(%o2) x

(%o3) $[\,t\,,\,r\,,\,\theta\,,\,\varphi\,]$

(%i4)
```
/* g1 is symm. metric with indices 1...4 */
g1: matrix(
  [-1,0,0,0],
  [0,1,0,0],
  [0,0,(abs(r-r0))^2,0],
  [0,0,0,(abs(r-r0))^2*sin(theta)^2]
);
```

(%o4)
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (r0-r)^2 & 0 \\ 0 & 0 & 0 & (r0-r)^2\,\sin(\theta)^2 \end{bmatrix}$$

(%i5)
```
/* contravariant g is inverse of g */
gContr1: ratsimp(invert(g1));
```

(%o5)
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{r0^2 - 2\,r\,r0 + r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{(r0^2 - 2\,r\,r0 + r^2)\,\sin(\theta)^2} \end{bmatrix}$$

(%i6)

```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g     [mu,nu]: g1     [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

```
(%i7)  /* computation of Christoffel symbols Gamma^sigma_mu_nu */
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           Gamma[sigma,mu,nu] :
           /* rho sum by function call: */
           sum(
               1/2 * gContr[sigma,rho]*(
               diff(g[nu,rho],x[mu] ) +
               diff(g[rho,mu],x[nu] ) -
               diff(g[mu,nu] ,x[rho])),
           rho, 0, 3),
           /* evaluate differentiation dy/dr */
           Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
       }}}$
```

```
(%i8)  /* dislplay Gamma's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
       for k:0 thru 3 do {
           if Gamma[i,j,k] # 0 then {
               display(Gamma[i,j,k])
           }}}}$
```

$$\Gamma_{1,2,2} = r0 - r$$

$$\Gamma_{1,3,3} = (r0 - r)\,sin(\theta)^2$$

$$\Gamma_{2,1,2} = -\frac{r0 - r}{r0^2 - 2\,r\,r0 + r^2}$$

$$\Gamma_{2,2,1} = -\frac{r0 - r}{r0^2 - 2\,r\,r0 + r^2}$$

$$\Gamma_{2,3,3} = -\frac{(r0 - r)^2\,cos(\theta)\,sin(\theta)}{r0^2 - 2\,r\,r0 + r^2}$$

$$\Gamma_{3,1,3} = -\frac{r0 - r}{r0^2 - 2\,r\,r0 + r^2}$$

$$\Gamma_{3,2,3} = \frac{(r0 - r)^2\,cos(\theta)}{(r0^2 - 2\,r\,r0 + r^2)\,sin(\theta)}$$

$$\Gamma_{3,3,1} = -\frac{r0 - r}{r0^2 - 2\,r\,r0 + r^2}$$

$$\Gamma_{3,3,2} = \frac{(r0 - r)^2\,cos(\theta)}{(r0^2 - 2\,r\,r0 + r^2)\,sin(\theta)}$$

```
(%i9)  /* compute Riemann tensor elements */
       for rho:0 thru 3 do {
       for sigma:0 thru 3 do {
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           R[rho,sigma,mu,nu] :
           diff(Gamma[rho,nu,sigma],x[mu]) -
           diff(Gamma[rho,mu,sigma],x[nu]) +
           /* lambda sums by function call: */
           sum(
               Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
               Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
               lambda, 0, 3)
       }}}}$

(%i10) /* dislplay R's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
       for k:0 thru 3 do {
       for l:0 thru 3 do {
           R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
           if R[i,j,k,l] # 0 then display(R[i,j,k,l])
           }}}}$

(%i11) /* Ricci tensor Ric[mu,nu] */
       for mu:0 thru 3 do {
       for nu:0 thru 3 do {
           Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
       }}$

(%i12) /* dislplay Ric's being different from zero */
       for i:0 thru 3 do {
       for j:0 thru 3 do {
           Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
           if Ric[i,j] # 0 then display(Ric[i,j])
           }}$

(%i13) /* Ricci Scalar */
       RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
           +  sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
           +  sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
           +  sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
       ;
(%o13) 0

(%i14) ratsimp(RicSc);
(%o14) 0
```

```
(%i15)  /* Raising of indices,
           contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
        /*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

        R0101: f(0,1);
        R0202: f(0,2);
        R0303: f(0,3);
(%o15)  0
(%o16)  0
(%o17)  0

(%i18)  R0101: factor(R0101);
        R0202: factor(R0202);
        R0303: factor(R0303);
(%o18)  0
(%o19)  0
(%o20)  0

(%i21)  R1010: f(1,0);
        R1212: f(1,2);
        R1313: f(1,3);
(%o21)  0
(%o22)  0
(%o23)  0

(%i24)  R1010: factor(R1010);
        R1212: factor(R1212);
        R1313: factor(R1313);
(%o24)  0
(%o25)  0
(%o26)  0

(%i27)  R2020: f(2,0);
        R2121: f(2,1);
        R2323: f(2,3);
(%o27)  0
(%o28)  0
(%o29)  0

(%i30)  R2020: factor(R2020);
        R2121: factor(R2121);
        R2323: factor(R2323);
(%o30)  0
```

```
(%o31)  0

(%o32)  0


(%i33)  R3030: f(3,0);
        R3131: f(3,1);
        R3232: f(3,2);

(%o33)  0

(%o34)  0

(%o35)  0


(%i36)  R3030: factor(R3030);
        R3131: factor(R3131);
        R3232: factor(R3232);

(%o36)  0

(%o37)  0

(%o38)  0


(%i39)  /* Coulomb law */
        DivE : R0101 + R0202 + R0303;

(%o39)  0


(%i40)  ratsimp(DivE);

(%o40)  0


(%i41)  /* J[r] */
        Jr : -(R1010 + R1212 + R1313);

(%o41)  0


(%i42)  ratsimp(Jr);

(%o42)  0


(%i43)  /* J[theta] */
        Jtheta : -(R2020 + R2121 + R2323);

(%o43)  0


(%i44)  ratsimp(Jtheta);

(%o44)  0


(%i45)  /* J[phi] */
        Jphi : -(R3030 + R3131 + R3232);

(%o45)  0
```

```
(%i46)  ev(ratsimp(Jphi),r);
(%o46)  0

(%i47)
```