RADIATIVELY INDUCED FERMION RESONANCE

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Several equations of motion are solved to give the ground state spin-spin coupling between a fermion (or classical electron or proton) and a classical, circularly polarized, electromagnetic field. On the simplest level this phenomenon can be understood as the $B^{(3)}$ to fermion spin spin coupling, and under the right conditions, ESR and NMR can be radiatively induced at visible frequencies, with unprecedented resolving power. The effect is illustrated briefly in the free electron, and in the H and Na atoms.

Key words: Radiatively induced fermion resonance, $B^{(3)}$ field; ESR, NMR.

1. INTRODUCTION

The Dirac equation was first solved to show the existence of radiatively induced fermion resonance (RFR) as reported in Ref. [1]. The term responsible for the effect was isolated to be the novel interaction energy, the real valued, and therefore physical, expectation value,

$$E_n = i\frac{e\hbar}{2m} \langle A \times A^* \rangle; \qquad (1)$$

where e/m is the charge to mass ratio of a fermion (electron or proton) and $iA \pounds A^{\alpha}$ the real valued conjugate product of complex vector potentials in a circularly polarized electromagnetic field, considered to be classical in the manner first proposed by Dirac [2]. In Eq. (1), σ is the Z component of the Pauli matrix [3-6]. The interaction energy can be expressed in terms of the $B^{(3)}$ field of the radiation as [1]

$$E_n = -\frac{e\hbar}{2m}\sigma \cdot B^{(3)}; \qquad (2)$$

and this is an ESR or NMR equation with the static magnetic field replaced by $B^{(3)}$. All known ESR and NMR effects can therefore be induced by radiation rather than by a static magnetic field. The technique produces unprecedented resolving power because the resonance frequencies are proportional [1] to I/ω^2, where I is the beam power density (or intensity in watts m^{-2}) and ω the beam angular frequency. With moderate microwave pumping, fermion resonance can be induced in theory in the visible, and picked up with an ordinary Fourier transform infra red / visible spectrometer acting as probe. This produces in theory a resolving power about one thousand to ten thousand times that available with magnet based ESR or NMR of any kind (including multi dimensional ESR and NMR) because the visible range is that much higher in frequency than the microwave (or gigahertz) range in which the current instruments operate.

In natural philosophy this result indicates the existence (and usefulness) of the $B^{(3)}$ field and is the fundamental spin spin coupling between the photomagneton [7] (the photon's $B^{(3)}$ field) and the fermion's half integral spin proposed by Pauli [8] and Dirac [9]. Indications of the existence of $B^{(3)}$ open the road to non Abelian electrodynamics and non local and superluminal interpretations [10] unknown in the traditional view [11].

In this Letter the above result is reproduced with several equations of motion, beginning with the Newton equation of a classical charged particle in a classical electromagnetic field; and ending with the quantum relativistic van der Waerden equation [12] for a two component spinor. The complete hierarchy of known equations of motion in physics produces the same RFR term, Eq. (1). It is a real, non-zero, and physical ground state term in Rayleigh Schrödinger perturbation theory [13]. The same type of coupling appears to have

been recognized in principle by Pershan et al. [14] in 1966, during their establishment of the inverse Faraday effect, but these authors used higher order perturbation theory near optical resonance as did Li et al. [15] and others [16–20] in recent papers confirming the original proposal of RFR [21]. The key $\propto \mathbf{A} \times \mathbf{A}^{*}$ coupling in higher order perturbation theory is clearly presented in Ref. [14], Eq. (8.6), and was confirmed by them empirically in paramagnetic, rare earth doped, glass samples. These authors did not appear to realize however that the ground state term (1) is non-zero. This is the fundamental $\mathbf{B}^{(3)}$ term discussed in this Letter and occurs independently of any optical resonance, as in ordinary magnet based ESR and NMR. The great beauty of the new theory therefore is that one merely replaces $\mathbf{B}$ of the magnet by $\mathbf{B}^{(3)}$ of the electromagnetic field [1]. One can then proceed to understand the gallery of consequences as in the highly developed theory of ESR and NMR, but with a potential resolving power up to ten thousand times greater. In analogy, successful development would be the metaphorical equivalent of replacing the optical with the scanning tunnelling electron microscope.

2. CLASSICAL, NON-RELATIVISTIC PHYSICS

In order to derive Eq. (1) in Newtonian physics write the kinetic energy in SU(2) topology through use of the Pauli matrix σ [8] and describe the field to particle interaction with the minimal prescription applied to a complex valued $\mathbf{A}$, representing the magnetic vector potential of the electromagnetic field [11]. Finally use ordinary complex algebra to extract the real-valued and physically meaningful interaction kinetic energy corresponding to equation (1). The newtonian kinetic energy of a classical charged particle interacting with the classical electromagnetic field in SU(2) topology is therefore the real part of

$$\begin{aligned} H_{KE} &= \frac{1}{2m}\sigma\cdot(\mathbf{p} - e\mathbf{A})\sigma\cdot(\mathbf{p} - e\mathbf{A}^{*}) \\ &= \frac{e^2}{2m}(\mathbf{A}\cdot\mathbf{A}^{*} + i\sigma\cdot\mathbf{A}\times\mathbf{A}^{*}) \\ &- \frac{e}{2m}(Re(\mathbf{A}\cdot\mathbf{p}) + Re(\mathbf{p}\cdot\mathbf{A}^{*}) \end{aligned} \qquad (3)$$

$$+\frac{1}{2m}\mathbf{p}\cdot\mathbf{p};$$

and contains term (1) in addition to the familiar and observable 0 (3) terms as found in a text such as that by Pike and Sarkav [22]. This is a term which relies for its existence on topology rather than quantum mechanics. It is well known [22] that $SU(2)$ is homomorphic with O(3), the usual rotation group of three dimensional space in Newtonian physics. However, the Cli®ord algebra underlying $SU(2)$ gives more information, as advocated by Bearden *et al.* [24]. Our Newtonian result is consistent with the fact that Eq. (1) was obtained in the non-relativistic limit of the Dirac equation as a real expectation value [1].

3. CLASSICAL, RELATIVISTIC PHYSICS

It is a straightforward matter to repeat this simple exercise for classical, relativistic physics because one can use the same minimal prescription in the Einstein equation written in SU(2) topology. For a free classical particle, the latter is

$$\circ^{\mu} p_{\mu} \circ^{\mu} p_{\mu} = m^2 c^2 \tag{4}$$

where $\circ^{\mu}$ is the Dirac matrix, p_{μ} the energy momentum four-vector, and c the speed of light in vacuo. The interaction of the classical electromagnetic field with the classical, relativistic particle is described therefore by the equation of motion,

$$\circ^{\mu}(p_{\mu} - eA_{\mu})\circ^{\mu}(p_{\mu} - eA^{\mu}) = m^2 c^2; \tag{5}$$

which in Feynman's slash notation [1] becomes

$$(\not{p} - e\not{A})(\not{p} - e\not{A}^{\mu}) = m^2 c^2. \tag{6}$$

The RFR term is [1] the real valued interaction energy,

$$E_n := \frac{e^2}{m} A_{\mu} A^{\mu}; \tag{7}$$

which includes term (1) of this paper as part of a fully relativistic treatment,

$$A_{\mu} A^{\mu} = A_0 A^0 - (\nabla \cdot \mathbf{A})(\nabla \cdot \mathbf{A}) = A_0 A^0 - \mathbf{A} \cdot \mathbf{A} - i\nabla \cdot \mathbf{A} \times \mathbf{A}. \tag{8}$$

4. NON-RELATIVISTIC QUANTUM PHYSICS

We can consider the Schrödinger Pauli equation [8],

$$\hat{H}|\psi\rangle = E_n|\psi\rangle; \tag{9}$$

in which the classical kinetic energy becomes an operator on a wavefunction which is a two component spinor in SU(2) topology. The

usual operator replacements are used as follows:

$$
\begin{aligned}
p^{\mu} &\rightarrow i\hbar\partial^{\mu}; & p_{\mu} &\rightarrow i\hbar\partial_{\mu};\\
E_n &\rightarrow i\hbar\frac{\partial}{\partial t}; & \mathbf{p} &\rightarrow -i\hbar\nabla;\\
p^{\mu} &:= \left(\frac{E_n}{c};\mathbf{p}\right); & p_{\mu} &:= \left(\frac{E_n}{c};-\mathbf{p}\right);\\
\partial^{\mu} &:= \left(\frac{1}{c}\frac{\partial}{\partial t};-\nabla\right); & \partial_{\mu} &:= \left(\frac{1}{c}\frac{\partial}{\partial t};\nabla\right).
\end{aligned}
\tag{10}
$$

It is interesting to note that for a real valued $\mathbf{A}$ (static magnetic field problem of ordinary ESR and NMR [13]), the Schrödinger-Pauli equation produces the famous real expectation value,

$$
E_n = -\frac{e\hbar}{2m}\langle\sigma\rangle\mathbf{B}; \qquad \mathbf{B} = \nabla\times\mathbf{A}; \tag{11}
$$

where $\hbar$ is the Dirac constant. This is the fundamental ESR or NMR term obtained in the non relativistic quantum limit and has no classical equivalent because it depends for its existence on the operator rules (10). The Hamiltonian operator that produces result (11) is,

$$\hat{H} = \frac{1}{2m}\sigma\cdot(\hat{\mathbf{p}} - e\mathbf{A})\sigma\cdot(\hat{\mathbf{p}} - e\mathbf{A}) + V;$$

$$\hat{\mathbf{p}} = -i\hbar\hat{\nabla} \qquad (12)$$

where V is a potential energy.

In order to obtain the new RFR term (1) this operator becomes,

$$\hat{H} = \frac{1}{2m}\sigma\cdot(\hat{\mathbf{p}} - e\mathbf{A})\sigma\cdot(\hat{\mathbf{p}} - e\mathbf{A}^{*}) + V; \qquad (13)$$

and leads to the classical, real valued term

$$\hat{H}_{RFR}|\rangle := i\frac{e^2}{2m}\sigma\cdot\mathbf{A}\times\mathbf{A}^{*}|\rangle; \qquad (14)$$

which obviously has the same expectation value, Eq. (1). Therefore, unlike ordinary ESR and NMR, RFR depends on a term which *does* have a classical equivalent if we treat the field classically as did Dirac [2].

5. RELATIVISTIC QUANTUM PHYSICS

The most straightforward route to relativistic quantum mechanics is through the replacement of p_μ in the Einstein equation (4) by its operator equivalent to give the van der Waerden equation of motion as detailed by Sakurai [8] for example,

$$\begin{pmatrix} i & \sigma\cdot(i\partial_t - e\mathbf{A}) \\ -\sigma\cdot(i\partial_t - e\mathbf{A}) & i \end{pmatrix}\begin{pmatrix} \phi \\ \chi \end{pmatrix} = \frac{m^2c^2}{\hbar^2}\begin{pmatrix} \phi \\ \chi \end{pmatrix}. \qquad (15)$$

Here $|\rangle$ is a two component spinor and the equation is well known to be equivalent to the much better known Dirac equation involving a four component spinor. The RFR term emerges from the van der Waerden equation in the form,

$$\begin{pmatrix} i & \sigma\cdot(i\partial_t - e\mathbf{A}) \\ -\sigma\cdot(i\partial_t - e\mathbf{A}^{*}) & i \end{pmatrix}\begin{pmatrix} \phi \\ \chi \end{pmatrix} = \frac{m^2c^2}{\hbar^2}\begin{pmatrix} \phi \\ \chi \end{pmatrix}. \qquad (16)$$

The real and classical $e^2\mathbf{A}\times\mathbf{A}^{*}$ is a simple multiplicative operator on the two component spinor, with the same, real expectation value.

This is also the case for the Dirac equation as given in ref. (1), and in general for all SU(2) topology quantum mechanical equations.

6. RAYLEIGH SCHRÖDINGER PERTURBATION THEORY

In perturbation theory [13] the RFR term is a non-zero ground state term,

$$E_n = i\frac{e^2}{2m}\langle 0|\nabla A - A\nabla|0\rangle + \text{second order terms:} \qquad (17)$$

As shown recently by Li et al. [15], and by others [16–20] small second order RFR shifts also occur in second order corrections in perturbation theory, but term (17) is of far greater practical interest, because as shown in Ref. 1, it produces fermion resonances in the visible. Second order perturbation theory was also used by Pershan et al. [14] to produce the paramagnetic inverse Faraday effect, which they confirmed experimentally.

7. DISCUSSION

In free space, the novel $B^{(3)}$ field of O(3) symmetry electrodynamics is defined for one photon by

$$B^{(3)} := -i\frac{e}{\hbar}A \wedge A^{*}; \tag{18}$$

where e is the elementary charge [1]. Substituting this definition into Eq. (17) we find that the RFR term takes the same form precisely as the spin Zeeman effect produced by a static magnetic field,

$$E_{n(RFR)} = -\frac{e\hbar}{2m}\langle 0|\sigma \cdot B^{(3)}|0\rangle. \tag{19}$$

We need only replace B by $B^{(3)}$ as defined in Eq. (18). Equation (19) is the fundamental spin spin interaction between one photon and one fermion. For a free electron, the resonance frequency is straightforwardly calculated [1] from Eq. (19) to be,

$$\omega_{res} = \frac{ie^2\mu_0 c}{\hbar m}\frac{I}{\omega^2} = 1.007 \times 10^{28}\frac{I}{\omega^2}; \tag{20}$$

where I is the pump beam power density in watts m^{-2} (10,000 watts m^{-2} = 1.0 watt cm^{-2}), μ_0 the free space permeability in SI units. For the H atom the Hamiltonian operator is well known to be

$$\hat{H}_{(H\ atom)} = -\frac{\hbar^2}{2\mu}\nabla^2 + V; \tag{21}$$

where V denotes the classical Coulomb interaction between electron and proton and μ is the reduced mass,

$$\mu = \frac{m_e m_p}{m_e + m_p} \approx m_e; \tag{22}$$

where m_e and m_p are respectively the electron and proton masses. The resonance frequency in atomic H from Eq. (20) is therefore slightly shifted away from the free electron resonance frequency because the reduced mass is slightly different from the electron mass. The Hamiltonian operator (21) for a monovalent alkali metal atom

such as sodium (Na) must take account of the fact the there are several protons, neutrons and electrons arranged in orbitals according to the Pauli Exclusion Principle [13]. This atomic structure gives rise to the possibility of spin orbit coupling, spin spin coupling between electrons, Fermi contact splitting and hyper¯ne splitting as in ESR or NMR [13]. However, as a rule of thumb estimate, the outer or valence electron can be considered as superimposed on closed shells of inner electrons and a nucleus made up of protons and neutrons of a given reduced mass. To a ¯rst approximation the Hamiltonian (21) can be used in which the sodium atom's reduced mass is slightly different from the free electron mass. This means that the main RFR resonance frequency in sodium is well estimated by Eq. (20) and so sodium vapor can be used in the experiment to detect RFR.

In order to detect RFR experimentally adjust conditions in the ¯rst instance so that

$$\omega_{res} = \omega; \tag{23}$$

which is *the autoresonance condition* in which the pump beam is absorbed at resonance because the pump frequency matches the resonance frequency precisely. Equation (20) simpli¯es to

$$\omega_{res}^3 = 1{:}007 \pounds 10^{28} I: \tag{24}$$

Therefore we can either tune ω_{res} for a given I or vice versa. Since autoresonance must appear in the GHz if the pump frequency is in the GHz it is convenient to slightly modify the set up used by Deschamps *et al.* [25] in their detection of the inverse Faraday e®ect in plasma. They used a pulsed microwave signal at $3{:}0\ GHz$ from a klystron delivering megawatts of power over $12\ ¹s$ with a repetition rate of $10\ Hz$. The TE_{11} mode was circularly polarized with a polarizer placed inside a circular waveguide of $7{:}5\ cm$ diameter. The plasma sample was created by a very intense microwave pulse and held in a pyrex tube inserted coaxially in the waveguide of $6{:}5\ cm$ diameter and $20\ cm$ long. The section of the waveguide surrounding the tube was made of nylon internally coated with a $20\ ¹m$ layer of copper. The inverse Faraday e®ect was then picked up with Faraday induction [25].

To detect RFR change the sample to sodium vapor, which is easily prepared and held in the sample tube. Equation (24) predicts that resonance occurs at $3{:}0\ GHz$ if I is tuned to $0{:}0665\ watts\ cm^{-2}$.

For a circular waveguide of 7:5 cm diameter this requires only 2:94 watts of CW power from the klystron at 3:0 GHz. In deriving Eq. (20) it has been assumed that [1,26]

$$I = \frac{c}{\mu_0} B^{(0)2}: \tag{25}$$

This is a simple theoretical estimate and it is strongly advisable that I can be tuned for a considerable range around 2:94 watts to allow for unforeseen discrepancies between Eq. (25) and the actual experimental beam intensity generated by the apparatus. Once the main resonance is detected, however, further re¯nements can follow, making full use of contemporary electronics. To repeat the experiment with H or with the free electron is likely to be more di±cult purely because of sample handling problems. The experiment should be repeated after autoresonance is detected to demonstrate the major advantage of RFR by pulsing the pump beam for increased power density at the same frequency and by using Eq. (24) to estimate the resonance frequency. A sample of expected results is given in Table 1. As can be seen it is possible in theory to produce ESR (and NMR) in the visible range, with a four order of magnitude increase in resolving power over current magnet based techniques.

Table 1. RFR Frequencies for a 3:0 GHz Pump for given I

Pump Intensity I watts=cm^2	Resonance Frequency	
10	451:1 GHz = 15:04 cm^{-1}	Far infra red
100	150:4 cm^{-1}	Far infra red
1;000	1504 cm^{-1}	Infra red
10;000	15;040 cm^{-1}	Visible
100;000	150;040 cm^{-1}	Ultra violet / X ray

For a 3:0 GHz circularly polarized pump pulse of 10;000 W cm^{-2} the RFR frequency is at 15;040 cm^{-1} in the visible, and can be detected with a Fourier transform infra red/visible spectrometer such as a fully computerized Bruker IFS 113v. The detector of the spectrometer must be fast enough to record an interferogram during the microsecond interval of the microwave pulse. Therefore pulse repetition and computer based re¯nement is needed for good quality data. The pump should be kept as noise free and homogeneous as possible, but because of the $I=!^2$ dependence of RFR, simple Maxwell Boltzmann theory [1] shows that conditions can be adjusted to produce a much larger population di®erence between up and down fermion spins than in magnet based ESR or NMR. Therefore this alleviates the well known problem of magnet homogeneity in magnet based ESR and NMR, a problem which is due to a small (one part in a million) population di®erence. In RFR the latter can easily exceed 20% [1] at a conservative estimate for moderate pump power of ten watts order of magnitude. The complete ESR spectrum of sodium vapor can therefore be taken, in theory, in the infra red or visible. This is *terra incognita* to magnet based technology, which is reaching its design limit. The whole process can then be repeated for NMR and MRI.

The characteristic and key $I=!^2$ coe±cient of our theory [1] appears also in the second order perturbation theory of Harris and Tinoco [17], their p. 9291, second column, premultiplied by a fac-

tor. These authors miss the first order or ground state term (1) and in consequence their theory falls short of some empirical indications by Warren et al. [27] by eight orders of magnitude. Straightforward estimates [1] based on Eq. (1) applied to NMR fall in the order of magnitude of the data obtained by Warren et al. by visible frequency irradiation of molecular liquids with various circularly polarized lasers, including an argon ion laser at 528:7 nm, 488 nm, and 476:5 nm. Accounting simply for the different g factors of the proton and electron, Eq. (1) applied to NMR [1] produces very tiny shifts of 0:12, 0:10 and 0:098 Hz respectively for the three argon ion frequencies quoted above and for an intensity of ten watts per square centimetre, approaching the highest CW intensity used by Warren et al. [27] in important and pioneering experiments at Princeton following our early theory [28], which also missed the key term (1) introduced finally in ref. [1]. Equation (1) shows now why Warren et al. [27] were not able to obtain more than indications of RFR shifts, both in proton and ^{13}C Fourier transform and two dimensional NMR. In ^{13}C NMR the mass of the ^{13}C nucleus is an order of magnitude heavier than in ^{1}H NMR, and the shifts from Eq. (1), all other factors being equal, are in consequence an order of magnitude smaller, in the 0:01 Hz range, and too small to be detected as found experimentally [27]. The remedy is also given by Eq. (1), which is to replace the lasers with pulsed or CW microwave generators for about the same I. Their effort [27] nevertheless remains as a landmark in the field, metaphorically akin to Shackleton's attempt at the Pole, just short of reaching the goal.

Finally, Li et al. [15] have shown that even in second order perturbation theory of the type used by Harris and Tinoco [16,17] or Buckingham et al. [18,19], large RFR shifts of up to 10 MHz are possible using pump lasers tuned near to optical resonance. Systematic development of RFR, first proposed by the present author in Ref. 21 and several consequent papers [28], is clearly going to be highly beneficial to chemical physics and medicine unless all the equations of physics are misleading or unless some unforeseen technical difficulty occurs. With contemporary technology it is unlikely that such a difficulty, if it occurred, could not be overcome. Philosophically, the whole process can be thought of as stemming from the $B^{(3)}$ (Evans Vigier) field of O (3) electrodynamics [1], which for one photon, is the fundamental photomagneton [29].

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