THE ECE2 UNIFIED FIELD EQUATIONS OF HYDRODYNAMICS, ELECTROMAGNETISM AND GRAVITATION.

by

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ABSTRACT

The field equations of hydrodynamics, electromagnetism and gravitation are developed as ECE2 relativity. The development shows that these subject areas can be developed using a single unified field. Spacetime turbulence can be observed in theory using a circuit that takes energy from spacetime. It is shown that Ohm's law and the Lorentz force law are intrinsic parts of the electromagnetic field equations, and have equivalents in hydrodynamics and gravitation. The geometrical condition is derived for zero magnetic charge / current density.

Keywords: ECE2 relativity, unified field equations of hydrodynamics, electromagnetism and gravitation.

UFT 349

1. INTRODUCTION

In recent papers of this series various theories of precession have been developed, including theories based on the gravitational Lorentz force equation {1 - 12}. In this paper the unified ECE2 field equations are developed of hydrodynamics, electromagnetism and gravitation, based on the same Cartan geometry. It is shown that the field equations are identical in mathematical structure, interpretations of this structure give hydrodynamics, electrodynamics and gravitation. The geometrical condition is derived for vanishing magnetic charge / current density, so ECE2 relativity allows the existence of these quantities. It is also shown that Ohm's Law and the Lorentz force equation are intrinsic parts of the ECE2 field equations in each subject area. It is possible to transfer concepts from one subject area to the others. For example the concept of spacetime turbulence enters into each subject area. Such turbulence may be observed in a circuit that takes energy from spacetime (for example UFT311 and UFT321 on www.aias.us).

This paper is a brief synopsis of detailed calculations in the notes accompanying UFT349 on www.aias.us.. Note 349(1) gives the field equations of each subject area, Note 349(2) develops work in the literature on turbulence as observed electrodynamically, Notes 349(3) - 349(5) define Ohm's Law and the Lorentz force equation from the ECE2 field equations and define the geometrical condition under which magnetic charge / current density vanishes.

Section 2 is a synopsis of the field equations, and Section 3 is a numerical and graphical development of new concepts.

2 FIELD EQUATIONS

Following work by Kambe, "On Fluid Maxwell Equations" on the net, Note 349(1) gives the following ECE2 field equations of hydrodynamics:

$$\underline{\nabla} \cdot \underline{B}_{FD} = 0 \quad -(1)$$

$$\underline{\nabla} \cdot \underline{E}_{FD} = \rho_{FD} \quad -(2)$$

$$\underline{\nabla} \times \underline{E}_{FD} + \frac{\partial \underline{B}_{FD}}{\partial t} = \underline{0} \quad -(3)$$

$$\underline{\nabla} \times \underline{B}_{FD} - \frac{1}{a_0^2} \frac{\partial \underline{E}_{FD}}{\partial t} = \mu_{0FD} \underline{J}_{FD} \quad -(4)$$

which are identical in structure both to the ECE2 field equations of electromagnetism and gravitation. All three sets of equations are Lorentz covariant in a space with finite torsion and curvature (ECE2 relativity). Here $\underline{B}_{FD}$ is the fluid magnetic flux density:

$$\underline{B}_{FD} = \underline{\nabla} \times \underline{v} \quad -(5)$$

and also the vorticity, where $\underline{v}$ is the fluid linear velocity. The fluid electric field strength is:

$$\underline{E}_{FD} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} h \quad -(6)$$

where h is the enthalpy per unit mass in joules per kilogram. The four potential of ECE2 fluid dynamics is:

$$\phi^{\mu}_{FD} = \left(\frac{h}{a_0}, \underline{v} \right) \quad -(7)$$

where a_0 is the speed of sound, and the d'Alembertian in fluid dynamics is:

$$\square = \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(8)$$

The hydrodynamic four potential is directly analogous with the four potential of ECE2 electrodynamics (UFT318 on www.aias.us):

$$\underline{W}^{\mu} = \left(\frac{\phi_W}{c}, \underline{W} \right) \quad -(9)$$
$$= \underline{W}^{(0)} \omega^{\mu}$$

where ω^{μ} is the spin connection four vector. Kambe derives the hydrodynamic Lorenz condition:

$$\partial_\mu \phi^{\mu}_{FD} = 0 \quad -(10)$$

where:

$$\partial_\mu = \left(\frac{1}{a_0} \frac{\partial}{\partial t}, \underline{\nabla} \right). \quad -(11)$$

In vector notation:

$$\frac{1}{a_0^2} \frac{\partial h}{\partial t} + \underline{\nabla} \cdot \underline{v} = 0. \quad -(12)$$

In ECE2 electrodynamics the Lorenz condition is:

$$\partial_\mu \underline{W}^{\mu} = 0 \quad -(13)$$

where

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right). \quad -(14)$$

In vector notation:

$$\frac{1}{c^2} \frac{\partial \phi_W}{\partial t} + \underline{\nabla} \cdot \underline{W} = 0. \quad -(15)$$

In Eq. (4), μ_{0FD} is the ECE2 hydrodynamic permeability in vacuo, and Note 349(1) gives the ECE2 electrodynamic and gravitational field equations for direct comparison (UFT317 and UFT318).

The ECE2 continuity equation of hydrodynamics is:

$$\frac{\partial q_{FD}}{\partial t} + \underline{\nabla} \cdot \underline{J}_{FD} = 0 \quad -(16)$$

where the hydrodynamic or fluid charge density and current density are defined by Kambe respectively as:

$$q_{FD} = \underline{\nabla} \cdot \left(\left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \right) \quad -(17)$$

and:

$$\underline{J}_{FD} = \frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla} \frac{\partial h}{\partial t} + a_0^2 \, \underline{\nabla} \times \left(\underline{\nabla} \times \underline{v} \right). \quad -(18)$$

In direct analogy, the continuity equation of ECE2 electrodynamics is:

$$\partial_\mu J^\mu = \frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{J} = 0 \quad -(19)$$

where the electromagnetic charge current density is:

$$J^\mu = \left(c\rho, \underline{J} \right). \quad -(20)$$

As shown in Note 349(1), there is also a direct analogy between the ECE2 Lorentz force equations of hydrodynamics and electrodynamics.

Note 349(2) summarizes and develops calculations based on an article on the net by A. Thess et al. (2006) of Stanford / NASA, (Google: turbulent flow Lorentz force, fourth site) and the note shows how transition to turbulence in hydrodynamics may be observed through the Lorentz force equation of electrodynamics. So it is possible to envisage transition to turbulence in ECE2 spacetime. Such turbulence could be observed with the circuit design of UFT311. This note considers Ohm's Law in the presence of a magnetic flux density B:

$$\underline{J} = \sigma \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(21)$$

where σ is the conductivity, with S. I. Units of $C^2 J^{-1} m^{-1} s^{-1}$. In the non relativistic limit, the Lorentz force equation for a charge density (charge continuum) is:

$$\underline{F}_0 = \rho \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(22)$$

where $\underline{F}_0$ is the force density (force per unit volume). The force is:

$$\underline{F} = \int \underline{F}_0 \, dV. \quad -(23)$$

Therefore the relation between the magnitudes of Lorentz force density and current density is

$$F_0 = \frac{\rho}{\sigma} J. \quad -(24)$$

With these definitions it can be shown as follows that the structure of the ECE2 field equations of electromagnetism (UFT317 and UFT318) contain both Ohm's Law and the Lorentz force equation. They are therefore preferred to the Maxwell Heaviside (MH) field equations, which neither contain nor derive the Ohm Law and the Lorentz force equation.

The complete set of ECE2 field equations of electrodynamics is as follows:

$$\underline{\nabla} \cdot \underline{B} = \underline{\kappa} \cdot \underline{B} \quad -(25)$$

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} \quad -(26)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = -\left(\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} \right) \quad -(27)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} \quad -(28)$$

where:

$$\kappa_0 = 2 \left(\frac{q_0}{r^{(0)}} - \omega_0 \right) \quad -(29)$$

$$\underline{\kappa} = 2 \left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega} \right). \quad -(30)$$

The spin connection four vector is:

$$\omega^\mu = (\omega_0, \underline{\omega}) \quad -(31)$$

and the tetrad four vector is:

$$q^\mu = (q_0, \underline{q}). \quad -(32)$$

Here $r^{(0)}$ is a coefficient with the units of metres. The charge current density is:

$$J^\mu = (c\rho, \underline{J}) \quad -(33)$$

and the A^μ and W^μ four potentials are:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right) \quad -(34)$$

and

$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W}\right). \quad -(35)$$

The field potential relations are:

$$\underline{E} = -\underline{\nabla}\phi_W - \frac{\partial \underline{W}}{\partial t} \quad -(36)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(37)$$

with

$$\phi_W = W^{(0)}\omega_0 = cW_0, \quad \underline{W} = W^{(0)}\underline{\omega}. \quad -(38)$$

In general:

$$\rho = \epsilon_0 \underline{\kappa} \cdot \underline{E} \quad -(39)$$

and

$$\mathbf{J} = \frac{1}{\mu_0}\left(\frac{\kappa_0}{c}\mathbf{E} + \boldsymbol{\kappa} \times \mathbf{B}\right). \quad -(40)$$

It follows immediately that the Lorentz force density equation has a geometrical structure:

$$\mathbf{F_0} = \frac{\rho}{\mu_0 \sigma}\left(\frac{\kappa_0}{c}\mathbf{E} + \boldsymbol{\kappa} \times \mathbf{B}\right) = \frac{\rho}{\mu_0 \sigma}\left(\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}\right) \quad -(41)$$

Complete self consistency requires that the relativistic Lorentz force be used on the left hand side of this equation, because ECE2 is automatically Lorentz covariant and relativistic. The MH field equations are:

$$\nabla \cdot \mathbf{B} = 0 \quad -(42)$$

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \quad -(43)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = \mathbf{0} \quad -(44)$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad -(45)$$

and do not contain the Ohm law and Lorentz force equation as is well known. The MH equations use zero torsion and curvature, and this is an incorrect geometry. The ECE2 field equations are defined with finite torsion and curvature, a valid geometry {1 - 12}.

In the presence of material polarization $\mathbf{P}$ and magnetization $\mathbf{H}$:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad -(46)$$

where ϵ_0 and μ_0 are the permittivity and permeability in vacuo, and the MH equations become the homogenous equations:

$$\nabla \cdot \mathbf{B} = 0 \quad -(47)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \quad -(48)$$

and the inhomogeneous equations:

$$\underline{\nabla} \cdot \underline{D} = \rho \quad -(49)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} . \quad -(50)$$

In ECE2 the homogenous field equations are (25) and (27), and the inhomogeneous equations are:

$$\underline{\nabla} \cdot \underline{D} = \underline{\kappa} \cdot \underline{D} \quad -(51)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \frac{\kappa_0}{c} \underline{D} + \underline{\kappa} \times \underline{H} = \underline{J} . \quad -(52)$$

Therefore in the presence of polarization and magnetization:

$$\underline{F}_0 = \frac{\rho}{\sigma} \left(\frac{\kappa_0}{c} \underline{D} + \underline{\kappa} \times \underline{H} \right) = \frac{\rho}{\sigma} \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) . \quad -(53)$$

In general, ECE2 unified field theory allows for the existence of magnetic charge current density. However if it is assumed that this is zero:

$$\underline{\kappa} \cdot \underline{B} = 0 \quad -(54)$$

and

$$\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} = \underline{0} . \quad -(55)$$

Note that Eq. (55) implies Eq. (54) because from Eq. (55):

$$\underline{B} = - \frac{1}{\kappa_0 c} \underline{\kappa} \times \underline{E} \quad -(56)$$

so:

$$\underline{\kappa} \cdot \underline{\kappa} \times \underline{E} = \underline{E} \cdot \underline{\kappa} \times \underline{\kappa} = 0 \quad -(57)$$

implying Eq. (54), Q. E. D. By definition:

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(58)$$

so if there is no magnetic charge density or magnetic monopole:

$$\underline{\kappa} \cdot \underline{\nabla} \times \underline{W} = 0 \quad -(59)$$

Using the vector identity:

$$\underline{\kappa} \cdot \underline{\nabla} \times \underline{W} = \underline{W} \cdot \underline{\nabla} \times \underline{\kappa} - \underline{\nabla} \cdot \underline{\kappa} \times \underline{W} \quad -(60)$$

it follows that the absence of a magnetic monopole requires:

$$\underline{W} \cdot \underline{\nabla} \times \underline{\kappa} = \underline{\nabla} \cdot \underline{\kappa} \times \underline{W} \quad -(61)$$

where:

$$\underline{W} = W^{(0)} \underline{\omega} \quad -(62)$$

and:

$$\underline{\kappa} = 2 \left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega} \right). \quad -(63)$$

The geometrical ECE2 condition for the absence of a magnetic monopole is therefore:

$$r^{(0)} \underline{\omega} \cdot \underline{\nabla} \times \underline{\omega} = \underline{\omega} \cdot \underline{\nabla} \times \underline{q} - \underline{\nabla} \cdot \underline{q} \times \underline{\omega} \quad -(64)$$

Now consider the Cartan identity in vector notation, (UFT350 chapter 3, "The Principles of ECE Theory"):

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_c - \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b \quad -(65)$$

The procedure of removing indices, the procedure that leads to ECE2 theory, reduces the identity to the well known vector identity:

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{q} = \underline{q} \cdot \underline{\nabla} \times \underline{\omega} - \underline{\omega} \cdot \underline{\nabla} \times \underline{q} \quad -(66)$$

From Eqs. (64) and (66), the geometrical condition for the absence of a magnetic monopole and magnetic current density becomes:

$$\underline{\nabla} \cdot \underline{q} \times \underline{\omega} = r^{(0)} \underline{\omega} \cdot \underline{\nabla} \times \underline{\omega} \quad -(67)$$

This geometry is transformed into electromagnetism using the definitions:

$$\underline{A} = A^{(0)} \underline{q} \quad -(68)$$

$$\underline{W} = W^{(0)} \underline{\omega} \quad -(69)$$

It follows that for the absence of a magnetic charge current density:

$$r^{(0)} \underline{\nabla} \cdot \underline{A} \times \underline{W} = \underline{W} \cdot \underline{\nabla} \times \underline{W} \quad -(70)$$

an equation which implies:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(71)$$

which in turn implies the Beltrami structure (UFT350):

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} \quad -(72)$$

so:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{W}) = \kappa \underline{\nabla} \times \underline{W} \quad -(73)$$

If it is assumed that $\underline{W}$ is a Beltrami potential, then:

$$\underline{\nabla} \times \underline{W} = k \underline{W} \quad -(74)$$

and the condition for vanishing magnetic charge current density reduces to:

$$\underline{\nabla} \cdot \underline{A} \times \underline{W} = \frac{k}{r^{(0)}} \underline{W} \cdot \underline{W} = x \quad -(75)$$

whose integral form is found from the divergence theorem:

$$\int_V \underline{\nabla} \cdot \underline{A} \times \underline{W} \, dV = \oint_S \underline{A} \times \underline{W} \cdot \underline{n} \, dA = \int x \, dV \quad -(76)$$

3. NUMERICAL AND GRAPHICAL ANALYSES

Section by Dr. Horst Eckardt

The ECE2 unified field equations of hydrodynamics, electromagnetism and gravitation

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3 Numerical and graphical analysis

3.1 Dynamic charge distribution

We investigate the dynamic charge density q derived from the velocity field $\mathbf{v}$ by Kambe (Eq. 17):

$$q = \boldsymbol{\nabla} \cdot (\mathbf{v} \cdot \boldsymbol{\nabla})\, \mathbf{v}. \tag{77}$$

For an incompressible fluid it is required that the velocity field is divergence-free:

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0. \tag{78}$$

We will inspect some velocity models by specifying $\mathbf{v}$ analytically. We use plane polar coordinates that are identical with cylindrical coordinates with $Z = 0$. Therefore we can use the differential operators of cylindrical coordinates (r, θ, Z). Then we have

$$\boldsymbol{\nabla}\psi = \begin{bmatrix} \frac{\partial \psi}{\partial r} \\ \frac{1}{r}\frac{\partial \psi}{\partial \theta} \\ \frac{\partial \psi}{\partial Z} \end{bmatrix} \tag{79}$$

and

$$\boldsymbol{\nabla} \cdot \mathbf{v} = \frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_Z}{\partial Z}, \tag{80}$$

$$\boldsymbol{\nabla} \times \mathbf{v} = \begin{bmatrix} \frac{1}{r}\frac{\partial v_Z}{\partial \theta} - \frac{\partial v_\theta}{\partial Z} \\ \frac{\partial v_r}{\partial Z} - \frac{\partial v_Z}{\partial r} \\ \frac{1}{r}\frac{\partial (r\, v_\theta)}{\partial r} - \frac{1}{r}\frac{\partial v_r}{\partial \theta} \end{bmatrix} \tag{81}$$

for a scalar function ψ and vector $\mathbf{v}$.

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A first simple case is

$$\mathbf{v}_1 = \begin{bmatrix} \frac{a}{r} \\ b \\ 0 \end{bmatrix} \tag{82}$$

with constants a and b from which follows

$$\boldsymbol{\nabla} \cdot \mathbf{v}_1 = 0 \tag{83}$$

and

$$(\mathbf{v}_1 \cdot \boldsymbol{\nabla})\, \mathbf{v}_1 = \begin{bmatrix} -\frac{a^2}{r^3} \\ 0 \\ 0 \end{bmatrix}, \tag{84}$$

$$\boldsymbol{\nabla} \times \mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{b}{r} \end{bmatrix}. \tag{85}$$

The result (84), representing the electric field, does not depend on b, and so does not the charge distribution:

$$q_1 = \frac{2a^2}{r^4}. \tag{86}$$

All results were obtained by computer algebra. Tests showed that a radial component dependent on r is necessary to give a vanishing divergence of $\mathbf{v}$. The velocity field $\mathbf{v}_1$ has been graphed in Fig. 1. This is a vortex around the coordinate origin where the angle of velocity is a tangent to a circle in the far field but not in the near field. This is the impact of $1/r$. The first component of (84) and the charge distribution (86) look similar but have higher exponents of r in the denominator. q_1 is graphed in Fig. 2 in a contour plot, showing the steep rise of charge density.

The next example is non-trivial. It was chosen so that the divergence vanishes, although this is not obvious from the velocity field:

$$\mathbf{v}_2 = \begin{bmatrix} \frac{a\cos\theta}{r^2} \\ \frac{a\sin\theta}{r^2} + b \\ 0 \end{bmatrix} \tag{87}$$

with constants a and b. Computer algebra gives the results:

$$\boldsymbol{\nabla} \cdot \mathbf{v}_2 = 0, \tag{88}$$

$$(\mathbf{v}_2 \cdot \boldsymbol{\nabla})\, \mathbf{v}_2 = -\frac{a}{r^5} \begin{bmatrix} a\sin^2\theta + b\,r^2\sin\theta + 2\,a\cos^2\theta \\ \cos\theta\,\left(a\sin\theta - b\,r^2\right) \\ 0 \end{bmatrix}, \tag{89}$$

$$\boldsymbol{\nabla} \times \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{b}{r} \end{bmatrix}, \tag{90}$$

$$q_2 = \frac{a}{r^6}\left(5\,a\sin^2\theta + b\,r^2\sin\theta + 7\,a\cos^2\theta\right). \tag{91}$$

The vector field (87) is shown in Fig. 3. There is a centre of rotation below the coordinate centre. The velocities are much higher above the centre than below. This leads to partially asymmetric electric field components E_r and E_θ, Eq. (89), which are graphed in Figs. 4 and 5. There are sharp peaks for E_θ at four sides. The E field has been converted to vector form in the XY plane and its (normalized) directional vectors are graphed in Fig. 6. The lower centre in Fig. 3 can be identified to produce a kind of "hole" in the electric field. There is a kind of flow along the Y axis which would not be expected from the form of the velocity field in Fig. 3. Despite these asymmetries, the charge distribution of this model velocity is mainly centrally symmetric as can be seen from Fig. 7. This result was not obvious from the formulas.

A more general example can be constructed by

$$\mathbf{v}_3 = \begin{bmatrix} \frac{a}{r^n} \\ f(r,\theta) \\ 0 \end{bmatrix} \tag{92}$$

with a general function $f(r,\theta)$. Then it follows

$$\boldsymbol{\nabla}\cdot\mathbf{v}_3 = r^{-n-1}\left(r^n \frac{d}{d\theta}\,\mathrm{f}(r,\theta) - a\,n + a\right), \tag{93}$$

$$(\mathbf{v}_3\cdot\boldsymbol{\nabla})\,\mathbf{v}_3 = \begin{bmatrix} -a^2\,n\,r^{-2\,n-1} \\ r^{-n-1}\left(r^n\,\mathrm{f}(r,\theta)\,\frac{d}{d\theta}\,\mathrm{f}(r,\theta) + a\,r\,\frac{d}{d\,r}\,\mathrm{f}(r,\theta)\right) \\ 0 \end{bmatrix}, \tag{94}$$

$$\boldsymbol{\nabla}\times\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{d}{d\,r}\mathrm{f}(r,\theta) + \frac{1}{r}\mathrm{f}(r,\theta) \end{bmatrix} \tag{95}$$

and the charge distribution takes the form

$$q_3 = r^{-2\,n-2}\left(r^{2\,n}\,\mathrm{f}(r,\theta)\,\frac{d^2}{d\,\theta^2}\,\mathrm{f}(r,\theta) + r^{2\,n}\left(\frac{d}{d\,\theta}\,\mathrm{f}(r,\theta)\right)^2 + a\,r^{n+1}\frac{d^2}{d\,r\,d\,\theta}\,\mathrm{f}(r,\theta) + 2\,a^2\,n^2\right). \tag{96}$$

The divergence of this velocity model vanishes if

$$r^{-n-1}\left(r^n\left(\frac{d}{d\,\theta}\,\mathrm{f}(r,\theta)\right) - a\,n + a\right) = 0, \tag{97}$$

which is a differential equation for $f(r,\theta)$ with the solution

$$\mathrm{f}(r,\theta) = \frac{a\,(n-1)\,\theta}{r^n} + c. \tag{98}$$

For $n = 1$ we obtain the model for $\mathbf{v}_1$ discussed above.

3.2 Solution of Ampère-Maxwell law

The complete Ampère-Maxwell law of ECE2 reads in the electrodynamic case:

$$\boldsymbol{\nabla}\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J} \tag{99}$$

with current density (Eq.(40)):

$$\mathbf{J} = \frac{1}{\mu_0}\left(\frac{\kappa_0}{c}\mathbf{E} + \boldsymbol{\kappa}\times\mathbf{B}\right) \tag{100}$$

which in in non-relativistic approximation is:

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right). \tag{101}$$

Here $\mathbf{v}$ is the velocity of charge carriers moving in a medium with conductivity σ. In the static case the law simplifies to

$$\boldsymbol{\nabla}\times\mathbf{B} = \frac{\kappa_0}{c}\mathbf{E} + \boldsymbol{\kappa}\times\mathbf{B} = \sigma\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right), \tag{102}$$

which is a destination equation for the magnetic field if all other quantities are given. In the absence of an electric field the equation

$$\boldsymbol{\nabla}\times\mathbf{B} = \boldsymbol{\kappa}\times\mathbf{B} \tag{103}$$

has to be solved. In the case of a constant $\boldsymbol{\kappa}$, this gives three differential equations for the three components of the $\mathbf{B}$ field, but computer algebra shows that these equations are under-determined. For example making an approach with oscillatory functions:

$$\mathbf{B} = \mathbf{A}\exp(-i\ (x\,\kappa_x + y\,\kappa_y + z\,\kappa_z)) \tag{104}$$

leads to a solution for the (complex-valued) vectorial amplitude

$$\mathbf{A} = \begin{bmatrix} \alpha^2/\kappa_y \\ i\,\alpha/\kappa_y \\ \alpha \end{bmatrix} \tag{105}$$

with an arbitrary constant α. If an electric field is included as in Eq.(102), there is no solution to that equation with the approach (104).

The situation changes if we assume a Beltrami solution for $\mathbf{B}$. Then we have

$$\boldsymbol{\nabla}\times\mathbf{B} = \kappa\mathbf{B} \tag{106}$$

with a constant κ, and for the pure magnetic case without electric field (non-relativistically):

$$\kappa\mathbf{B} = \mu_0\mathbf{J} = \mu_0\sigma\mathbf{v}\times\mathbf{B}. \tag{107}$$

However, for a fixed $\mathbf{v}$, there is no solution of this equation for $\mathbf{B}$. This is an important result, showing that not all equations being derivable from simplifications of ECE2 theory are deployable for solving real world problems. This situation changes as soon as the electric field is included in the Beltrami structure for $\mathbf{B}$:

$$\kappa\mathbf{B} = \mu_0\sigma\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right). \tag{108}$$

This gives an inhomogeneous linear equation system for the components of $\mathbf{B}$, in contrast to Eq. (107), which is a homogeneous system. Defining a velocity constant

$$w = \frac{\mu_0\sigma}{\kappa}, \tag{109}$$

the general solution is

$$\mathbf{B} = \frac{1}{w(w^2+v^2)} \begin{bmatrix} (v_x\, v_z + w\, v_y)\; E_z - w\, E_y\, v_z + v_x\, v_y\, E_y + \left(v_x^2 + w^2\right)\; E_x \\ (v_y\, v_z - w\, v_x)\; E_z + w\, E_x\, v_z + \left(v_y^2 + w^2\right)\; E_y + v_x\, E_x\, v_y \\ \left(v_z^2 + w^2\right)\; E_z + (v_y\, E_y + v_x\, E_x)\; v_z + w\, v_x\, E_y - w\, E_x\, v_y \end{bmatrix}. \tag{110}$$

Trivially, Eq. (108) can also be resolved for **E**, giving

$$\mathbf{E} = \begin{bmatrix} -v_y\, B_z + B_y\, v_z + w\, B_x \\ v_x\, B_z - B_x\, v_z + w\, B_y \\ w\, B_z - v_x\, B_y + B_x\, v_y. \end{bmatrix}. \tag{111}$$

It is important to note that the same equation is not a valid equation for **v**, there is no solution. Therefore it is not possible to specify **E** and **B** a priori and find a suitable charge carrier velocity **v**.

Finally we give a numerical example for the Beltrami solution (110). Setting

$$w = 1, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{112}$$

results to

$$\mathbf{B} = \begin{bmatrix} 0 \\ -1/2 \\ 1/2 \end{bmatrix}. \tag{113}$$

Defining this a little bit more general:

$$w = 1, \quad \mathbf{v} = \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix} \tag{114}$$

this leads to

$$\mathbf{B} = \begin{bmatrix} 0 \\ -\frac{v_x E_z}{v_x^2+1} \\ \frac{E_z}{v_x^2+1} \end{bmatrix}. \tag{115}$$

For a better understanding, a vector map $(v_x, E_z) \to (B_y, B_z)$ has been constructed in Fig. 8 to show the resulting magnetic field in the YZ plane. There is a zero field for $E_z = 0$. Directions of the **B** field are maintained by crossing this line but the signs differ.

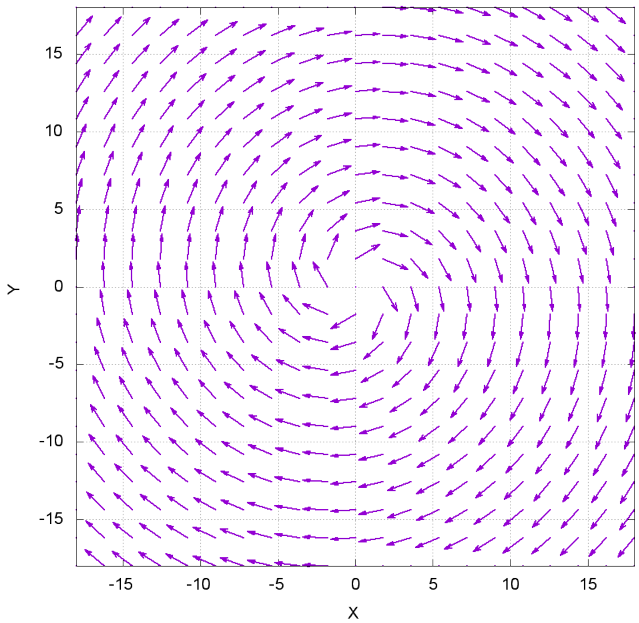

Figure 1: Velocity model $\mathbf{v}_1$.

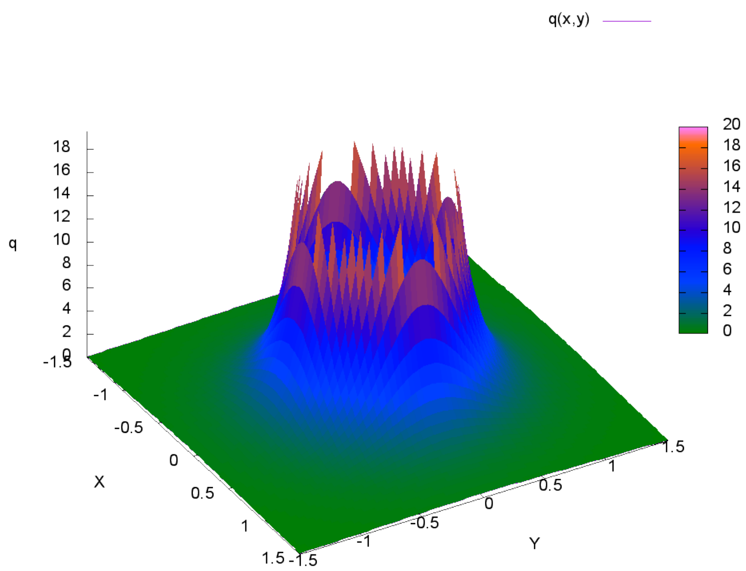

Figure 2: Velocity model $\mathbf{v}_1$, charge distribution q_1.

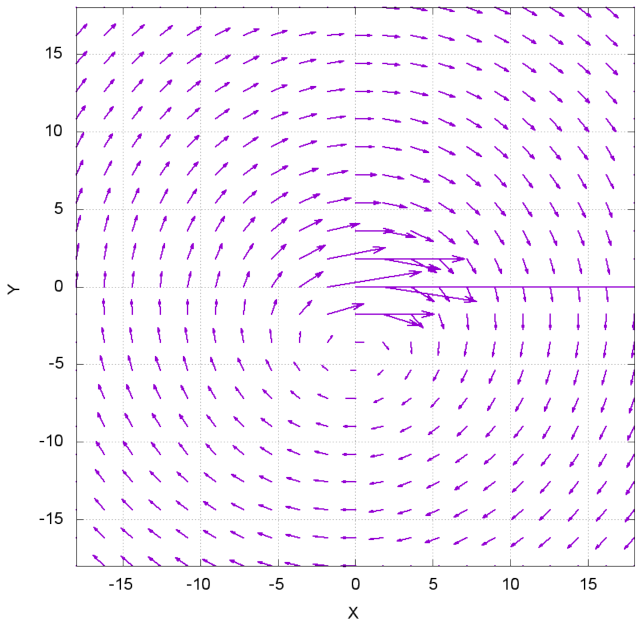

Figure 3: Velocity model $\mathbf{v}_2$.

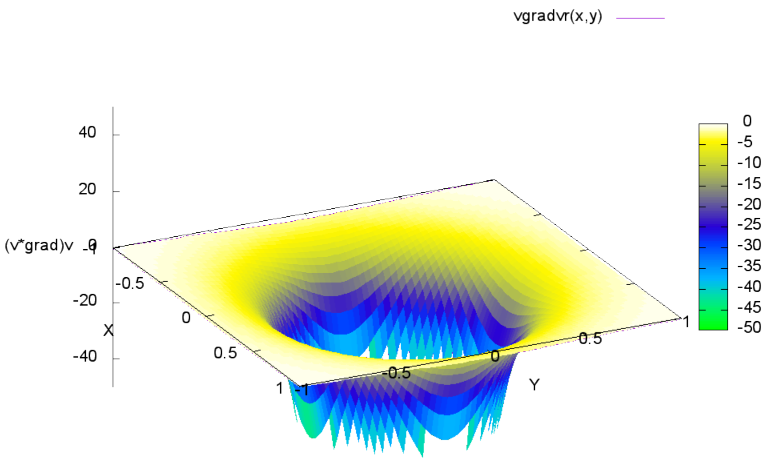

Figure 4: Velocity model $\mathbf{v}_2$, first component (E_r) of Eq. (89).

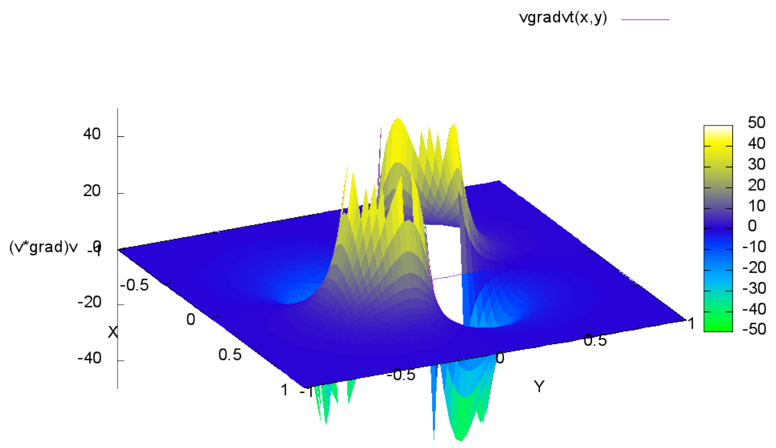

Figure 5: Velocity model $\mathbf{v}_2$, second component (E_θ) of Eq. (89).

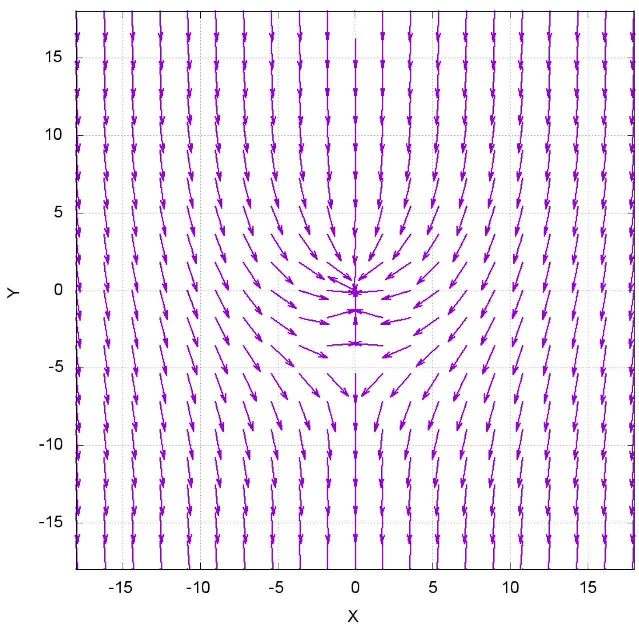

Figure 6: Velocity model $\mathbf{v}_2$, directional vectors of $\mathbf{E}$ field.

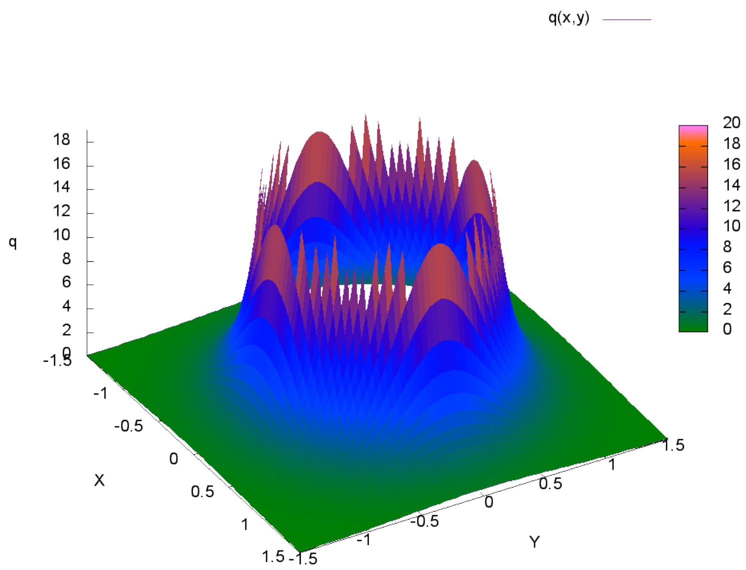

Figure 7: Velocity model $\mathbf{v}_2$, charge distribution q_2.

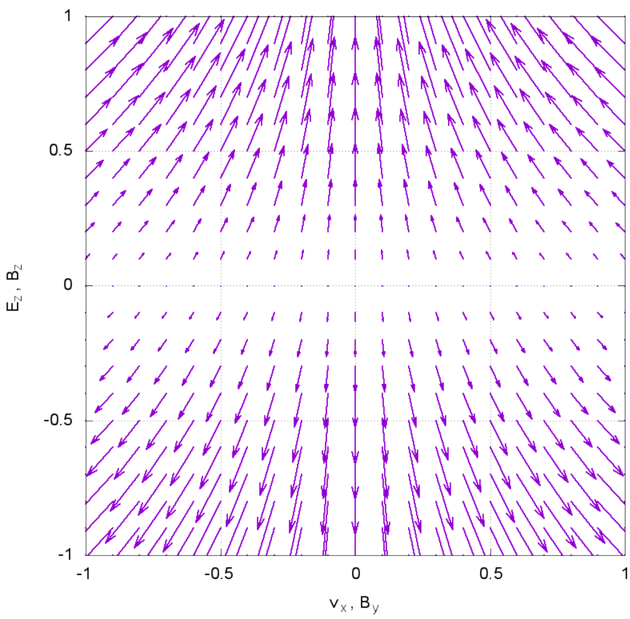

Figure 8: Vector map $(v_x, E_z) \to (B_y, B_z)$ for model of Eq. (114).

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