ORBITAL PRECESSION AS A LORENTZ FORCE EQUATION

by

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ABSTRACT

Using ECE2 relativity and the minimal prescription, the hamiltonian is defined of a particle in the presence of a gravitomagnetic vector potential with the units of velocity. The lagrangian is calculated from the hamiltonian using the canonical momentum, and the relevant Euler Lagrange equation used to derive the gravitomagnetic Lorentz force equation. In the absence of gravitomagnetism this equation reduces to the Newton equation. The precession frequency of the Lorentz force equation is any orbital precession frequency of any kind.

Keywords: ECE2 relativity, gravitomagnetic minimal prescription, Lorentz force equation, general theory of precession.

UFT 347

1. INTRODUCTION

In recent papers of this series {1-12} the gravitomagnetic theory of orbital precession has been developed in several ways, giving self consistent and accurate results. In this paper a synthesis of concepts is attempted by deriving the gravitomagnetic Lorentz force equation form the minimal prescription. The precession frequency of the Lorentz force equation is orbital precession of any kind. This method gives a simple general theory of precession in ECE2 special relativity, which is Lorentz covariant in a space with finite torsion and curvature.

This paper is a concise summary of detailed calculations found in the notes accompanying UFT347 on www.aias.us. Note 347(1) defines the gravitomagnetic field $\mathbf{\Omega}_g$ as a vorticity, the curl of an orbital velocity due to precession $\mathbf{v}_g$. The orbital velocity is identified with the gravitomagnetic vector potential of ECE2 relativity. For a uniform gravitomagnetic field it is possible to calculate the magnitude v_g, and the note shows that for the earth's perihelion precession v_g is about seven orders smaller that the orbital velocity of the earth about the sun. In this view any precession is the vorticity of ECE2 spacetime. In Note 347(2) the vorticity is calculated of the Newtonian orbit, and it is shown that precession increments the vorticity. Note 347(3) introduces the gravitomagnetic minimal prescription, and it is shown that it leads to a hamiltonian that incorporates the gravitomagnetic field. The observed precession is half the magnitude of the gravitomagnetic field. Note 347(4) develops the hamiltonian of Note 347(3). Section 2 of this paper is based on Note 347(5), which gives the general theory of orbital precession in terms of the gravitomagnetic Lorentz force equation.

Section 3 is a numerical and graphical analysis.

2. DERIVATION OF THE LORENTZ FORCE EQUATION

Consider the gravitomagnetic minimal prescription:

$$\underline{p} \rightarrow \underline{p} + m\underline{v}_g \quad -(1)$$

in which the linear momentum of a particle of mass m is incremented by the gravitomagnetic vector potential:

$$\underline{W}_g = \underline{v}_g \quad -(2)$$

where $\underline{v}_g$ is the gravitomagnetic linear velocity. The free particle hamiltonian becomes:

$$H = \frac{1}{2m}\left(\underline{p} + m\underline{v}_g\right)\cdot\left(\underline{p} + m\underline{v}_g\right) = \frac{p^2}{2m} + \frac{1}{2}mv_g^2 + \frac{1}{2}\underline{L}\cdot\underline{\Omega}_g \quad -(3)$$

where the orbital angular momentum is:

$$\underline{L} = \underline{p} \times \underline{r} \quad -(4)$$

and where $\underline{r}$ is the radial vector. Here $\underline{\Omega}_g$ is the gravitomagnetic field:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g . \quad -(5)$$

Any observed orbital precession frequency is defined as half the magnitude of the gravitomagnetic field:

$$\Omega = \frac{1}{2}\Omega_g . \quad -(6)$$

Consider an object of mass m in orbit around an object of mass M. The central gravitational potential is:

$$U(r) = -\frac{mMG}{r} \quad -(7)$$

where G is Newton's constant. In the presence of U the hamiltonian is:

$$H = \frac{1}{2m} \left(\underline{p} + m\underline{v}_g \right) \cdot \left(\underline{p} + m\underline{v}_g \right) + U(r) \quad -(8)$$

which reduces to the Newtonian:

$$H = \frac{p^2}{2m} + U(r) \quad -(9)$$

in the absence of a gravitomagnetic field. It is well known that the Newtonian hamiltonian produces the conic section orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(10)$$

in plane polar coordinates (r, θ). Here α is the half right latitude and ϵ the eccentricity.

The hamiltonian (8) can be developed as:

$$H = \frac{p^2}{2m} + \frac{1}{2} m v_g^2 + \frac{1}{2} \underline{L} \cdot \underline{\Omega}_g + U(r) \quad -(11)$$

as shown in detail in the notes accompanying UFT347. Therefore the minimal prescription (1) is the origin of the gravitomagnetic field appearing in the orbital hamiltonian (11) The minimal prescription therefore produces any observed orbital precession.

The lagrangian is calculated from the hamiltonian using the well known canonical momentum where q is a generalized coordinate:

$$p_q = \frac{\partial \mathcal{L}}{\partial \dot{q}} \quad -(12)$$

Denote

$$\dot{\underline{r}} = \frac{1}{m} \left(\underline{p} + m\underline{v}_g \right) \quad -(13)$$

then:

$$H = \underline{p} \cdot \underline{\dot{r}} - \mathcal{L}. \quad -(14)$$

The lagrangian is therefore:

$$\mathcal{L} = \frac{1}{2}m\left(\underline{p} + m\underline{v}_g\right)\cdot\left(\underline{p} + m\underline{v}_g\right) - U(r) - m\underline{\dot{r}} \cdot \underline{v}_g. \quad -(15)$$

The relevant Euler Lagrange equation is:

$$\underline{\nabla}\mathcal{L} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}}\right). \quad -(16)$$

The left hand side of this equation is:

$$\underline{\nabla}\mathcal{L} = \underline{\nabla}\left(\frac{1}{2}m\underline{\dot{r}}\cdot\underline{\dot{r}} - U(r)\right) - m\underline{\nabla}\left(\underline{\dot{r}}\cdot\underline{v}_g\right). \quad -(17)$$

In general:

$$\underline{\nabla}\left(\underline{\dot{r}}\cdot\underline{v}_g\right) = \left(\underline{\dot{r}}\cdot\underline{\nabla}\right)\underline{v}_g + \left(\underline{v}_g\cdot\underline{\nabla}\right)\underline{\dot{r}} + \underline{\dot{r}}\times\left(\underline{\nabla}\times\underline{v}_g\right) + \underline{v}_g\times\left(\underline{\nabla}\times\underline{\dot{r}}\right) \quad -(18)$$

and reduces to:

$$\underline{\nabla}\left(\underline{\dot{r}}\cdot\underline{v}_g\right) = \left(\underline{\dot{r}}\cdot\underline{\nabla}\right)\underline{v}_g + \underline{\dot{r}}\times\left(\underline{\nabla}\times\underline{v}_g\right) \quad -(19)$$

if it is assumed that:

$$\left(\underline{v}_g\cdot\underline{\nabla}\right)\underline{\dot{r}} = \underline{0} \quad -(20)$$

and:

$$\underline{\nabla}\times\underline{\dot{r}} = \underline{0}. \quad -(21)$$

So

$$\underline{\nabla}\mathcal{L} = -\underline{\nabla}U(r) - m\left(\left(\underline{\dot{r}}\cdot\underline{\nabla}\right)\underline{v}_g + \underline{\dot{r}}\times\left(\underline{\nabla}\times\underline{v}_g\right)\right) \quad -(22)$$

The right hand side of Eq. (16) is:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}}\right) = \frac{d}{dt}\left(m\dot{\underline{r}} - m\underline{v}_g\right) \quad -(23)$$

In order to evaluate the total derivative of $\underline{v}_g$, consider one component, e. g.

$$\frac{dv_{gx}}{dt} = \frac{\partial v_{gx}}{\partial t} + \left(\frac{\partial x}{\partial t}\right)\left(\frac{\partial v_{gx}}{\partial x}\right) + \ldots \quad -(24)$$

to first order. Therefore in three dimensions:

$$\frac{d\underline{v}_g}{dt} = \frac{\partial \underline{v}_g}{\partial t} + \left(\dot{\underline{r}} \cdot \underline{\nabla}\right)\underline{v}_g \quad -(25)$$

and:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}}\right) = m\ddot{\underline{r}} - m\left(\frac{\partial \underline{v}_g}{\partial t} + \left(\dot{\underline{r}} \cdot \underline{\nabla}\right)\underline{v}_g\right). \quad -(26)$$

The Euler Lagrange equation is therefore:

$$-\underline{\nabla}U(r) - m\dot{\underline{r}} \times \left(\underline{\nabla} \times \underline{v}_g\right) = m\ddot{\underline{r}} - m\frac{\partial \underline{v}_g}{\partial t} \quad -(27)$$

i .e. :

$$m\ddot{\underline{r}} = -\underline{\nabla}U(r) + m\frac{\partial \underline{v}_g}{\partial t} - m\dot{\underline{r}} \times \left(\underline{\nabla} \times \underline{v}_g\right) \quad -(28)$$

Now define:

$$m\phi_g = -U(r) \quad -(29)$$

and:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g \quad -(30)$$

to find the gravitomagnetic Lorentz force equation:

$$\underline{F} = m\underline{\ddot{r}} = -m\left(\underline{E}_g + \underline{\dot{r}} \times \underline{\Omega}_g\right) \quad -(31)$$

where:

$$\underline{E}_g = -\underline{\nabla}\phi_g - \frac{\partial \underline{v}_g}{\partial t} \quad -(32)$$

is the gravitomagnetic analogue of the acceleration due to gravity in ECE2 relativity.

The precession of any orbit is therefore governed by the gravitomagnetic Lorentz force law (31) with precession frequency (6).

From Eqs. (29) and (7):

$$\phi_g = \frac{MG}{r} . \quad -(33)$$

It follows that:

$$\underline{\nabla}\phi_g = -\frac{MG}{r^2}\underline{e}_r \quad -(34)$$

so:

$$\underline{E}_g = \frac{MG}{r^2}\underline{e}_r - \frac{\partial \underline{v}_g}{\partial t} . \quad -(35)$$

The gravitomagnetic Lorentz force is therefore:

$$\underline{F} = m\underline{\ddot{r}} = -m\frac{MG}{r^2}\underline{e}_r + m\frac{\partial \underline{v}_g}{\partial t} - m\underline{\dot{r}} \times \underline{\Omega}_g \quad -(36)$$

where:

$$\underline{\dot{r}} = \frac{1}{m}\left(\underline{p} + m\underline{v}_g\right) . \quad -(37)$$

In the absence of a gravitomagnetic field, Eq. (36) reduces to the Newtonian:

$$\underline{F} = m\ddot{\underline{r}} = -\frac{mMG}{r^2}\underline{e}_r . \quad -(38)$$

For a planar orbit it is well known {1 - 12} that:

$$\underline{v} = \dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{d}{dt}\left(r\underline{e}_r\right) = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta \quad -(39)$$

in the absence of a gravitomagnetic field. The acceleration in the absence of a gravitomagnetic field is:

$$\underline{a} = \ddot{\underline{r}} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta\right), \quad -(40)$$

an expression which gives rise {1 -12} to the well known Coriolis and centripetal forces, as developed in detail in previous UFT papers. So the gravitomagnetic force terms occur in addition to these well known forces. The gravitomagnetic force terms result in a precessing orbit, whereas the centripetal and Coriolis forces describe a non precessing orbit.

The gravitomagnetic field $\underline{\Omega}_g$ is governed by the gravitational equivalent of the Ampere Law of ECE2 electrodynamics:

$$\underline{\nabla} \times \underline{\Omega}_g = \frac{4\pi G}{c^2}\underline{J}_g \quad -(41)$$

where $\underline{J}_g$ is a localized current density of mass, analogous to electric current density in electrodynamics. The vacuum gravitomagnetic permeability is:

$$\mu_{0g} = \frac{4\pi G}{c^2} \quad -(42)$$

and the gravitomagnetic four potential is:

$$W_g^\mu = \left(\phi_g, c\underline{W}_g\right) \quad -(43)$$

In UFT328 it was shown that the simultaneous solution of the hamiltonian and lagrangian of ECE2 theory leads to orbital precession. This paper confirms that finding in the sense that the same underlying structure is used: ECE2 relativity.

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