THE DESCRIPTION OF ANY ORBIT IN TERMS OF SPECIAL AND GENERAL RELATIVITY

by

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ABSTRACT

It is shown that the preferred description of any orbit is one based on the Minkowski metric. The orbit is described straightforwardly in terms of the ratio of the relativistic linear and angular momentum. Any other description, such as Einsteinian general relativity (EGR), is necessarily more complicated and so must be rejected by Ockham's Razor. The new theory is compared with the Crothers general relativity, which is the only correct metrical theory of general relativity, and is applied to precessing planetary orbits and the Thomas Precession.

Keywords: ECE theory, the description of any orbit in terms of special and general relativity.

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1. INTRODUCTION.

Special relativity is generally thought of as a theory of flat spacetime without curvature or torsion and applicable only to linear motion without acceleration. However, in previous work {1 - 10} it has been shown that the Minkowski metric expressed in plane polar coordinates produces an orbit in which angular momentum may be defined clearly. The Minkowski metric diag (1, -1, -1, -1) also produces spacetime torsion and curvature through the Cartan structure equations, because it may be factorized into phase dependent tetrads. In Section 2 it is shown that the Minkowski metric must always be the preferred description of any orbit, and produces a straightforward description of any orbit in terms of the ratio p / L of the relativistic linear momentum p and relativistic angular momentum L. It is shown that the ratio p / L can also produce a description of any orbit in general relativity, for example Einsteinian general relativity (EGR), but that description is always more complicated and so must always be rejected by Ockham's Razor. Previous work has shown that the only valid description of general relativity based on an infinitesimal line element is the Crothers general relativity (CGR) and the CGR description of any orbit is compared with this new description. In Section 3 this new theory is applied to precessing planetary orbits and the Thomas Precession. In Section 4, some graphical results are discussed.

2. THE ORBITAL EQUATIONS OF SPECIAL AND GENERAL RELATIVITY

Consider the infinitesimal line element in plane polar coordinates produced by the Minkowski metric:

$$ds^2 = c^2 d\tau^2 - dr^2 - r^2 d\theta^2 \quad -(1)$$

where (r, θ) is the coordinate system, τ is the proper time, t the observer time and c is the speed of light in a vacuum. It follows that:

$$mc^2 = \frac{E^2}{mc^2} - \frac{p^2}{m} \quad -(2)$$

which is the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(3)$$

Q.E.D. A lagrangian analysis produces the energy E:

$$E = \gamma mc^2 = \left(\frac{dt}{d\tau}\right) mc^2 \quad -(4)$$

and the angular momentum:

$$L = \gamma m r^2 \omega = \gamma m r^2 \frac{d\theta}{dt} \quad -(5)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad -(6)$$

The latter is calculated by expressing Eq. (1) as:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - \underline{dr} \cdot \underline{dr} \quad -(7)$$

with

$$\underline{dr} \cdot \underline{dr} = v^2 dt^2. \quad -(8)$$

The linear momentum in Eq. (3) is defined as:

$$p^2 = m^2 \left(\left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2 \right) \quad -(9)$$

$$= \gamma^2 m^2 \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) = (\gamma m v)^2 \quad -(10)$$

from which the relativistic linear momentum is defined:

$$\underline{p} = \gamma m \underline{v}. \quad -(11)$$

Squaring Eq. (11) gives;

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{v^2}{c^2} \right), \quad -(12)$$

from Eq. (6):

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \quad -(13)$$

so:

$$p^2 c^2 = \gamma^2 m^2 c^4 - m^2 c^4 \quad -(14)$$

giving Eq. (3), Q. E. D. Therefore the analysis is internally consistent.

Eq. (2) may be written as:

$$mc^2 = \frac{E^2}{mc^2} - m \left(\frac{dr}{d\tau} \right)^2 - \frac{L^2}{mr^2} \quad -(15)$$

where:

$$\frac{dr}{d\tau} = \frac{dr}{d\theta} \frac{d\theta}{d\tau} = \frac{L}{mr^2} \frac{dr}{d\theta} \quad -(16)$$

It follows that the orbit is defined by:

$$\left(\frac{dr}{d\theta} \right)^2 = \frac{mr^4}{L^2} \left(\frac{E^2 - m^2 c^4}{mc^2} - \frac{L^2}{mr^2} \right). \quad -(17)$$

Using Eq. (3):

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\left(\frac{p}{L}\right)^2 - \frac{1}{r^2}\right) \quad -(18)$$

so:

$$\left(\frac{p}{L}\right)^2 = \frac{1}{r^4}\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) \quad -(19)$$

Any observed orbit expressed by $dr/d\theta$ may be described by the ratio:

$$\left(\frac{p}{L}\right)^2 = \left(\frac{v}{\omega r}\right)^2 \quad -(20)$$

and this is the result of the Minkowski metric.

Newtonian dynamics is a well known limit of special relativity {11} usually described merely by v << c. However Newtonian dynamics expresses the orbit as follows:

$$\left(\frac{d\theta}{dr}\right)^2 = \frac{L^2}{r^4}\left(\frac{1}{2m\left(E - U - \frac{L^2}{2mr^2}\right)}\right) \quad -(21)$$

where E is the non relativistic total energy and where U is the potential energy. Eq. (18) may be written as:

$$\left(\frac{d\theta}{dr}\right)^2 = \frac{1}{r^4} \cdot \frac{1}{\left(\frac{p}{L}\right)^2 - \frac{1}{r^2}} \quad -(22)$$

Eqs. (21) and (22) are the same if:

$$r^2 p^2 = 2mr^2(E - U) \quad -(23)$$

i.e. if:

$$E = \frac{p^2}{2m} + U \quad -(24)$$

which is the well known expression for non relativistic total energy as the sum of the non relativistic kinetic energy:

$$T = \frac{p^2}{2m} \quad -(25)$$

and the potential energy. As first shown by Robert Hooke, an elliptical orbit is produced by:

$$U = -\frac{k}{r} \quad -(26)$$

and Bernoulli later showed that all conical sections or conic sections are produced by a potential energy of type (26). The insight produced by the new theory of this paper is that this procedure is equivalent to expressing the ratio p / L as:

$$\left(\frac{p}{L}\right)^2 = \frac{2m}{L^2}(E - U) \quad -(27)$$

so the Newtonian theory expresses linear momentum p in terms of two parameters E and U and is a more complicated theory that is rejected by Ockham's Razor. The theory of this paper~~s~~ is preferred to general relativity and Newtonian dynamics by Ockham's Razor.

EGR is merely a simple adjustment of the Minkowski metric. In a spherically symmetric spacetime {12} EGR can be expressed most generally as:

$$ds^2 = c^2 d\tau^2 = Ac^2 dt^2 - Bdr^2 - r^2 d\theta^2 \quad -(28)$$

where A and B are in general functions of r and t. Eq. (21) has the structure:

$$mc^2 = \frac{E^2}{mc^2} - \frac{p^2}{m} \quad -(29)$$

where:

$$E = A^{1/2} \frac{dt}{d\tau} mc^2 \quad -(30)$$

and:

$$p^2 = m^2 \left(B \left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2 \right) \quad -(31)$$

This means that general relativity produces the same type of energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(32)$$

as special relativity. Eq. (28) may be written as:

$$ds^2 = c^2 d\tau^2 = A c^2 dt^2 - v^2 dt^2 \quad -(33)$$

where:

$$v^2 = B\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \quad -(34)$$

so:

$$\gamma = \frac{dt}{d\tau} = \left(A - \frac{v^2}{c^2}\right)^{-1/2} \quad -(35)$$

which is the Lorentz factor with 1 replaced by A. Therefore all theories of general relativity of type (28) reduce to the relativistic linear momentum equation:

$$\underline{p} = \gamma m \underline{v} \quad -(36)$$

and have a very simple structure, the same structure as special relativity. Squaring Eq. (36) produces:

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{v^2}{c^2}\right) \quad -(37)$$

Using Eq. (35) in the form:

$$\frac{1}{\gamma^2} = A - \frac{v^2}{c^2} \quad -(38)$$

produces:

$$p^2 c^2 = A\gamma^2 m^2 c^4 - m^2 c^4 \quad -(39)$$

$$= E^2 - m^2 c^4$$

So Eq. (36) produces Eq. (32), Q.E.D. and the theory is internally consistent.

The orbit in any theory of general relativity of type (28) is given by writing Eq. (32) as:

$$mc^2 = \frac{E^2}{mc^2} - mB\left(\frac{dr}{d\tau}\right)^2 - \frac{L^2}{mr^2} \quad -(40)$$

so:

$$mB\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - mc^2 - \frac{L^2}{mr^2} \quad -(41)$$

and:

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{L^2}{m^2 r^4}\left(\frac{dr}{d\theta}\right)^2 \quad -(42)$$

giving the orbital equation:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^4}{BL^2}\left(p^2 - \frac{L^2}{r^2}\right). \quad -(43)$$

It follows that the ratio of p to L in any theory of general relativity of this type is:

$$\left(\frac{p}{L}\right)^2 = \frac{B}{r^4}\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right). \quad -(44)$$

In the Minkowski metric:

$$B = 1 \quad -(45)$$

so:

$$\left(\frac{p}{L}\right)^2_{GR} = B\left(\frac{p}{L}\right)^2_{SR} \quad -(46)$$

The only effect of general relativity is to make the analysis more complicated by introducing the parameter B. Therefore the theory of this paper is preferred to general relativity by Ockham's Razor.

An observable orbit of any type anywhere in the universe can always be expressed as $dr/d\theta$. Its simplest relativistic description is always the theory of this paper, based on the Minkowski metric. This procedure is in agreement with the Baconian principles and Ockham's Razor.

The only correct theory of metric based general relativity is that given by Crothers, in which the infinitesimal line element is:

$$ds^2 = c^2 d\tau^2 = AC^{1/2} c^2 dt^2 - BC^{1/2} dr^2 - C(r)\, d\theta^2 \quad -(47)$$

and in which C is defined by:

$$C(r) = \left(|r - r_0|^n + \alpha_1^n \right)^{2/n} \quad -(48)$$

where n is an integer and in which α_1 is a parameter. The CGR theories do not produce non existent singularities such as black holes and big bang, obsolete and incorrect concepts. In CGR theories the Lorentz factor is given by:

$$\gamma = \frac{dt}{d\tau} = \left(AC^{1/2} - \frac{v^2}{c^2} \right)^{-1/2} \quad -(49)$$

where the velocity v is defined by

$$v^2 = BC^{1/2} \left(\frac{dr}{dt} \right)^2 + C(r) \left(\frac{d\theta}{dt} \right)^2 \quad -(50)$$

CGR theories can all be reduced to:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(51)$$

in the same way as EGR theories, provided that the energy is defined as:

$$E = \gamma A^{1/2} C^{1/4} mc^2 \quad -(52)$$

and the momentum as:

$$p^2 = m^2 \left(BC^{1/2} \left(\frac{dr}{d\tau} \right)^2 + C(r) \left(\frac{d\theta}{d\tau} \right)^2 \right) \quad -(53)$$

Eq. (51) in CGR theories is equivalent to:

$$\underline{p} = \gamma m \underline{v} . \quad -(54)$$

In CGR theories the ratio of p to L is defined by:

$$\left(\frac{p}{L} \right)^2 = \frac{BC^{1/2}}{r^4} \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right) \quad -(55)$$

and CGR theories correct EGR theories by this factor. Unlike EGR theories, CGR theories reduce correctly to the present Minkowski theory if

$$BC^{1/2} = \underline{1} \quad -(56)$$

and so are valid theories by Ockham's Razor provided that Eq. (56) is applied.

3. APPLICATIONS OF THE MINKOWSKI THEORY.

In the solar system the orbit is observed to be the precessing ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(57)$$

where x - 1 is of the order of 10^{-6}. It is observed experimentally in the solar system that x is a constant independent of r and θ. The half right latitude is defined by {11}:

$$\alpha = (1+\epsilon) r_{min} = (1-\epsilon) r_{max} \quad -(58)$$

where ϵ is the eccenticity, r_{min} the distance of closest approach in the orbit, and r_{max} the maximum separation of a mass m orbiting a mass M. Differentiating Eq. (57) produces:

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon x}{\alpha}\right)^2 r^4 \sin^2(x\theta)$$
$$= \left(\frac{\epsilon x}{\alpha}\right)^2 r^4 \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)$$
$$= x^2 \left(\frac{r}{\alpha}\right)^2 \left(\epsilon^2 r^2 - (\alpha - r)^2\right) \quad -(59)$$
$$= x^2 r^2 \left(\frac{1-\epsilon}{1+\epsilon}\right)\left(\frac{(r_{max} - r)(r - r_{min})}{r_{min}^2}\right)$$
$$= x^2 r^2 \left(\frac{1+\epsilon}{1-\epsilon}\right)\left(\frac{(r_{max} - r)(r - r_{min})}{r_{max}^2}\right).$$

In the Newtonian theory:

$$x = 1 \quad -(60)$$

and the ellipse is static. From Eqs. (19) and (60) the ratio p / L is:

$$\left(\frac{p}{L}\right)^2 = \frac{1}{r^2}\left(1 + x^2\left(\frac{1+\epsilon}{1-\epsilon}\right)\left(\frac{(r_{max} - r)(r - r_{min})}{r_{max}^2}\right)\right) \quad -(61)$$

and is plotted in Section 4 of this paper. By Ockham's Razor this is the simplest description of planetary orbits based on a metric or infinitesimal line element. It is a very simple and yet very profound description which removes all the fallacies of EGR theories such as black holes and big bang. The previous paper UFT232 on www.aias.us shows that EGR theories give an incorrect description of planetary precession. CGR theories give a correct description provided that Eq. (56) is used. The purpose of Ockham's Razor is to give the simplest possible theory of natural philosophy consistent with all the data, and Eq (19) is precisely this. It is well known that EGR theories fail qualitatively in galaxies of any type. The dogma of standard physics claims absurdly that EGR is precise in the solar system even though it fails completely in galaxies. The truth is that EGR fails completely as a theory, and this is very easy to show.

In general, x in Eq. (57) is a function of θ . Previous work in this series has shown that any orbit can be described by a conical section in which x depends on θ . EGR theories therefore become irrelevant as well as incorrect. CGR theories still remain valid given Eq. (56). When x depends on θ the Leibnitz theorem produces:

$$\frac{dr}{d\theta} = \left(x(\theta) + \theta \frac{dx}{d\theta} \right) \frac{\epsilon}{\alpha} r^2 \sin\left(\theta x(\theta)\right) \quad -(62)$$

Denote:

$$y = x + \theta \frac{dx}{d\theta} \quad -(63)$$

and denote:

$$r = f(\theta) = \frac{\alpha}{1 + \epsilon \cos\left(\theta x(\theta)\right)} \quad -(64)$$

then any orbit may be synthesized by:

$$f(\theta) = \epsilon \alpha \left(\int \frac{x(\theta) \sin\left(\theta x(\theta)\right)}{1 + \epsilon \cos\left(\theta x(\theta)\right)} d\theta + \int \frac{\theta \sin\left(\theta x(\theta)\right)}{1 + \epsilon \cos\left(\theta x(\theta)\right)} dx \right) \quad -(65)$$

In the limit of constant x then:

$$f(\theta) = \epsilon \alpha x \int \frac{\sin(x\theta)}{1+\epsilon\cos(x\theta)} d\theta \quad -(66)$$

because for a constant x:

$$dx = 0. \quad -(67)$$

Eq. (66) is integrated numerically in Section 4. In this general case the ratio of p to L is given by Eq. (61) with x replaced by y.

Finally consider the Thomas precession, which was explained in UFT110 of this series by a rotating Minkowski metric defined by:

$$d\theta' = d\theta + \Omega \, dt \quad -(68)$$

where Ω is the angular velocity of the rotation. From Eq. (68) in Eq. (1) it follows that:

$$d\tau'^2 = \left(1 - \left(\frac{r\Omega}{c}\right)^2\right)\left(dt^2 + \frac{2\Omega r^2 d\theta dt}{c^2\left(1-\left(\frac{r\Omega}{c}\right)^2\right)}\right) - dr^2 - r^2 d\theta^2 \quad -(69)$$

As in UFT110 the rotation of the Minkowski metric produces a precession:

$$\alpha = 2\pi\left(\left(1-\left(\frac{r\Omega}{c}\right)^2\right)^{1/2} - 1\right) \quad -(70)$$

for a rotation of 2π. For planetary orbits the precession of the ellipse produces:

$$\alpha := 2\pi(x - 1) \quad -(71)$$

so the precession factor x is:

$$x = \left(1 - \left(\frac{r\Omega}{c}\right)^2\right)^{1/2} \quad -(72)$$

if the planetary precession is thought of as a Thomas precession of the metric itself.

4. SECTION BY DR HORST ECKARDT.

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