LOW ENERGY NUCLEAR FUSION REACTIONS: QUANTUM TUNNELLING.

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ABSTRACT

A new linear equation is developed of relativistic quantum mechanics and the equation is applied to the theory of quantum tunnelling based on the Schroedinger equation in the non relativistic limit. Using a square barrier model in the first approximation, it is shown that low energy nuclear fusion occurs as a result of the Schroedinger equation, which is a limit of the ECE fermion equation. It is shown that for a thin sample and a given barrier height, 100% transmission occurs by quantum tunnelling even when the energy of the incoming particle approaches zero. This is therefore a plausible model of low energy nuclear reaction. The new relativistic equation is used to study relativistic corrections. Absorption of quanta of spacetime may result in enhancement of the quantum tunnelling process.

Keywords: Limits of ECE theory, linear relativistic quantum mechanics, low energy nuclear reaction, quantum tunnelling

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1. INTRODUCTION

Recently in this series of papers {1 - 10} the ECE fermion equation has been used to give an explanation of low energy nuclear reaction (LENR {11}), which has been observed experimentally to be reproducible and repeatable, and which has been developed into a new source of energy. In this paper the plausibility of LENR is examined with a new linear type of relativistic quantum mechanics which can be derived straightforwardly from classical special relativity, a well defined limit of ECE theory. It is shown in Section 2 that the Einstein energy equation can be quantized directly into a new type of linear, relativistic Schroedinger equation which reduces in the non relativistic limit to the Schroedinger equation. It is well known {12} that the latter is the basis for quantum tunnelling theory, and can be solved to give the transmission coefficient of quantum tunnelling. In Section 2, the well known standard theory {12} of quantum tunnelling is used with a rectangular barrier of thickness 2a and height V_o where V_o is the potential energy. This is a simple but instructive model of nuclear fusion in which an incoming atom meets the Coulomb barrier of a second atom and tunnels into it, causing nuclear fusion. It is shown that 100% transmission (complete tunnelling) can occur for a thin sample when the energy E of the incoming particle approaches zero for a finite V. This process is graphed in Section 3. Relativistic corrections of this simple theory can be developed from the new linear equation derived in this paper of relativistic quantum mechanics. Relativistic corrections are graphed and discussed in Section 3.

2. LINEAR EQUATION FOR RELATIVISTIC QUANTUM MECHANICS AND APPLICATION TO THE TRANSMISSION COEFFICIENT OF QUANTUM TUNNELLING.

Consider the Einstein equation of special relativity {12}:

$$E^2 = p^2c^2 + m^2c^4 \quad -(1)$$

where E is the total relativistic energy:

$$E = \gamma mc^2 \quad -(2)$$

and where $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \gamma m\underline{v}. \quad -(3)$$

Here m is the particle mass, c the speed of light in vacuo, and γ is the Lorentz factor

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(4)$$

where $\underline{v}$ is the particle velocity. The classical relativistic hamiltonian is

$$H = \gamma mc^2 + V \quad -(5)$$

where V is the potential energy.

The problem faced by the pioneers of relativistic quantum mechanics was the quantization of Eq. (1) using the Schroedinger postulate:

$$p^\mu = i\hbar\partial^\mu \quad -(6)$$

i.e. :

$$\hat{E} = i\hbar\frac{\partial}{\partial t}, \quad \hat{\underline{p}} = -i\hbar\underline{\nabla}. \quad -(7)$$

Eq. (1) produced the Klein Gordon equation:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad -(8)$$

and the Dirac equation, which has been recently developed into the ECE fermion equation in UFT172 ff. of this series. These are all non-linear in E because of the structure of Eq. (1).

Consider Eq. (1) in the format:

$$E = \gamma mc^2 = \frac{1}{\gamma m} \left(p^2 + m^2 c^2 \right). \quad -(9)$$

By using the momentum operator:

$$\hat{\underline{p}} = -i\hbar \underline{\nabla} \quad -(10)$$

Eq. (9) becomes a linear, relativistic Schroedinger equation of a new type

$$\hat{H} \phi = E \phi \quad -(11)$$

where the relativistic hamiltonian eigenoperator for a free particle is:

$$\hat{H} = \frac{1}{\gamma m} \left(\hat{p}^2 + m^2 c^2 \right) \quad -(12)$$

and where the total energy eigenvalues are:

$$E = \gamma mc^2. \quad -(13)$$

The eigenfunction ϕ is the wave function of the Schroedinger equation generalized to relativistic quantum mechanics.

It follows that:

$$\hat{p}^2 \phi = -\hbar^2 \nabla^2 \phi = m^2 c^2 \left(\gamma^2 - 1 \right) \phi \quad -(14)$$

The Schroedinger postulate (6) combined with the de Broglie Einstein postulate is:

$$p^\mu = i\hbar \partial^\mu = \hbar \kappa^\mu \quad -(15)$$

where:

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \; \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right), \; \kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa}\right). \quad -(16)$$

Here ω is the frequency of the matter wave, and $\underline{\kappa}$ is its wavenumber. Therefore:

$$\hat{p}^2 \psi = \hbar^2 \kappa^2 \psi = m^2 c^2 (\gamma^2 - 1)\psi = \left(\frac{E^2}{c^2} - m^2 c^2\right)\psi. \quad -(17)$$

For a free wave / particle:

$$\kappa = \frac{mc}{\hbar}\left(\gamma^2 - 1\right)^{1/2} \quad -(18)$$

and in the non-relativistic limit:

$$m^2 c^2 (\gamma^2 - 1) = m^2 c^2 \left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - 1\right) \xrightarrow[v \ll c]{} m^2 v^2 \quad -(19)$$

so:

$$p \rightarrow mv \quad -(20)$$

which is the classical relation between momentum and velocity. For the purposes of quantum tunnelling theory denote:

$$k = \frac{mc}{\hbar}\left(\gamma^2 - 1\right)^{1/2}. \quad -(21)$$

In the presence of potential energy V the operator (12) becomes:

$$\hat{H} = \frac{1}{\gamma m}\left(\hat{p}^2 + m^2 c^2\right) + V \quad -(22)$$

so:

$$\hat{p}^2\phi = \hbar^2\kappa^2\phi = \left(\gamma m(E-V) - m^2c^2\right)\phi \quad -(23)$$

and:

$$\kappa^2 = \frac{1}{\hbar^2}\left(\gamma m(E-V) - m^2c^2\right). \quad -(24)$$

In quantum tunnelling theory we wish to consider:

$$E < V \quad -(25)$$

so we define:

$$\kappa = \frac{1}{\hbar}\left(\gamma m(V-E)\right)^{1/2}. \quad -(26)$$

Denote the rest wavenumber by:

$$\kappa_0 = \frac{mc}{\hbar} \quad -(27)$$

then arrive at the definition:

$$\kappa^2 + \kappa_0^2 = \frac{\gamma m}{\hbar^2}(V-E). \quad -(28)$$

Eq. (21) can be written as:

$$k^2 + \kappa_0^2 = \gamma^2\left(\frac{mc}{\hbar}\right)^2 \quad -(29)$$

where

$$E = \gamma mc^2 \quad -(30)$$

so we arrive at the definition:

$$k^2 + \kappa_0^2 = \frac{\gamma m E}{\hbar^2} . \quad -(31)$$

In order to forge a precise analogy with the Schroedinger equation write Eq. (14) as

$$\frac{\hat{p}^2}{2m}\phi = \frac{mc^2}{2}\left(\gamma^2 - 1\right)\phi . \quad -(32)$$

In the non relativistic limit:

$$\frac{mc^2}{2}\left(\gamma^2 - 1\right) = \frac{mc^2}{2}\left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - 1\right) \xrightarrow[v \ll c]{} \frac{1}{2}mv^2 \quad -(33)$$

and reduces to the classical kinetic energy of a free particle:

$$E = T = \frac{1}{2}mv^2 \quad -(34)$$

so:

$$\nabla^2\phi = -\left(\frac{2mE}{\hbar^2}\right)\phi \quad -(35)$$

and:

$$k^2 = \frac{2mE}{\hbar^2} \quad -(36)$$

which is the non relativistic limit of :

$$k^2 = \frac{2mE}{\hbar^2}, \quad E = \frac{mc^2}{2}\left(\gamma^2 - 1\right) . \quad -(37)$$

In the presence of a potential, Eq. (32) becomes:

$$\left(\frac{\hat{p}^2}{2m} + V\right)\phi = E\phi = \frac{mc^2}{2}\left(\gamma^2 - 1\right)\phi \quad -(38)$$

so:

$$-\hbar^2 \nabla^2 \phi = \kappa^2 \hbar^2 \phi = 2m(E - V)\phi \quad -(39)$$

and:

$$\kappa^2 = \frac{2m}{\hbar^2}(V - E), \quad E = \frac{mc^2}{2}(\gamma^2 - 1), \quad -(40)$$

$$V > E.$$

It is well known {12} that the transmission coefficient of quantum tunnelling is:

$$T = 8\kappa^2 k^2 / \left[(k^2 + \kappa^2)^2 \cosh(4\kappa a) - (\kappa^4 + k^4 - 6\kappa k) \right] \quad -(41)$$

for a potential of the type:

$$\left.\begin{aligned} V &= 0, \quad x < -a, \\ V &= V_0, \quad -a < x < a, \\ V &= 0, \quad x > a, \end{aligned}\right] -(42)$$

$$E < V_0, \quad -(43)$$

in which:

$$k^2 = 2mE/\hbar^2, \quad E = mc^2(\gamma^2 - 1)/2, \quad -(44)$$

$$\kappa^2 = 2m(V_0 - E)/\hbar^2, \quad E = mc^2(\gamma^2 - 1)/2. \quad -(45)$$

In Section 3, various results from the standard equation (41) are graphed with the intention of finding the optimal condition for low energy nuclear reaction described through the process of quantum tunnelling. These results are augmented by considerations based on the new relativistic Schroedinger equation (38). This is a simple first theory, contemporary supercomputers and code packages can be applied to the problem of simulating the fusion of one atom with another. The analysis in Section 3 shows that the single most important factor is the mass m of the incoming particle. The extra ingredient given by ECE theory is the possibility of augmenting this standard quantum tunnelling theory with resonant absorption of quanta of spacetime energy. That will be the subject of future work.

3. GRAPHICAL ANALYSIS AND DISCUSSION

Section by Horst Eckardt and Douglas Lindstrom

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3 Graphical analysis and discussion

We start the graphical analysis with the transmission coefficient T (Eq.(41)) for the rectangular barrier. The coefficient depends on wave vectors k and κ and barrier half-width a. In the 3D plot of Fig. 1 the κ dependence is plotted for three values of k with constant a. One sees that T is maximal for k and κ going to zero. In Fig. 2 both a and k have been varied. It can be concluded that T is at maximum when ka as as well as κ are minimal; this corresponds to quantum waves with lowest energy.

Since k and κ depend on the energy E and height of the potential well V_0 (Eqs. (44,45)), it is more conclusive to study the dependence on these parameters. For Fig. 3 the parameters were chosen so that T is near to zero in the range $E < V_0$ which corresponds to the classical limit. Above V_0 the transmission oscillates as can be expected from wave mechanics. For a different parameter set (Fig. 4), T is quite high in the "forbidden" region, showing the quantum mechanical tunneling behaviour. This can also be seen from Fig. 5 in a 3D representation.

In the remaining figures the relativistic effects are studied. According to Eqs. (44,45) the total energy E depends on γ, therefore it is of interest to study the dependence $T(\gamma)$ or $T(v/c)$. The latter is graphed in Figs. 6 and 7 for $a = 0.1$ and $a = 1$, for three values of V_0 each, all constants set to unity. This shows the principal behaviour of the transmission coefficient. It depends highly on the potential barrier. In all cases T drops to zero for $v \to c$. For high V_0 values it is constant in a broader range, denoting that relativistic effects decrease with increasing V_0.

Fig. 8 describes tunneling of an electron through another electron. We had to use atomic units in the calculation, otherwise the arithmetic explodes because of the high values of mc^2. V_0 is interpreted as the Coulomb barrier and kept fix now at a value of

$$V_0 = \frac{1}{r_{electron}} = 18797.0$$

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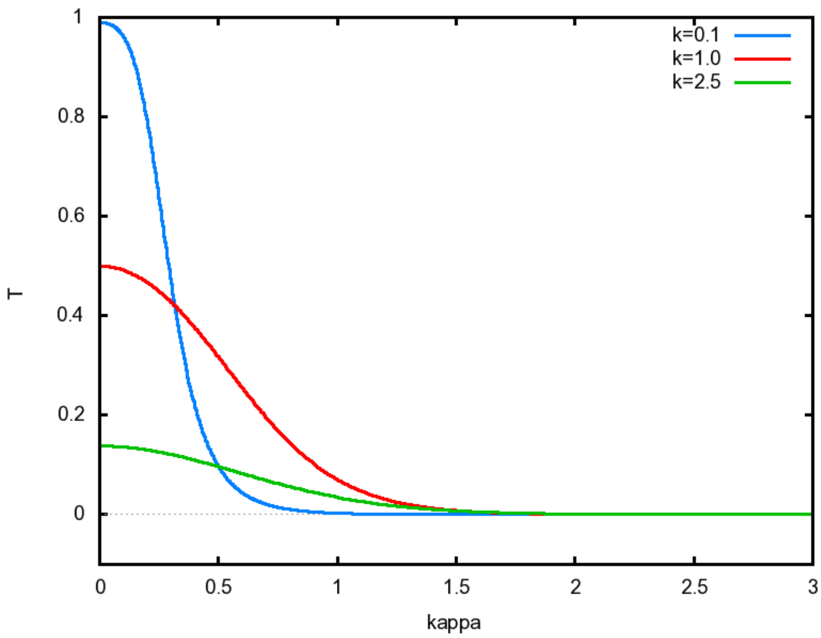

Figure 1: Transmission coefficient $T(\kappa)$ for three k values and $a = 1$.

in atomic uits. The curves are shown for three mass values, where the electron mass is $m = 1$. The tunnelling probability decreases drastically with slightly enhanced masses. Mass is a very sensitive parameter. This can also be seen from Fig. 9 where we have graphed the mass dependence directly with v/c as a curve parameter. For $v \to c$ the transmission coefficient degenerates to a delta function at $m = 0$.

Finally we considered proton-proton tunnelling (Fig. 10). This is impossible because the transmission is practically zero for $m > 4$ and the proton mass is 1836 electron masses. The Coulomb barrier is similar as for an electron as the particle radius for both particles is in the same order of magnitude. Tests showed that the barrier value is not dicisive, it is the particle mass.

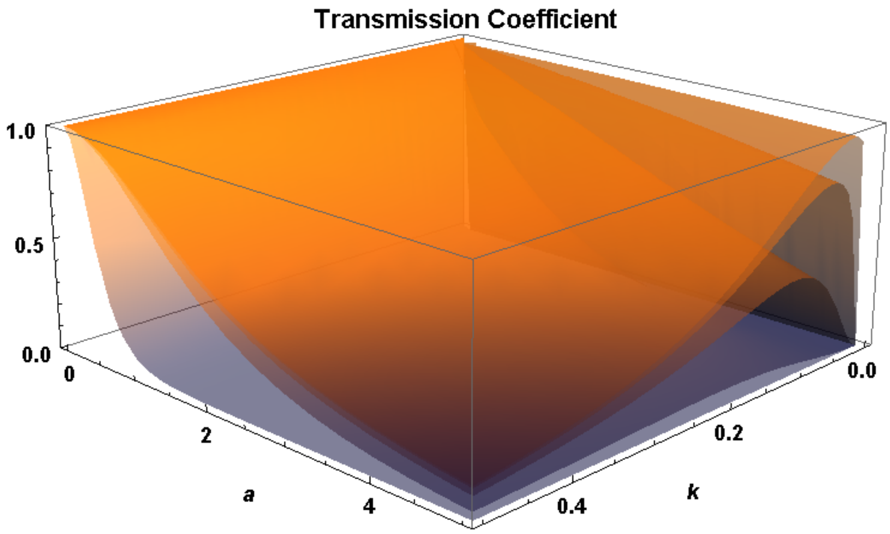

Figure 2: Transmission coefficient $T(k, a)$ for five values of κ.

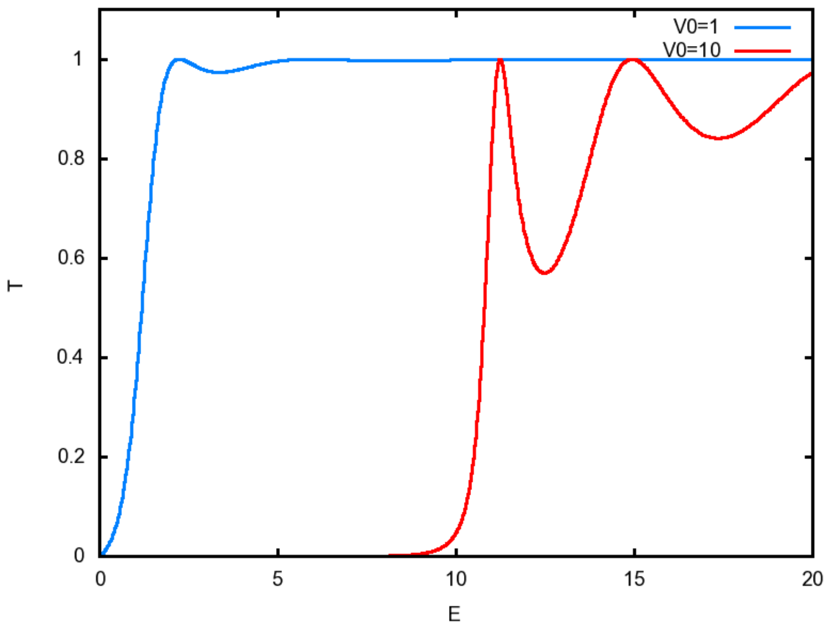

Figure 3: Transmission coefficient $T(E)$ for $m = \hbar = 1,\ a = 1$.

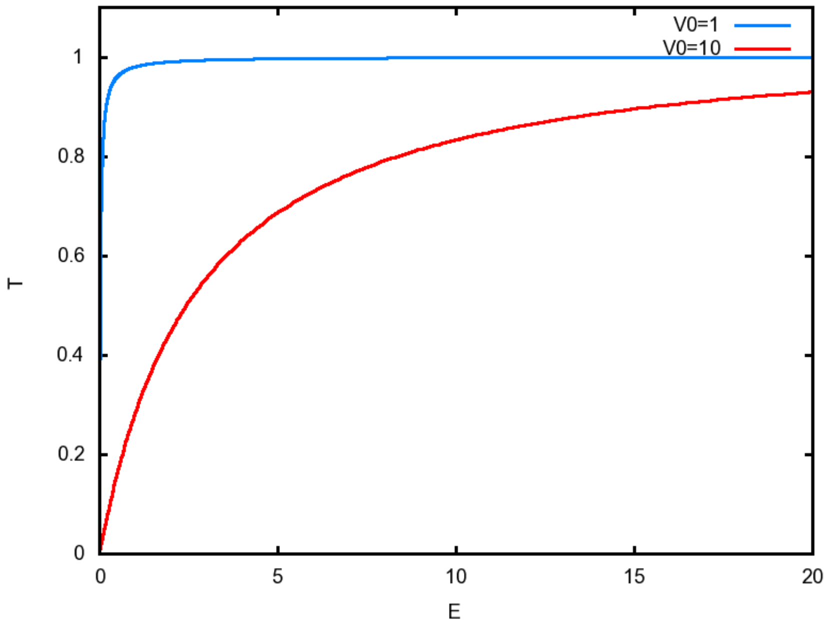

Figure 4: Transmission coefficient $T(E)$ for $m = \hbar = 1,\ a = 0.1$.

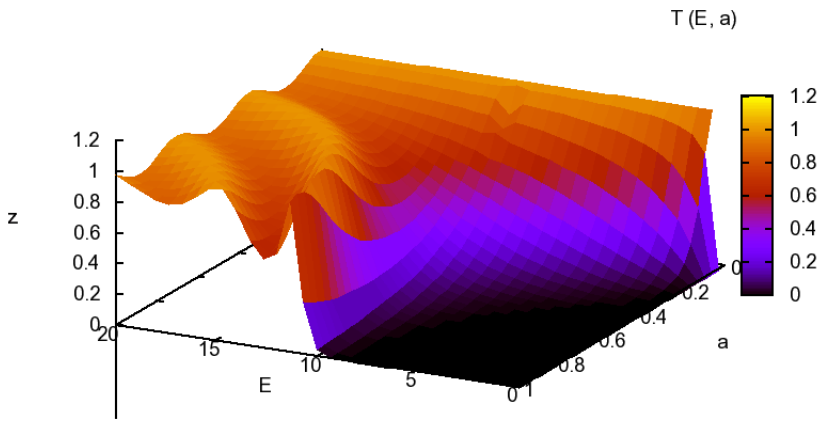

Figure 5: Transmission coefficient $T(E, a)$ for $m = \hbar = 1,\ V_0 = 10$.

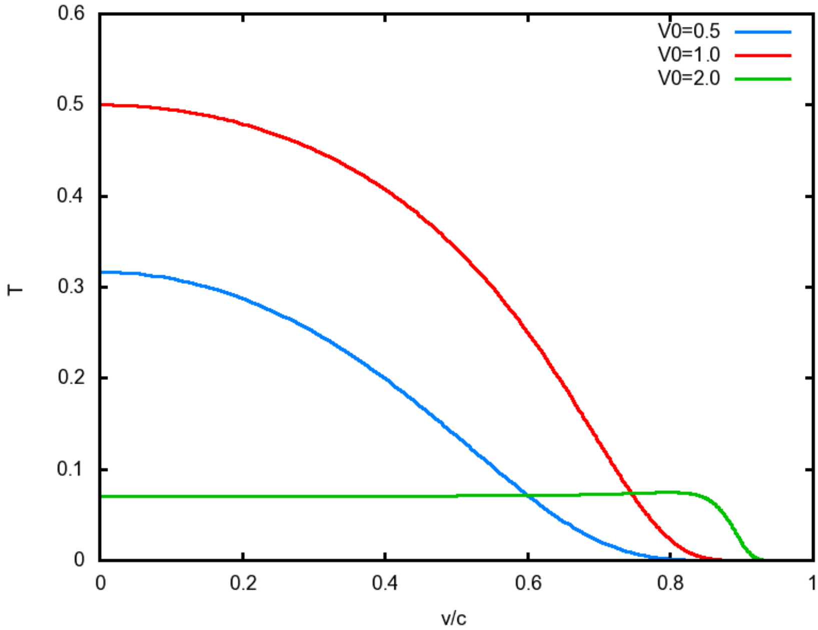

Figure 6: Relativistic transmission coefficient $T(v/c)$ for $c = m = \hbar = 1,\ a = 1$.

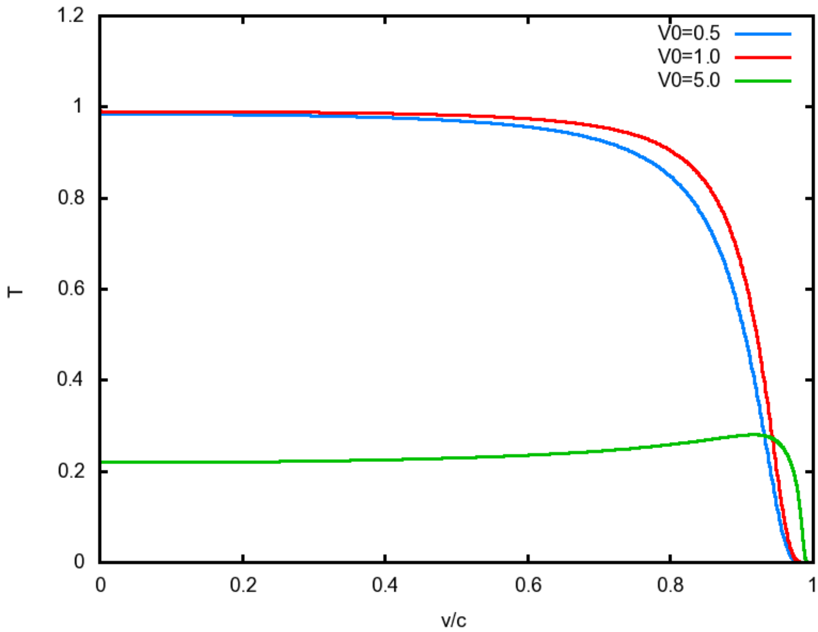

Figure 7: Relativistic transmission coefficient $T(v/c)$ for $c = m = \hbar = 1,\ a = 0.1$.

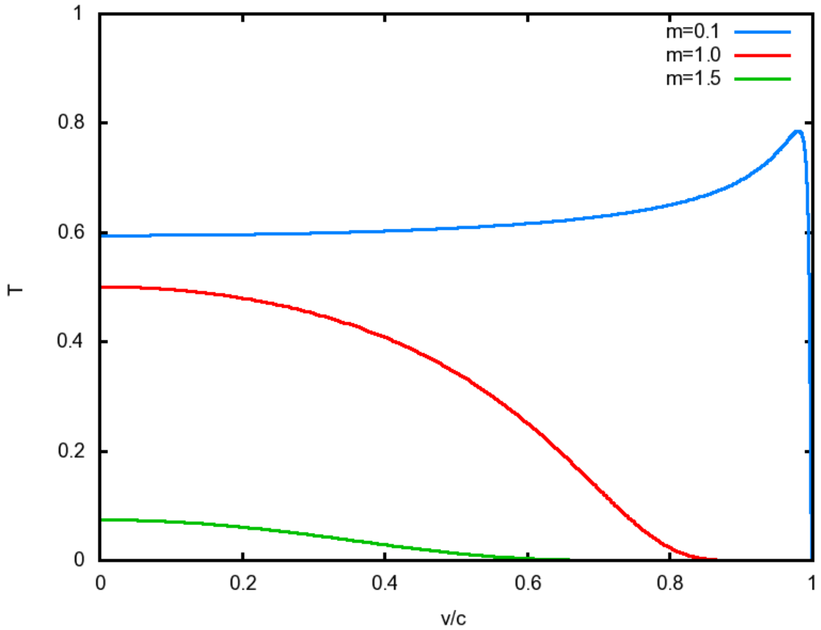

Figure 8: Relativistic transmission coefficient $T(v/c)$ for electron-electron tunneling, electrom mass is $m = 1$.

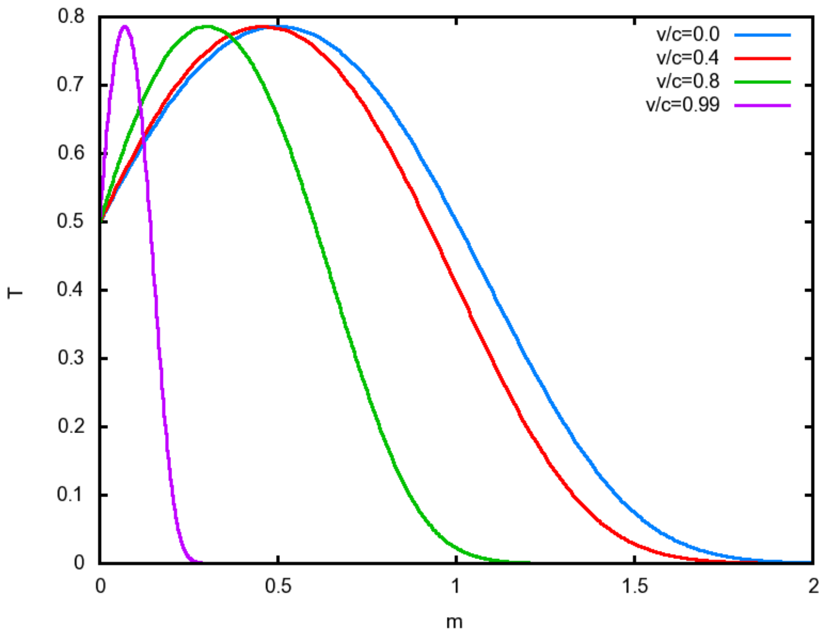

Figure 9: Mass dependence of the realtivistic transmission coefficient $T(m)$ for electron-electron tunneling, electrom mass is $m = 1$.

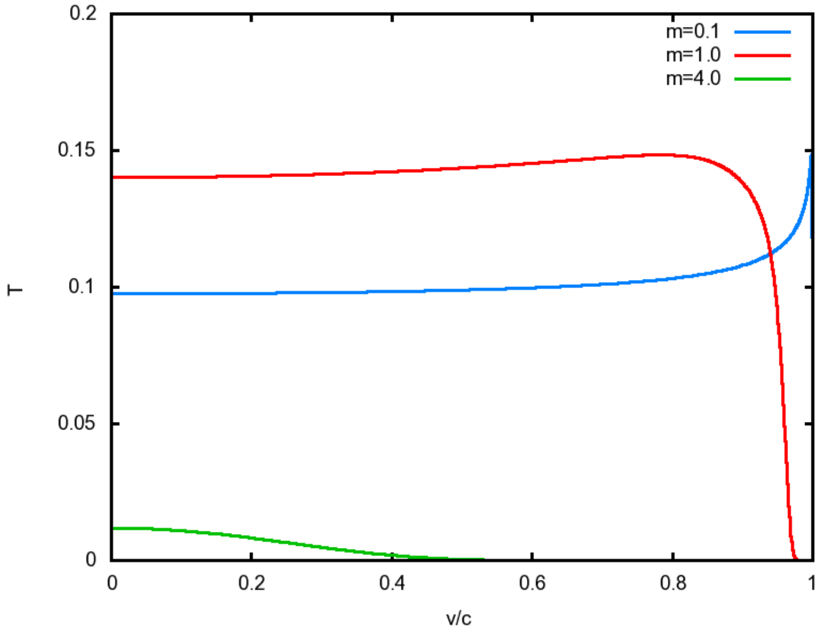

Figure 10: Relativistic transmission coefficient $T(v/c)$ for proton-proton tunneling, proton mass is $m = 1836$.

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