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Conservation Theorem of Einstein Cartan Evans Field Theory

by

Myron W. Evans,
Alpha Institute for Advanced Study, Civil List Scientist.
(emyrone@aol.com and www.aias.us)

Abstract

The conservation theorems of physics are based on the tetrad postulate of differential geometry. It is shown that the tetrad postulate is invariant under the general coordinate transformation and that a frame invariant conservation theorem of physics can be based directly on the invariant tetrad postulate of geometry, as required by the philosophy of relativity. In special cases the conservation theorem reduces to the various conservation laws of physics, notably the conservation of canonical energy/momentum density. The conservation theorem and conservation laws apply to all the equations of physics derivable from ECE field theory, these include the wave equations of physics, also derivable from the tetrad postulate.

Keywords: Tetrad postulate, conservation theorem of ECE theory, conservation laws of physics.

28.1 Introduction

In gauge theory the conservation laws of physics are based on the Noether Theorem, which is derived from the invariance of action under various symmetry operations [1] in a Lagrangian formalism. These laws include the conservation of canonical energy momentum density, the covariant form of which is used in the Einstein field equation. The latter is well known [2–10] to be obsolete because of its neglect of space-time torsion, an essential part of differential geometry, and has been replaced by the well accepted ECE engineering model. The latter is based directly on Cartan's differential geometry both in respect of dynamics and electrodynamics, and the torsion plays a central role in both subjects. The ECE model allows new technologies to be developed using the concept of spin connection resonance (for example ECE papers 63, 92, 94 and 107 on www.aias.us). The physics of ECE theory is shown in Section 28.2 to be based on a general conservation theorem constructed in turn from the tetrad postulate [11] of differential geometry. The tetrad postulate is the most fundamental theorem of differential geometry, and states that the complete vector field is independent of its components and basis elements. In three dimensions for example a complete vector field V is the same if expressed in cartesian or circular polar coordinates. The same is true of a complete vector field in n dimensions. The tetrad postulate is used throughout differential geometry, and all the equations of Cartan geometry depend on it. The postulate may be seen as the link between Cartan and Riemann geometry. In view of its importance a proof of it is given in Section 28.2 with all details. In Section 28.3, it is shown that the tetrad postulate is invariant under the general coordinate transformation, i.e. is frame invariant. This property means that the postulate is the same for an observer moving arbitrarily with respect to another. In Section 28.4 the fundamental conservation theorem of ECE theory is based directly on the tetrad postulate, and is developed to give the conservation laws and wave equations of physics. In ECE theory the latter therefore obey the conservation laws by construction, because in ECE theory the fundamental wave equations are derived from the tetrad postulate by developing the latter into the ECE lemma and wave equation [2–10].

28.2 Proof of the Tetrad Postulate

Consider the complete vector field X in n dimensions and in a space-time with torsion and curvature. Denote the covariant derivative of the compete vector field by DX. In Riemannian geometry this quantity is expressed as [2–11]:

$$DX = D_\mu X^\nu dx^\mu \otimes \partial_\nu \tag{28.1}$$

where X^ν are the components of X, D_μ are the components of D, dx^μ are the basis elements of X, and ∂_ν are the basis elements of X^ν. The covariant derivative is defined as:

$$D_\mu X^\nu = \partial_\mu X^\nu + \Gamma^\nu_{\mu\lambda} X^\lambda \tag{28.2}$$

where $\Gamma^\nu_{\mu\lambda}$ is the connection. Therefore:

$$DX = (\partial_\mu X^\nu + \Gamma^\nu_{\mu\lambda} X^\lambda) dx^\mu \otimes \partial_\nu . \tag{28.3}$$

In Cartan geometry a tangent Minkowski space-time is defined at point P to the base manifold and the covariant derivative is defined in terms of the spin connection $\omega^a_{\mu b}$:

$$D_\mu X^a = \partial_\mu X^a + \omega^a_{\mu b} X^b. \tag{28.4}$$

The quantity DX is the same in Riemannian and Cartan geometry, so:

$$DX = (\partial_\mu X^a + \omega^a_{\mu b} X^b) dx^\mu \otimes \hat{e}_a \tag{28.5}$$

where $\hat{e}_a$ is the basis element of the component X^a. By construction:

$$\hat{e}_a = q^\sigma_a \partial_\sigma \tag{28.6}$$

$$X^a = q^a_\nu X^\nu \tag{28.7}$$

where q^a_ν is the Cartan tetrad [2–11] and where q^σ_a is the inverse tetrad. These are related by:

$$q^\sigma_a q^a_\nu = \delta^\sigma_\nu \tag{28.8}$$

where:

$$\delta^\sigma_\nu = 1, \quad \sigma = \nu, \tag{28.9}$$

$$\delta^\sigma_\nu = 0, \quad \sigma \neq \nu. \tag{28.10}$$

Note carefully that by convention, there is no summation over repeated a indices in Eq. (28.8), the notation of which means that when:

$$\sigma = \nu \tag{28.11}$$

then:

$$q^\sigma_a q^a_\sigma = 1. \tag{28.12}$$

using Eqs. (28.6) and (28.7) in Eq. (28.5):

$$DX = (\partial_\mu (q^a_\nu X^\nu) + \omega^a_{\mu b} q^b_\nu X^\nu) dx^\mu \otimes (q^\sigma_a \partial_\sigma) \tag{28.13}$$

which may be re-expressed as:

$$DX = (q_a^\sigma q_\nu^a \partial_\mu X^\nu + q_a^\sigma X^\nu \partial_\mu q_\nu^a + q_a^\sigma \omega_{\mu b}^a q_\lambda^b X^\lambda) dx^\mu \otimes \partial_\sigma \tag{28.14}$$

Now compare Eqs. (28.3) and (28.14) when:

$$\sigma = \nu. \tag{28.15}$$

In this case, using Eq. (28.12), Eq. (28.14) becomes:

$$DX = (\partial_\mu X^\nu + q_a^\nu \; X^\lambda \; \partial_\mu \; q_\lambda^a + q_a^\nu \; \omega_{\mu b}^a \; q_\lambda^b \; X^\lambda) dx^\mu \otimes \partial_\nu. \tag{28.16}$$

Therefore we obtain:

$$\Gamma_{\mu\lambda}^\nu = q_a^\nu \partial_\mu q_\lambda^a \; + \; q_a^\nu \omega_{\mu b}^a q_\lambda^b. \tag{28.17}$$

Multiply both sides of Eq. (28.17) by q_λ^a to find:

$$\partial_\mu q_\lambda^a + \omega_{\mu b}^a q_\lambda^b = q_\nu^a \Gamma_{\mu\lambda}^\nu \tag{28.18}$$

i.e.:

$$\partial_\mu q_\lambda^a + \omega_{\mu b}^a q_\lambda^b - q_\nu^a \Gamma_{\mu\lambda}^\nu = 0. \tag{28.19}$$

Using the rule [2–11] for the covariant derivative of the tetrad, a rank two mixed-index tensor, Eq. (28.19) is:

$$D_\mu q_\lambda^a = 0 \tag{28.20}$$

which is the tetrad postulate Q.E.D.

28.3 Invariance of the Tetrad Postulate

In this section the tetrad postulate is subjected to a well defined general coordinate transformation [2–11]. The fundamental idea of relativity theory is that the equations of physics retain their format under the general coordinate transformation. They must be generally covariant. In Riemann geometry there are base manifold indices (labelled by Greek subscripts and superscripts), and in Cartan geometry there are additional Latin indices of the tangent space-time. So in Cartan geometry the general coordinate transformation consists in general of transformation matrices with base manifold and tangent indices [2–11]. The transformation matrix in the tangent space-time is the Lorentz transformation matrix $\Lambda_a^{a'}$. In the base manifold there occur transformation matrices such as $\partial x^{\mu}/\partial x^{\mu'}$. Therefore Eq. (28.20) transforms according to the rule for the coordinate transformation of a rank three mixed index tensor:

$$(D_\mu q^a_\nu)' = \left(\Lambda^{a'}_a \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}}\right) D_\mu q^a_\nu = 0 \tag{28.21}$$

It follows that the tetrad postulate is true in any frame of reference, and is an invariant under the general coordinate transformation, Q.E.D.

28.4 Conservation Theorem of ECE Theory

For practical applications the tetrad postulate is developed in the base manifold using Eq. (28.8). Therefore:

$$q^\mu_\nu = \delta^\mu_\nu \tag{28.22}$$

in the base manifold. The contravariant and covariant form of Eq. (28.22) are obtained using the inverse metric and metric respectively, to give:

$$q^{\mu\nu} = g^{\mu\sigma}\delta^\nu_\sigma = g^{\mu\nu}, \tag{28.23}$$

$$q_{\mu\nu} = g_{\mu\nu}. \tag{28.24}$$

The general conditions of metric compatibility [2–11] follow:

$$D^\sigma g_{\mu\nu} = 0, \tag{28.25}$$

$$D_\sigma g^{\mu\nu} = 0, \tag{28.26}$$

and these are forms of the ECE conservation theorem. The fundamental and most general conservation theorem of ECE theory is therefore the tetrad postulate itself.

The conservation of canonical energy momentum density used in the Einstein field equation is derived as a special case of Eq. (28.25) when:

$$\sigma = \mu. \tag{28.27}$$

In general, the canonical energy momentum density is defined in ECE theory as:

$$T^a_\mu = T^{(0)} q^a_\mu \tag{28.28}$$

so using Eq. (28.25):

$$D^{\mu}T_{\mu\nu} = 0. \tag{28.29}$$

This is the covariant law for conservation of energy momentum density which is obtainable from the invariance of action under space-time translation in the Noether Theorem formalism [1]. In ECE theory it is derived straightforwardly from geometry. During the course of development of ECE theory it has been proven rigorously [2–10] that the tetrad postulate may be developed into the ECE Lemma:

$$\Box q^a_{\mu} = R q^a_{\mu} \tag{28.30}$$

where R is a well defined scalar eigen-value with the units of curvature (inverse square metres). Therefore the general law for conservation of canonical energy - momentum density is also a wave equation:

$$\Box T^a_{\mu} = R T^a_{\mu} \tag{28.31}$$

This may be a wave equation of quantum mechanics or statistical mechanics. It follows that all process of quantum mechanics and statistical mechanics in ECE theory automatically obey all the conservation theorems of physics.

The law of conservation of canonical angular energy momentum density follows from the definition [1–10]:

$$J^{\kappa\mu\nu} = -\frac{1}{2}(T^{\kappa\mu}x^{\nu} - T^{\kappa\nu}x^{\mu}) \tag{28.32}$$

which is a rank three tensor density in the base manifold. The space-time torsion tensor in the base manifold [2–11] is also a rank three tensor related to a curvature tensor through the Cartan Evans dual identity:

$$D_{\mu}T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{28.33}$$

By index contraction (summation over internal indices), a rank two curvature tensor may be defined as follows:

$$R^{\kappa\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{28.34}$$

and by hypothesis similar to that of Einstein this tensor is made proportional to the rank two canonical energy momentum density tensor through Einstein's constant k:

$$R^{\kappa\nu} = kT^{\kappa\nu} = D_{\mu}T^{\kappa\mu\nu}. \tag{28.35}$$

By similar hypothesis:

$$T^{\kappa\mu\nu} = kJ^{\kappa\mu\nu} \tag{28.36}$$

so we obtain:

$$D_\mu J^{\kappa\mu\nu} = T^{\kappa\nu}. \tag{28.37}$$

This is a field equation that automatically includes a non-zero torsion as required by the dual identity (28.33) [2–10]. It follows that the law for conservation of canonical angular energy momentum density is:

$$D^\mu T_{\mu\nu} = D^\mu (D^\kappa J_{\mu\kappa\nu}) = 0. \tag{28.38}$$

Similarly the charge current density in general is defined as:

$$J_\mu^a = J^{(0)} q_\mu^a \tag{28.39}$$

and leads to the covariant continuity equation:

$$D^\mu J_{\mu\nu} = 0 \tag{28.40}$$

(see also paper 116 of www.aias.us). Finally the fundamental hypothesis leading to the equations of classical dynamics in ECE tehory [2–10] is:

$$A_\mu^a = A^{(0)} q_\mu^a \tag{28.41}$$

where A_μ^a is the electromagnetic potential field. It follows that the electromagnetic potential in the base manifold is the metric within a factor $A^{(0)}$, where $cA^{(0)}$ is the primordial voltage of ECE theory [2–10]. The electromagnetic field tensor in ECE theory is a rank three tensor proportional to the space-time torsion, so from Cartan's structure equation:

$$F^{\kappa\mu\nu} = \partial^\mu A^{\kappa\mu} - \partial^\nu A^{\kappa\mu} + \omega^{\kappa\mu}{}_\lambda A^{\lambda\nu} - \omega^{\kappa\nu}{}_\lambda A^{\lambda\mu}. \tag{28.42}$$

For example, the electric field in Coulomb's law [2–10] is:

$$\boldsymbol{E} = E^{010}\boldsymbol{i} + E^{020}\boldsymbol{j} + E^{030}\boldsymbol{k} \tag{28.43}$$

in the Cartesian basis. Each electric field component is a component of orbital torsion, and in general:

$$F^{010} = \partial^1 A^{00} - \partial^0 A^{01} + \omega^{01}{}_\lambda A^{\lambda 0} - \omega^{00}{}_\lambda A^{\lambda 1} \tag{28.44}$$

where there is summation over repeated λ indices. Therefore the components of the potential tensor appearing in Eq. (28.44) are components of the inverse metric tensor in a space-time with both curvature an torsion present in general:

$$A^{\mu\nu} = A^{(0)} g^{\mu\nu}. \tag{28.45}$$

In vector format, Eq. (28.44) reduces to:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t} + \phi\boldsymbol{\omega} - \omega\boldsymbol{A}. \tag{28.46}$$

where ω is the spin connection scalar and $\boldsymbol{\omega}$ is the spin connection vector. The Coulomb law itself reduces to:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0 \tag{28.47}$$

and Eqs. (28.46) and (28.47) give rise to spin connection resonance [2–10]. It is seen that this process obeys the conservation theorems of physics. The same is true for the other ECE equations of classical electrodynamics, and for all the ECE equations of classical dynamics. The same is also true for the ECE equations of wave mechanics and statistical mechanics. At spin connection resonance the primordial voltage $cA^{(0)}$ (which fills the vacuum in ECE theory and which is observable in the well known radiative corrections) is greatly amplified, giving rise to the possibility of electric power from space-time. This process obeys all the conservation theorems of physics.

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