GENERAL COVARIANCE AND COORDINATE TRANSFORMATION IN CLASSICAL AND QUANTUM ELECTRODYNAMICS.

by

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ABSTRACT

In a unified field theory classical and quantum electrodynamics must be generally covariant, and not Lorentz covariant as in the contemporary standard model. This means that electrodynamics must be objective under the general coordinate transformation: equivalently the effect of gravitation on electrodynamics must be considered. As an illustration of this general principle the Lorentz force law is derived from a general coordinate transformation of the torsion tensor of standard differential geometry. In the limit of special relativity the general coordinate transformation becomes a Lorentz transformation and the Lorentz force law is recovered in the absence of gravitation.

Keywords: Evans unified field theory, general coordinate transformation, general covariance, Lorentz covariance, Lorentz force law.

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1. INTRODUCTION

In the contemporary standard model neither classical nor quantum electrodynamics is an objective investigation in natural philosophy. In consequence the effect of gravitation on electromagnetism cannot be investigated in the standard model, a major weakness of contemporary physics. Recently an objective or generally covariant unified field theory has been developed {1-20}, a theory which shows how gravitation and electromagnetism may be able to influence each other mutually. In this paper the Evans unified field theory is illustrated through the general coordinate transformation of the torsion tensor in differential geometry {21}. Within a factor $A^{(0)}$, the torsion tensor is the electromagnetic field tensor. In Section 2 the generally covariant form of the Lorentz force law is obtained through a general coordinate transformation of the electromagnetic field tensor. In the limit of special relativity Section 3 shows that Lorentz force law of the standard model is obtained as a well defined limit of the generally covariant, or objective, Lorentz force law of the Evans unified field theory. The correctly objective Lorentz force law shows how gravitation affects the Lorentz force law of the standard model.

2. GENERAL COORDINATE TRANSFORMATION

The vector transformation law of general relativity shows that the vector field:

$$V = V^{\mu} \hat{e}_{(\mu)} \qquad -(1)$$

is invariant under the general coordinate transformation

$$V^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} V^{\mu}, \quad \partial_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \partial_{\mu} \qquad -(2)$$

where

$$\hat{e}_{(\mu)} = \partial_\mu . \qquad -(3)$$

Here V^μ denotes the vector components {21} and $\hat{e}_{(\mu)}$ the set of basis vectors. The vector components x^μ are those of the position four vector, and ∂_μ is the partial derivative four vector. Therefore Eq. (3) defines the coordinate basis. In Eqs. (1) to (3) the primed frame is related to the unprimed frame through the general coordinate transformation. The coordinate basis (3) is used conventionally {21} in gravitational general relativity. This is the Einstein Hilbert variation of general relativity, where the fundamental field is the symmetric metric tensor. The Lorentz transform of special relativity is the special case of Eq. (2) where:

$$V^{\mu'} = \Lambda^{\mu'}{}_{\mu} V^\mu , \quad x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^\mu . \qquad -(4)$$

Here $\Lambda^{\mu'}{}_{\mu}$ is the well known {21} Lorentz transform matrix.

The tensor transformation law of general relativity {21} is

$$T^{\mu_1' \cdots \mu_k'}{}_{\nu_1' \cdots \nu_l'} = \left(\frac{\partial x^{\mu_1'}}{\partial x^{\mu_1}} \cdots \frac{\partial x^{\mu_k'}}{\partial x^{\mu_k}} \right) \left(\frac{\partial x^{\nu_1}}{\partial x^{\nu_1'}} \cdots \frac{\partial x^{\nu_l}}{\partial x^{\nu_l'}} \right) T^{\mu_1 \cdots \mu_k}{}_{\nu_1 \cdots \nu_l} \qquad -(5)$$

in a notation which can be built up from the notation of Eq. (1). An important example of Eq. (5) is the metric transformation law:

$$g_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} g_{\mu\nu} \qquad -(6)$$

Eq (6) is the fundamental axiom of relativity theory {1-21}, a tensor transforms generally and covariantly, producing a new tensor. In Eq. (6) the tensor is the fundamental field, implying that the field is covariant to an observer moving arbitrarily with respect to another observer. In the Evans unified field theory this axiom is applied to all radiated and matter fields {1-20} self consistently using Cartan geometry.

The covariant and exterior derivatives {21} of a vector transform covariantly in relativity theory, whereas the ordinary partial derivative does not. For example, the covariant derivative transforms covariantly as:

$$D_{\mu'} V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} D_{\mu} V^{\nu} \quad -(7)$$

provided that the Christoffel symbol transforms as:

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \Gamma^{\nu}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial^2 x^{\nu'}}{\partial x^{\mu} \partial x^{\lambda}} . \quad -(8)$$

The Christoffel symbol itself does not transform as a tensor, as is well known. As a final example the torsion tensor {1-21} transforms covariantly as a three index tensor:

$$T^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} T^{\lambda}_{\mu\nu} . \quad -(9)$$

In the unified field theory {1-20} the Palatini variation of general relativity is used, a variation in which the fundamental field is the tetrad, a vector-valued one-form of Cartan differential geometry {21}. The general transformation law for forms is:

$$T^{a'\mu'}{}_{b'\nu'} = \Lambda^{a'}{}_{a} \frac{\partial x^{\mu'}}{\partial x^{\mu}} \Lambda_{b'}{}^{b} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a\mu}{}_{b\nu} \quad -(10)$$

where $\Lambda^{a'}{}_{a}$ is a Lorentz transform defined in the tangent spacetime by {21}:

$$\eta_{a'b'} = \Lambda_{a'}{}^{a} \Lambda_{b'}{}^{b} \eta_{ab} \quad -(11)$$

Here $\Lambda_{a'}{}^{a}$ and $\Lambda_{b'}{}^{b}$ are inverse Lorentz transforms. From Eq. (10) a vector valued one-form $X^{a}{}_{\mu}$ transforms as:

$$X^{a'}{}_{\mu'} = \Lambda^{a'}{}_{a} \frac{\partial x^{\mu}}{\partial x^{\mu'}} X^{a}{}_{\mu} \quad -(12)$$

where the Lorentz transform $\Lambda^{a'}{}_{a}$ in the tangent spacetime is defined by {21}:

$$x^{a'} = \Lambda^{a'}{}_{a} x^{a}. \quad -(13)$$

If μ is fixed then:

$$A^{a'}{}_{\mu} = \Lambda^{a'}{}_{a} A^{a}{}_{\mu} \quad -(14)$$

and if a is fixed:

$$A^{a}{}_{\mu'} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}} \right) A^{a}{}_{\mu}. \quad -(15)$$

The torsion form in Cartan differential geometry is a vector valued two-form defined by:

$$T^{a}{}_{\mu\nu} = \partial_{\mu} q^{a}{}_{\nu} - \partial_{\nu} q^{a}{}_{\mu} + \omega^{a}{}_{\mu b} q^{b}{}_{\nu} - \omega^{a}{}_{\nu b} q^{b}{}_{\mu} \quad -(16)$$

where q^a_μ is the tetrad and where $\omega^a_{\mu b}$ is the spin connection. The torsion form transforms as a tensor:

$$T^{a'}_{\mu'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} T^a_{\mu\nu}. \quad -(17)$$

In the unified field theory {1-20} the electromagnetic field tensor is also a vector valued two-form defined by:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad -(18)$$

where $A^{(0)}$ is the vector potential magnitude. The generally covariant Lorentz force law is therefore expressed most generally as:

$$F^{a'}_{\mu'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} F^a_{\mu\nu}. \quad -(19)$$

The mutual effect of gravitation and electromagnetism within this law is contained within Eq. (19). For fixed a:

$$F^a_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} F^a_{\mu\nu} \quad -(20)$$

and so for fixed a the generally covariant Lorentz force law is described by the metric transformation law (6). In order to calculate the effect of gravitation on the Lorentz force law we need know only the metric transformation law for a given metric, defined by:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab}. \quad -(21)$$

Here η_{ab} is the Minkowski metric {21} of the tangent spacetime.

3. SPECIAL RELATIVISTIC LIMIT.

In the special relativistic limit of Eq. (20) we obtain the Lorentz transformation:

$$F^a_{\mu'\nu'} = \Lambda^\mu_{\ \mu'}\Lambda^\nu_{\ \nu'}F^a_{\mu\nu} \quad -(22)$$

for each index a. The latter is a polarization index. Since a appears on both sides of Eq. (22) it may be omitted for ease of notation. The conventional Lorentz transform of the electromagnetic field {22} is therefore obtained:

$$F_{\mu'\nu'} = \Lambda^\mu_{\ \mu'}\Lambda^\nu_{\ \nu'}F_{\mu\nu}. \quad -(23)$$

In vector notation it is well known {22} that Eq. (23) is:

$$\underline{E}' = \gamma\left(\underline{E} + \frac{\underline{v}}{c}\times\underline{B}\right) + \cdots \quad -(24)$$

$$\underline{B}' = \gamma\left(\underline{B} - \frac{\underline{v}}{c}\times\underline{E}\right) + \cdots \quad -(25)$$

where $\underline{E}$ denotes electric field strength in volt m^{-1} and $\underline{B}$ is magnetic flux density. Here

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(26)$$

originates in the Lorentz transform matrix of special relativity. The Lorentz force law in S.I. units is:

$$\frac{d\underline{p}}{dt} = e\left(\underline{E} + \frac{\underline{v}}{c}\times\underline{B}\right) - (27)$$

where $\underline{p}$ is linear momentum and e is electric charge, and the Lorentz force law holds at non-relativistic velocities, where

$$\gamma \sim 1 \quad -(28)$$

From Eq. (23) the magnetic induction due to the Lorentz transformation at non-relativistic velocities is {22}:

$$\underline{B} = \frac{e}{c} \frac{\underline{v} \times \underline{r}}{r^3} \quad -(29)$$

which is the Ampère Biot Savart law. It is seen that the Lorentz force law is built up from a sum of $\underline{E}$ and $\frac{\underline{v}}{c} \times \underline{B}$ in Eq. (24) in the non-relativistic limit.

The correct laws of electrodynamics are therefore obtained from the Evans unified field theory and from the generally covariant transformation (19) of the electromagnetic field tensor. The effect of gravitation on these well known laws of electrodynamics may therefore be calculated for a given metric.

Finally, in quantum electrodynamics {1-20} the tetrad is the fundamental field and the tetrad transforms according to Eq. (12).

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