1)

GENERALLY COVARIANT QUANTUM MECHANICS

Generally covariant quantum mechanics does not exist in the standard model, but is required by the experimental precision both of general relativity and of quantum mechanics. In order to construct such a theory the Heisenberg uncertainty principle must be made generally covariant. The standard uncertainty principle is illustrated by:

$$[x, p_x]\phi = i\hbar\phi \quad -(1)$$

where:

$$p_x = -i\hbar\frac{\partial}{\partial x}. \quad -(2)$$

Eqn (1) is simply a restatement of the operator equivalence (2). Eqn (1) is deduced as follows:

$$\begin{aligned}
[x, p_x]\phi &= (xp_x - p_x x)\phi \\
&= x(p_x\phi) - p_x(x\phi) \\
&= x(p_x\phi) - (p_x x)\phi - x(p_x\phi) \\
&= -(p_x x)\phi \\
&= \left(i\hbar\frac{\partial x}{\partial x}\right)\phi \\
&= i\hbar\phi. \quad -(3)
\end{aligned}$$

Standard arguments then show that:

$$\delta x \, \delta p_x \geq \frac{\hbar}{2} \quad — (4)$$

and eqn (4) is the standard expression of the Heisenberg uncertainty principle. Eqn. (1) is the Heisenberg equation of motion. It is seen that eqns (1) and (4) are direct mathematical consequences of eqn. (2).

Recent experimental results of Croca et al., using advanced microscopy, show that for moderate resolution:

$$\delta x \, \delta p_x \sim 10^{-9} \frac{\hbar}{2} \quad — (5)$$

therefore the Heisenberg uncertainty principle (4) is violated experimentally by at least nine orders of magnitude. At higher resolution the experimental results show that:

$$\delta x \, \delta p_x \rightarrow 0, \quad — (6)$$

in complete contradiction to the standard model.

3) These results produce a crisis in the standard model and lead to the abandonment of the Heisenberg Bohr complementarity principle.

The error in the Copenhagen School's philosophy is traced in these notes to the fact that the fundamental operator equivalence (2) is not generally covariant. In order to make it generally covariant the momentum p_x has to be replaced by a momentum density and the angular momentum $\hbar$ by an angular momentum density. The reason is that the fundamental law of general relativity is:

$$R = -kT \quad -(7)$$

where T is a canonical energy-momentum density. In the rest frame T reduces to a mass density:

$$T \rightarrow \frac{m}{V_0} \quad -(8)$$

and within a factor c^2 this is the rest energy density. Here V_0 is the Evans rest volume:

$$V_0 = \frac{\hbar^2 k}{mc^2} \quad -(9)$$

where m is the elementary particle mass.

4) Define the experimental momentum density $\overline{p_x}$ by:

$$\overline{p_x} = \frac{p_x}{V} \quad -(10)$$

and the fundamental angular momentum density $\overline{\hbar}$ by:

$$\overline{\hbar} = \frac{\hbar}{V_0}. \quad -(11)$$

Here V is the volume of the apparatus, or the volume occupied by the momentum p_x. The $\overline{\hbar}$ is the density of the reduced Planck constant. Eq. (11) means that the quantum of action $\hbar$ occupies a volume V_0, the Evans rest volume. This deduction follows from the equivalence principle of the Evans wave equation:

$$kT \rightarrow \left(\frac{mc}{\hbar}\right)^2 = \frac{km}{V_0} \quad -(12)$$

in the limit of special relativity. The quantum $\hbar$ is defined by the volume V_0.

In general relativity therefore eqn. (2) becomes:

$$\overline{p_x} = -i\overline{\hbar}\frac{\partial}{\partial x} \quad -(13)$$

i.e.

$$\boxed{\overline{p_x}\psi = -i\overline{\hbar}\frac{\partial\psi}{\partial x}} \quad -(14)$$

5)

Eqn. (2), which works very precisely in quantum mechanics, is therefore the same as:

$$\bar{P}_x = -i \left(\frac{\bar{V}_0}{\bar{V}} \right) \bar{\hbar} \frac{\partial}{\partial x} \quad -(15)$$

which is a special case of:

$$\bar{p}^\mu = i \left(\frac{\bar{V}_0}{\bar{V}} \right) \bar{\hbar} \partial^\mu \quad -(16)$$

The Heisenberg equation in generally covariant form is therefore:

$$\boxed{[x, \bar{P}_x] = i \left(\frac{\bar{V}_0}{\bar{V}} \right) \bar{\hbar}} \quad -(17)$$

and the fundamental conjugate variables are x and $\bar{P}_x$. The fundamental quantum is therefore $\bar{\hbar}$, and not $\hbar$.

Experimentally, for a macroscopic volume $\bar{V}$:

$$\bar{V}_0 << \bar{V} \quad -(18)$$

and so:

$$[x, \bar{P}_x] \sim 0 \quad -(19)$$

$$\Rightarrow \quad \delta x = 0, \quad \delta \bar{P}_x = 0 \quad -(20)$$

is quite possible experimentally.

This means that a particle and matter wave co-exist experimentally - the point of view of de Broglie and Einstein. For electromagnetism this coexistence has been observed by Afshar at Harvard and elsewhere in quite simple experiments (New Scientist, 2004).

The fundamental conjugate variables are therefore position and momentum density, or time and energy density. The wave-function is always the tetrad, and this is always governed by the causal and objective Evans wave equation:

$$(\square + kT)\, q^a_\mu = 0. \quad -(21)$$

Eqn (21) is the fundamental wave equation of generally covariant quantum mechanics.